ARITHMETICK

Both in the

Theory *and* Practice,

Made Plain and Easie,

In all the Common and Useful Rules, both in *Whole Numbers*, and *Fractions*, VULGAR and DECIMAL.

ALSO

Interest {Simple and Compound,} and *Annuities*.

Likewise

Extraction of the *Square* and *Cube-Roots*;

AS ALSO

The TABLES and Construction of LOGARITHMS, with their Use in ARITHMETICK, and COMPOUND INTEREST:

Together with

Arithmetical and *Geometrical Progression*; and the *Combination* and *Election*, *Permutation* and *Composition* of NUMBERS and QUANTITIES.

The like not Extant.

By *JOHN HILL* Gent.
Recommended by Mr. HUMPHREY DITTON.

The Second Edition; With the Addition of several ALGEBRAICAL QUESTIONS.

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m P.

TO THE READER.

IT being a sort of Disparagement, to Things which are evidently very Good, to say much in Commendation of them; I shall therefore, in Justice to this Book, give but a very short Account of it.

It appears to me, upon the Perusal of it, to be a curious Piece: 'Tis CLEAN, METHODICAL, *and handsomely Dress'd: So* PLAIN, *that the dullest Person may learn by it; and so* COMPLEAT, *that he need learn no more.*

The Author (whoever he was) has in this Treatise, gone much beyond the Bounds, which the common Writers of this SCIENCE *use to advance to.*

And tho' many of the Practices *he delivers,* Ex. gr. *the making of* LOGARITHMS, INTEREST, and COMBINATION of QUANTITIES, *are*

to be done with greater Advantage and Exactness, by the Help of Superior Methods, *as* ALGEBRA, *&c. yet take him purely* as an Arithmetician, *and he has not only done more, and much better than* WINGATE, COCKER, LEYBOURN, *or any other of the Writers in our Tongue; but indeed all that can be done by Arithmetick: And therefore if no other Book on this Subject comes out, till this Performance* is really mended; *I am satisfy'd, we shall have no new Book of Arithmetick very soon.*

Christ's Hospital,
Novemb. 28. 1712.

H. DITTON.

ARITHMETICK,

Both in the

Theory and *Practice.*

The INTRODUCTION.

SECTION I.

Containing the General Præcognita.

1. ARITHMETICK is an Art or Science that Teacheth us the dextrous Handling of Numbers, and contains three Branches.

Viz. VULGAR, LOGARITHMICAL, ANALYTICAL.

2. For the well Managing of which, the *Arabians*, as may be supposed by their way of Reading, Invented the following Symbols or Characters, commonly call'd Digits, (as may be reasonably guess'd from the Fingers of the Hand) which tho' but few in Number, are sufficient for managing of the vastest Calculations.

See here their Names and Characters.

1	2	3	4	5	6	7	8	9	0
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Null

B 3. The

3. The Cypher, Null, standing by it self signifieth Nothing; but being joyned with other Numbers, increaseth or decreaseth their Value, and is indeed the beginning of all Number, as we shall elsewhere Demonstrate, contrary to what *Tacquet*, and some other modern Artists affirm.

4. Number is composed of a Multitude of Units, and is that by which we say any Thing is Numbred; as 1 Acre, 4 Crowns, 7 Days in a Week.

5. Of Numbers are several Sorts; as Digits, Articles, Compounds, Whole, Broken, Mix'd, *&c.*

6. Digits are such Numbers as are under Ten, as, 2, 3, 4, 5, *&c.*

7. Articles are such Numbers as are Composed of a Digit and a Cypher, as 10, 20, 30, *&c.*

8. Compound Numbers are such as are compounded of many Numbers, as 144, 282, 1728, 1702, *&c.*

9. A whole Number either contains Unity, or some Number thereof; as 7, 21, 512, 2056, *&c.*

10. A Fraction, or broken Number, is always less than Unity, as $\frac{3}{4}$ represents three Quarters of any Thing, or Unity; and $\frac{6}{10}$, or .6, is six tenth Parts of Unity, *&c.*

11. A mixt Number is always greater than Unity, as $2\frac{1}{2}$ represents 2 Integers, and one half of an Integer or Unity; and $7\frac{75}{100}$, or 7.75, betokens 7 Integers, and 75 Hundred Parts of an Integer or Unity.

12. According to the Division of Unity, a Fraction comes to be stiled Vulgar or Decimal.

13. A Vulgar Fraction is divided into two Parts, one above another, with a small Line drawn betwixt, of which the lower is call'd the Denominator, and the higher the Numerator, shewing how many of those Parts are signify'd by the Fraction. So if we divide Unity into 12 Parts, 5 of those Parts will be express'd thus $\frac{5}{12}$ $\frac{\text{Numerator}}{\text{Denominator}}$, and 7 Parts thus, $\frac{7}{12}$, and so of others.

14. A Decimal Fraction (which is the most Genuine and Natural Way of dividing Unity, and perhaps the most Ancient) always supposes the Integer to be divided into 10, 100, 1000 Parts, *&c.* as you covet Preciseness in your Operation. Hence the Denominator being known, needs not to be express'd, but you may place your Fraction as an Integer, by taking Care to prefix its distinguishing Point, or Comma; so $\frac{5}{10}$ will be express'd thus, .5; and $\frac{05}{100}$ thus, .05; $\frac{5}{1000}$ thus, .005; $\frac{75}{100}$ thus, .75, *&c.*

15. Num-

15. Numbers are said again to be Equal, Unequal, Even, Odd, Evenly even, Evenly odd, Oddly odd, Composite, Prime, Plain, Solid, Perfect, Harmonick, Square, Cube, &c.

16. Equal Numbers are such as contain an equal Number of Units.

17. Unequal Numbers are such, whose Number of Units differ.

18. An even Number is such as may be divided into 2 equal Parts.

19. An odd Number is such as cannot be so divided.

20. A Number evenly even, is such as is composed of two even Numbers, such a one is 8, for $2 \times 4 = 8$.

21. A Number evenly odd, is such as is Composed of an even and an odd Number; such is 18, Composed of 6 and 3, for $6 \times 3 = 18$; or of 9 and 2, for $9 \times 2 = 18$.

22. A Number oddly odd, is such as is composed of 2 odd Numbers; such is 21, Composed of 7 and 3, for $7 \times 3 = 21$.

23. Some Numbers are both evenly even, and evenly odd, as 24 Composed of 6 and 4, for $6 \times 4 = 24$, and so is evenly even; and it may be Composed of 8 and 3, for $8 \times 3 = 24$, and so is evenly odd.

24. Composite Numbers are such as are Measured by some other Number than Unity, such are 8, 12, 15, 25, &c.

25. Prime or Incomposite Numbers, are such as Unity only Measureth; such are 3, 5, 7, 11, 17, 19, &c.

26. Plane Numbers are such as are made by the Multiplication of two Numbers, as 12, 18, 36; the first made up of 6 and 2, and the second of 6 and 3, and the third of 9 and 4.

27. Solid Numbers are such as are made by the Multiplication of three Numbers into one another; such are 24, made by the Multiplication of 2 into 3, into 4; and 60, made of 3 into 4, into 5; whence you may infer, that all Plane and Solid Numbers are Composite.

28. Perfect Numbers are such, whose Aliquot Parts added, are equal to themselves; the first of which is 6, whose Aliquot Parts are $3, 2, 1 = 6$: The Second is 28, whose Aliquot Parts are $14, 7, 4, 2, 1 = 28$; of these Numbers are but few, only Nine in a Hundred Thousand Million.

29. Harmonick Numbers are such, that the Aliquot Parts of the one Collected, make a Sum equal to the other Number.

30. Square Numbers are such as are made by the Multiplication of some Number into it self; so 4 is the Square of 2, 9 of 3, 16 of 4, and so on *ad infinitum*.

31. Cube Numbers are made by the Multiplication of some Number twice into it self, such a one is 8, made by the Multiplication of 2 into 2 into 2, such another is 27, and infinite more.

32. Numbers to one another may be termed Aliquot Parts, Aliquant Parts, Prime, Composite.

33. One Number is said to be an Aliquot Part to another, when the first precisely Measures the second; so 6 is an Aliquot Part of 18, and 7 of 28; for 6 Measures 18 by 3, and 7 Measures 28 by 4; &c.

34. One Number is said to be an Aliquant Part of another Number, when the first Measures not the second without a Remainder; so 5 is an Aliquant Part of 18, and 9 of 25, &c.

35. One Number is said to be Prime to another, when no Number can be found to Measure both precisely, excepting Unity; so 11 and 15 are Prime to one another, so are 13 and 36, and many more.

36. One Number is Composite to another, when a Number can be found that Measures both exactly besides Unity, such are 12 and 36; 15 and 75; since 3 Measures the first Pair, and 5 the second; and so in many more.

37. Numbers to one another may be said to have Reason or Habitude, and may be twofold, either in respect of Quantity or Quality.

38. In respect of Quantity, it is consider'd only betwixt two Numbers, of which the first is called the Antecedent, the second the Consequent, and is either equal, as 3 to 3, or 7 to 7; or Unequal, which may be of the Greater to the Less, as 6 to 4, or of the Less to the Greater, as 4 to 6.

39. Reason, as well of the Greater to the Less, as of the Less to the Greater, is fivefold, *viz.* First *Multiple*, Secondly *Superparticular*, Thirdly *Superpartiens*, Fourthly *Multiplesuperparticular*, Fifthly and lastly, *Multiplesuperpartiens*. The three first of which are called Simple, the two last Mixt, Reason or Habitude; to give a Name to their Op-

posites

jective or Contraries, we joyn the Preposition *Sub*, then they are called *Submultiple*, *Subsuperparticular*, &c.

40. First, *Multiple* Reason, is when the Antecedent or Greater Number contains the Consequent or less Number, some certain Number of times, without a Remainder, as 6 to 3, commonly called *Duple*, 21 to 7, commonly called *Triple* Reason; their Opposites is of the Less to the Greater, as 3 to 6, 7 to 21, that is *Sub-duple*, *Sub-triple* Reason.

41. Secondly, *Superparticular* Reason, is when the Antecedent or Greater Number contains the Consequent or Less Number, but once with a Fraction, whose Numerator is always Unity; such are 3 to 2, 4 to 3, 5 to 4, &c. Commonly called, *Sesquialtera*, *Sesquitertia*, *Sesquiquarta*, Reason or Proportion; Its Opposite is, *Subsuperparticular*, as of 2 to 3, 3 to 4, 4 to 5, &c. Commonly called *Subsesquialtera*, *Subsesquitertia*, *Subsesquiquarta*, &c.

42. Thirdly, *Superpartient* Reason is, when the Antecedent or Greater Number contains the Consequent or Less Number, once with a Fraction, whose Numerator is always more than Unity, such as 5 to 3, 7 to 4, &c. commonly called *Superdupartiens tres*, and *Supertripartiens quartas*, &c. Its Opposite is, *Subsuperpartiens*, as of 3 to 5, 4 to 7; or *Subsuperdupartiens tres*, *Subsupertripartiens quartas*, &c.

43. Fourthly, *Multiplesuperparticular* Reason, is when the Antecedent or Greater Number contains the Consequent or Less Number, divers times with a Fraction, whose Numerator is always Unity; such as 9 to 4, or *Duplasesquiquarta*, 9 to 2, or *Quadruplasesquialtera*, 26 to 5, or *Quintuplasesquiquinta*, &c. Its Opposite is, *Submultiplasuperparticular*, as 4 to 9, 2 to 9, 5 to 26, &c.

44. Fifthly, *Multiplesuperpartiens* Reason, is when the Antecedent or Greater Number contains the Consequent or Less Number divers times with a Fraction; whose Numerator is always greater than Unity; as 8 to 3, commonly called, *Duplasuperdupartiens tertia*; 19 to 5, termed *Triplasuperquadripartiens quinta*, &c. Its Opposite is, *Submultiplasuperpartiens*, as 3 to 8, 5 to 19, &c. Under some of these five Species are comprehended all the Variety that can happen betwixt two Numbers, in respect of Quantity; the same holds also in Fractions, as well as mix'd Numbers.

45. In respect of Quality; which is only a Similitude of Reasons, commonly called Proportion; it is considered betwixt more than two Numbers: For tho' the Reason of two Numbers may be had as before; yet a Similitude of Reasons cannot be found, unless the Number be more than two, and is Threefold. First, In respect of their *Differences*: Secondly, In respect of their *Quote*. Thirdly, In respect of both. Of the first, springeth Arithmetical; of the second, Geometrical; of the third, Harmonical Proportion.

46. Arithmetical Proportion, is an Equality of Differences; that is to say, When a Rank of Numbers have one and the same Difference, and this is two-fold; Continued, or Discontinued.

47. First, Continued; when of several, the 2d exceeds or is less than the first, by the same Numbers of Units, as the 3d doth exceed or is less than the 2d; or as the 4th doth exceed or is less than the 3d, *&c.* So 1, 3, 5, 7, 9, 11, *&c.* are Numbers in Arithmetical Proportion, increasing by 2. And 16, 13, 10, are Numbers in Arithmetical Proportion decreasing by 3. And 1, 2, 3, 4, 5, 6, 7, are Numbers in Arithmetical Proportion, Continued, increasing by Unity; and these are what is commonly called, *Arithmetical Progression.*

48. Secondly, Discontinued, that is, when there is the same difference betwixt the first and 2d, as there is betwixt the 3d and 4th, but not as between the 2d and 3d. So 1, 3 ∴ 7, 9 are four Numbers in Arithmetical Proportion. The Difference of 1 and 3, and of 7 and 9, being 2; which is not the Difference of 3 and 7, which is 4.

49. Geometrical Proportion, is an Equality of *Ratio's*; that is to say, when several Numbers being divided by one another have several Quotients, and is either Continued, or Discontinued.

50. Continued, when of several Numbers, the 1st bears the same *Ratio* or Proportion to the 2d, as the 2d doth to the 3d, and as the 3d doth to the 4th, *&c.* Thus 2, 3, 4, 6, are Geometrical Proportionals, Continued; since there is the same Reason of 2 to 3, as of 4 to 6; each being *Subsesquialtera.* 1, 2, 4, 8, 16, 32, *&c.* are Numbers in Geometrical Proportion, for the same Reason; and this is what is commonly called, *Geometrical Progression.*

51. Second, Discontinued or Interrupted, when the Proportion of the 1st to the 2d, is the same as that of the 3d to the 4th, but not of the 2d to the 3d. Thus, 3 : 6 :: 16 : 32, are Geometrical Proportions, Discontinued; 3 being contained in 6, as oft as 16 in 32, that is twice, which is not the Proportion of 6 to 16; and this is what is commonly called, *The Golden Rule.*

52. Harmonick or Musical Proportion, is when the 1st Term is to the last, as the Difference of the 1st and 2d to the Difference of the two last. So these three Numbers, 2, 3, 6, are in Musical Proportion, since 2 is to 6, as 1 the Difference of the two first, to 3 the Difference of the two last. Thus also these 4 Numbers are in Harmonical Proportion, *viz.* 2, 3, 6, 12. Since the first is to the last, as the Difference of the two first, to the Difference of the two last.

SECT. II.

The Division of a Pound Sterling.

4 Farthings	Make	1 Penny
12 Pence, or 3 Groats	Make	1 Shilling
5 Shillings	Make	1 Crown
4 Crowns, or 20 Shillings	Make	1 Pound

20 Groats, or 6 *s.* and 8 *d.*	Make	1 Noble
2 Nobles	Make	1 Mark
3 Marks	Make	2 Pounds
1 Mark 1 Noble	Make	1 Pound
240 Pence	Make	1 Pound
960 Farthings	Make	1 Pound

The Division of a Pound Troy.

24 Grains	Make	1 Penny Wt.
20 Penny Weight		1 Ounce
12 Ounces		1 Pound Troy
14 Oun. 12 Penny Wt.		1 Pound Aver.

240 Peny Wt. make a Pound. So will 5760 Grains.

Averdupoise Weight.

16 Drams	Make	1 Ounce
16 Ounces		1 Pound
14 Pound		1 Stone
2 Stone, or 28 Pound		$\frac{1}{4}$ Hundred
4 Stone, or 56 Pound		$\frac{1}{2}$ Hundred
8 Stone, or 112 Pound		1 Hundred
5 Hundred		1 Hogshead
10 Hundred		1 Pipe or Butt
20 Hundred		1 Tunn or Load

256 Drams make a lb. 1792 Ounc. make a C. Wt. 28672 Dr. make a C. Wt. 3184 Gr. make a Stone, so will 224 Ounces.

Apothecaries Weight.

24 Grains of Wheat	Make	1 Scruple
3 Scruples		1 Dram
8 Drams		1 Ounce
12 Ounces		1 Pound

96 Drams in a Pound, 288 Scruples in a Pound; 6912 Grains in a Pound.

Long Measure.

3 Barley Corns	Make	1 Inch	190080 Bar. Cor. make a Mile, 63360 Inches make a Mile, 5280 Feet make a Mile, 1760 Yards make a Mile, 1056 Paces make a Mile, 320 Perch. make a Mile.
4 Inches.		1 Palm	
12 Inches, or 3 Palms		1 Foot	
3 Feet		1 Yard	
3 Feet 9 Inches		1 Ell	
5 Feet		1 Geomet. Pace	
6 Feet, or 2 Yards		1 Fathom	
5 ½ Yards		1 Perch	
40 Perch or 132 Paces		1 Furlong	
8 Furl. or 320 Perches		1 Mile	
3 Miles		1 League	

Cloath Measure.

4 Nails	Make	1 Quarter	16 Nails one Yard.
4 Quarter		1 Yard	20 Nails one Ell *English*.
5 Quarter		1 Ell *English*	12 Nails one Ell *Flemish*.
3 Quarter		1 Ell *Flemish*	

Dry Measure.

2 Pints or Pounds	Make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Peck		1 Bushel Land Measure
5 Peck		1 Bushel Water Measure
4 Bushels		1 Coomb
2 Coombs		1 Quarter
4 Quarters		1 Chalder
5 Quarters		1 Tunn, or Wey

Liquid Measure.

2 Pints	Make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of Ale, Soap, or Herrings
9 Gallons		1 Firkin of Beer
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel, or 36 Gallons
42 Gallons		1 Tierce
63 Gallons		1 Hogshead
2 Hogsheads		1 Pipe or Butt
2 Butts, or 250 Gallons		1 Tunn

TIME.

60 Seconds	Make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day Natural
4 Days		1 Week
4 Weeks		1 Month
13 Months and one Day		1 Year, or 365 Days

Sometimes a Fraction is expressed Decimally; and in this Case an Unit is supposed to be divided into 10 Parts, and every one of those 10 Parts, into other 10 Parts, whereby Unity is divided into 100 Parts. Again, every of those Parts are supposed to be divided into other 10 Parts, and then Unity is divided into 1000 Parts; and so as far as you please.

In any Decimal Fraction, the Denominator is not express'd, but understood; and the Numerator hath a Point, or Comma prefixed, to distinguish it from an Integer.

So if a Pound be divided into 10 Parts, 10 Shillings, or $\frac{1}{2}$, will be thus express'd, $\frac{5}{10}$, but more accurately thus, ,5.

Again, If a Pound Sterling be divided into a 100 Parts, 5 Shillings, or $\frac{1}{4}$ of a Pound, will be express'd thus, $\frac{25}{100}$, or ,25.

Thus

Thus you see the Denominator of a Decimal Fraction may very well be omitted, because easily known, being always an Unit with as many Cyphers annex'd, as there are Places in the Numerator.

Note also, That Cyphers placed to the Left-hand of an Integral Number, or to the Right-hand of a Decimal, neither increase nor decrease the Value; but placed contrary, work contrary Effects; for as Cyphers placed to the Right-hand of an Integer, increase the Value in a Tenfold Proportion; so Cyphers placed to the Left-hand of a Decimal Fraction, decrease the Value in the same Proportion.

So 5 Pound, by annexing a Cypher to the Right-hand, becomes 50 Pound, Ten times more than before: So .5 *l*, or 10 *s*, by annexing a Cypher to the Left-hand, becomes .05, or 1 Shilling, Ten times less than before. But more of this in Decimal Arithmetick.

NUMERATION.

BY *Numeration* we know how to Place, or give a juſt Value to any Number propounded; which that you may do, obſerve the Nature of the following Table.

The TABLE.

Hundreds of Millions	Tens of Millions	Millions	Hundreds of Th.	Tens of Thouſ.	Thouſands	Hundreds	Tens	Units
								1
							2	1
						3	2	1
					4	3	2	1
				5	4	3	2	1
			6	5	4	3	2	1
		7	6	5	4	3	2	1
	8	7	6	5	4	3	2	1
9	8	7	6	5	4	3	2	1

In the foregoing Table you may ſee how each Place exceeds the former Ten times, increaſing in Value towards the Left-hand.

The firſt Place is the Place of Units, the ſecond Tens, the third Hundreds, the fourth Thouſands; and ſo on as far as you pleaſe.

The Denomination of the firſt Period, or of the firſt 3 Places, is Hundreds; the Denomination of the ſecond Period, or of the ſecond 3 Places, is Thouſands, of the third is Millions.

The fourth Period, if it had been annex'd, would have been Thouſands of Millions; but in Practice we have ſeldom Uſe for ſo large a Number.

In reading the Numbers, it is convenient the young Learner exercise himself in the smallest first, and so proceed to the greater, till he be perfect.

The Value of 7654321, being the 7th Number in the Table, will be found to be in Words at length, seven Millions, six hundred fifty four Thousand, three hundred twenty one. Of the fourth, to wit of 4321, the Value in Words at length will be, four Thousand, three Hundred twenty one; and so of any other.

And though the former Table go but to 9 Places, yet it is sufficient to find the Value of any Number, though it consist of 90 Thousand Places.

A ready way in long Numbers, is, by pointing the Places of Millions, as in the Number underneath.

M. of M. of M. of M. — Mill. of Mill. of Mill. — Millions of Millions — Millions

765432345678987654323456789142

A D D I T I O N.

ADDITION is the Gathering together of divers Numbers, into one Total Sum.

Addition of Integers.

Take care to place Units under Units, Tens under Tens, &c. And for every 10, carry one to the next place.

To

Example.	
4216	
2974	
2967	
10157	Total.

To work this Example, I begin in the Units place, and say, 7 and 4 is 11, and 6 is 17; place 7 under the place of Units, and for the Ten carry one to the next place; then going to the place of Tens, I say, 1 that I carried and 6 is 7, and 7 is 14, and 1 is 15; set down 5, and carry one; then 1 I carried and 9 is 10, and 9 is 19, and 2 is 21; set down one, and carry two; then, 2 I carried and 2 is 4, and 2 is 6, and 4 is 10, which being the last, set down, and the Total Sum will be 10157, as in the Example may be seen.

Other Examples for Practice.

46725632		4126
12982624		1234
37890167		7162
34256782		3242
42167142		421
46300001		219
29067892		265
		398
		789
		672
		17
		72
		81
		67
32142	72900	50
12162	4678	2
42164	290	4
39786	46	6
21214	7	

Addition of Money.

In Addition of Numbers of divers Denominations, Money, Weight, Measure, &c.

We shall first begin with Money, that being the mo[st] common.

Havi[ng]

Having placed the Numbers given, to be added in their Order, (*viz.*) Pounds under Pounds, Shillings under Shillings, and Pence under Pence, *&c.*

Then,

For every 4 Farthings carry one Penny, for 12 Pence carry one Shilling, and for 20 Shillings carry one Pound.

Example.

l.	*s.*	*d.*	*q.*
21	12	7	1
36	13	8	3
14	12	7	2
18	15	2	2
91	16	2	0

Begin with the Farthings, and say, 2 and 2 is 4, and 3 is 7, and 1 is 8 Farthings, set down a Cypher, and carry 2 Pence to the place of Pence; then, 2 I carried and 2 is 4, and 7 is 11, and 8 is 19, and 7 is 26 Pence; set down 2, and carry 2 Shillings; then 2 I carried and 5 is 7, and 2 is 9, and 5 is 14, and 2 is 16; set down 6 Shillings and carry 1 Angel, which with the other 4 Angels make 5 Angels; set down 1 Angel and carry 2 *l.* Then in Pounds work as in Integers, and the Sum will be 91 *l.* 16 *s.* 2 *d.* 0 *q.*

Other Examples for Practice.

l.	*s.*	*d.*	*q.*
42	16	9	1
36	18	2	1
34	17	9	2
16	16	8	1
74	17	7	3
18	12	5	1
224	19	6	1

l.	*s.*	*d.*	
	17	9	½
	12	7	½
	13	9	¼
	7	2	¾
2	11	5	0

l.	*s.*	*d.*	*q.*
365	16	8	1
321	12	5	2
178	18	8	3
641	12	5	1
129	13	6	3
421	12	7	1
624	13	8	3
424	12	7	1
100	00	0	0
724	16	0	3
146	17	10	1
741	18	8	2
178	12	3	2
246	16	8	2
146	17	11	3
424	12	5	0
129	18	8	0
5949	03	6	0

If

If your Sum be long, you may point it, or divide it into Parts; and the Parts added together, will be equal to the whole, which proves the Work.

Addition of Troy-Weight.

Having placed your Numbers in order, that is, each under its own Denomination; then, for every 24 Grains, carry one Penny-weight, for 20 Penny-weight carry one Ounce, for 12 Ounces carry one Pound.

Example.

lb.	ou.	pw.	gr.
24	7	11	15
36	5	15	13
64	2	14	12
125	4	01	16

Begin with the Grains, and say, 12 Gr. and 13 is 25, and 15 is 40, which is one Penny-weight, and 16 Grains; set down 16 Grains, and carry one Penny Weight to the place of Penny-weights: In Penny-weights work as in Shillings; in the Ounces Work as in Pence; and in the Pounds as in Integers, and the Total will be 125 *lb.* 4 *ou.* 1 *pw.* 16 *gr.*

Other Examples.

lb.	ou.	pw.	gr.
364	7	17	11
142	8	18	10
219	6	10	14
216	7	12	10
943	6	18	21

lb.	ou.	pw.	gr.
4216	7	10	19
1216	5	7	05
7146	8	11	16
2162	5	13	10
14742	3	03	02

Addition of Averdupois Weight.

Having placed your Numbers in their true places; for 16 Drams carry one Ounce, for 16 Ounces carry one Pound, for 28 pound carry one Quarter, for 4 Quarters carry one Hundred Weight.

Be

Begin with the Ounces and say, 10 Ounces and 5 is 15, and 8 is 23, set down 7 Ounces, and carry 1 Pound to the Pound; then 1 Pound I carryed and 13 is 14, and 17 is 31, and 11 is 42 Pound, which is 1 Quarter and 14 Pound; set down 14 and carry 1 Quarter: In the Quarters Work as in the Farthings, and in the Hundreds as in Integers; and the Sum will be found to be, 115 *C.* 2 *q.* 14 *lb.* 7 *ou.*

Example.

C.	*q.*	*lb.*	*ou.*
36	2	11	8
14	1	17	5
64	2	13	10
115	2	14	7

Other Examples.

lb.	*ou.*	*dr.*
71	11	10
36	8	12
14	5	10
36	5	6
14	7	5
173	6	11

C.	*q.*	*lb.*	*ou.*
142	2	11	7
678	1	14	10
241	2	19	8
362	3	10	5
176	1	15	6
1601	3	15	4

There are other Weights and Measures: But he that understands these cannot be ignorant of any other; if he but take notice of the Tables of Weights and Measure, in the Introduction, where he may see how much of one Denomination make one of another; then the Work will be easy enough.

We shall therefore shew the Learner the Proof of Addition, and so conclude this Rule.

Proof of Addition.

In *Proof of Addition,* add your Numbers downward, contrary to the common way, carrying as usually; so will you avoid making a mistake in the same Place, if the Total Sum be the same both Ways, you are right, else not.

Example.

In Money.

	l.	*s.*	*d.*
	146	7	9
	362	14	2
	174	11	5
Sum	683	13	4
Proof	683	13	4

In Averdupois wt.

	C.	*q.*	*lb.*	*ou.*
	142	2	11	6
	178	1	19	10
	242	2	18	5
	426	3	11	5
Sum	990	2	04	10
Proof	990	2	04	10

Questions in Addition.

A Man at *Manchester* demands how far to *London*; and was anſwered, from hence to *Derby*, is 38 Miles, thence to *Harborough* 32 Miles, thence to St. *Albans* 46 Miles, and ſo to *London* 20 Miles.

What is the Diſtance from *Manchester* to *London*?

Facit 136, as in the Work.

38
32
46
20
136

An Old Man's Age was required, and he anſwered, I have 5 Sons and 3 Daughters, betwixt the Birth of each of my Sons was two Years; betwixt my laſt Son and firſt Daughter 4 Years; and 4 Years apiece betwixt the reſt of my Daughters; in my 20th Year was my firſt Son born, and that is the Age of my Youngeſt Daughter.

What is the Father's Age?

Anſwer, Sixty Years.

SUB-

SUBTRACTION.

BY *Subtraction*, we find the Difference of any two Numbers, by taking or drawing the lesser from the greater, whereby the Difference will appear.

Subtraction in Integers.

Take care to place Units under Units, Tens under Tens; and in case of Want, in Subtracting, borrow 10, and for every 10 so borrowed, pay one in the next place.

EXAMPLE.

Bought 7126 Bundles of Yarn, of which I have sold 1693 out again. What remains to sell?

Bought	7126
Sold	1693
Rest	5433

Place your Numbers as in the Margent, and beginning at the Right-hand, say; 3 from 6 and there remains 3, 9 from 2 I cannot, but 9 from 12 (for borrowing 10 makes the 2 12) rest 3, then go on, saying, 1 I borrowed and 6 is 7, from 1 I cannot, but 7 from 11, rest 4; Lastly, One I borrowed and 1 is 2 from 7, rest 5: So will the remain be found 5433, as in the Work.

Other Examples for Practice.

Lent	457256	From	67254246
Paid	414663	Sub.	6109826

Lent at one time	4246462 *l.*
at another	124216
at another	62142
at another	4215
Lent in all	4437035
Paid at one time	1263125
at another	642162
at another	82425
Paid in all	1987712
Rest to pay	2449323

In this last Example I add the Summs lent into one Summ and likewise what was paid; then Subtracting as before, the Remainder will be found to be 2449323.

Subtraction in Money.

In Subtraction of Numbers of divers Denominations, we shall, as in *Addition*, begin with Money in the first Place, and of the rest in their Order.

Subtraction in Money is not much different from *Integers*; only Note, That having placed your Numbers right, the Less under the Greater, and Pounds under Pounds, Shillings under Shillings, *&c.* You must in Case of Want in the Farthings, borrow 4, or one Penny; and in the Pence borrow 12, or one Shilling; and in the Shillings, borrow 20 Shillings, or one Pound, remembring always to pay what you borrowed to the next Place, by calling the lower Figure one more than it is.

EXAM-

EXAMPLE.

		l.	s.	d.	q.
	Lent	67	12	07	01
	Paid	18	14	09	02
	Rest	48	17	09	03

Begin with the Farthings, and say, 2 Farthings from 1 I cannot, but 2 from 4, rest 2, and 1 is 3, which set down; then go to the Pence, saying 1 I borrowed, and 9 is 10, from 7 I cannot, but 10 from 12 rest 2 and 7 is 9 Pence, which set down; then proceed to the Shillings, say 1 Shilling I borrowed and 14 is 15, from 12 I cannot, but from 20, rest 5 and 12 is 17, which set down, and going to the Pounds, Work as in Integers, and the remain will be 48 *l.* 17 *s.* 9 *d.* 3 *q.*

Other Examples for Practice.

	l.	s.	d.	q.
Lent	142	16	9	1
Paid	79	13	8	2

	l.	s.	d.
Lent	416	16	7
Paid	198	14	9

	l.	s.	d.
	21	14	9
	36	18	2
	14	17	3
Lent at several times	91	14	6
	71	12	8
	83	16	7
	14	14	2
Paid	333	12	5 ½
Rest to pay			

Subtraction

Subtraction of Troy-Weight.

In *Subtraction of Troy-Weight* in case of Want in the Grains, borrow 24, in the Penny-Weight 20, in the Ounces 12, and in the Pounds as in Integers; remembring still to pay what you borrow to the next Place.

Begin with the Grains, and say 16 Grains from 14 Grains I cannot, but 16 from 24, rest 8, and 14 is 22, which set down; then proceeding to the Penny-weight, there you may Work as in Shillings, and in the Ounces as in Pence, and in the Pounds as in Integers.

	l.	*ou.*	*pw.*	*gr.*
Bought	672	11	12	14
Sold	149	08	13	16
Rest				

Other Examples.

	l.	*ou.*	*pw.*	*gr.*
Bought	674	07	04	10
Sold	194	08	11	06
Rest				

	l.	*ou.*	*pw.*	*gr.*
Bought	4216	05	07	11
Sold	1982	08	10	14
Rest				

Subtraction of Averdupoise-Weight.

Having placed your Numbers in Order, as was intimated before, Subtract as usually; but in Case of Want in the Drams or Ounces borrow 16, in the Pounds 28, in the Quarters 4, and in the Hundreds as in Integers.

Begin with the Ounces, and say, 8 Ounces from 5 Ounces I cannot, but 8 from 16 rest 8, and 5 is 13, which set down; then 1 I borrowed and 10 is 11, which Subtracted from 11, rest 0, which set down; then proceed to the Quarters, where Work as in Farthings, and in the *Cs.* Work as in Integers.

	C.	*q.*	*l.*	*ou.*
Bought	142	2	11	5
Sold	79	3	10	8
Rest				

Other

Other Examples.

	C.	q.	l.
Bought	426	2	19
Sold	198	3	25
Rest			

	C.	q.	l.	ou.
Bought	144	2	14	05
Sold	79	3	19	10
Rest				

Proof of Subtraction.

To prove Subtraction do thus: Add the Sum to be Subtracted to the Remainder, the Total will be equal to the Number from which you were to Subtract, if your Work be right.

Example in Money.

	l.	s.	d.	
Lent	42	16	09	
Paid	18	16	11	Add
Rest	23	19	10	
Proof	42	16	09	

In Troy-Weight.

	C.	q.	ou.	gr.	
Bought	142	12	11	14	
Sold	79	08	15	17	Add
Rest	63	03	15	21	
Proof	142	12	11	14	

A Bond

A Bond dated in the Year 1685, How many Years are ſpent this preſent 1712?

From	1712
Subt.	1685
Reſt	27 Years, the Anſwer.

The Author hereof was born in the Year of our Lord 1660, How old is he this preſent Year 1712?

From	1712
Subt.	1660
Reſt	52 Years, the Anſwer.

What Number of Pounds Shillings and Pence, added to 34 *l.* 16 *s.* 9 *d.* 1 *q.* will make 100 *l*?

	l.	*s.*	*d.*	*q.*	
From	100	00	00	00	
Subt.	34	16	09	01	
Reſt	65	03	02	03	The Anſwer.

MUL-

MULTIPLICATION.

BY *Multiplication*, we Increase or Multiply one Number by another, as oft as there are Units in either of the Numbers, and it ought to be perfectly understood by the Learner, who would know any thing of Arithmetick; Thousands of Questions in a great many Parts of the Mathematicks being resolved thereby.

In Multiplication are three Numbers or Members to be well taken notice of.

First, The Multiplicand, or Number to be multiplied.

Secondly, The Multiplier, or Number by which we multiply.

Thirdly, The Product, or the Number proceeding, or produced from both.

In Multiplication it holds,
As an Unit : to the Multiplier : :
So is the Multiplicand : to the Product.

So if one Yard Cost 5 Shillings, what will 64 Yards Cost?

Here One Yard bears such proportion to 5 Shillings, as 64 Yards will bear to the Product.

To work this Question, place your Numbers in order, as in the Example following.

	Yd.	s.	Yds.	
If	1 :	5 ::	64	Multiplicand.
			5	Multiplier.
		Product	320	*Facit* 320 Shillings, or 16 Pounds.

Here I multiply 64 by 5, saying, 5 times 4 is 20, set down a Cypher and carry 2; then 5 times 6 is 30, and 2 carried is 32, which set down to the Left-hand, the Cypher makes the Sum 320 for the Product: And so many Shillings will 64 Yards cost, at 5 Shillings the Yard.

But before we proceed any farther, it will be convenient to give you a Table of Multiplication, which the Learner ought to get perfectly by Heart.

A Table of Multiplication.

1	2	3	4	5	6	7	8	9	12
2	4	6	8	10	12	14	16	18	24
3	6	9	12	15	18	21	24	27	36
4	8	12	16	20	24	28	32	36	48
5	10	15	20	25	30	35	40	45	60
6	12	18	24	30	36	42	48	54	72
7	14	21	28	35	42	49	56	63	84
8	16	24	32	40	48	56	64	72	96
9	18	27	36	45	54	63	72	81	108
12	24	36	48	60	72	84	96	108	144

The foregoing Table contains the Multiplication of the 9 Digits, one by another, or by themselves, to which we have added a Column of 12 by the Digits, for the usefulness thereof; the reading whereof is easy; for suppose the Product of 7 by 9 were required; look for a Number at the Top, as suppose 9, and the other, to wit, 7, in the Side, and in the Angle or Meeting is 63, the Product required; so 8 times 6 will be 48, look 8 in the Top, and 6 in the Side, and in the Angle of meeting, you wll find 48, and so of any other.

In *Multiplication* it mattereth not whether of the Numbers is made the Multiplicand, or whether the Multiplier, for the Product is the same.

Only its more convenient to make the less the Multiplier, and then proceed to the Work by the following Rule.

First, Set down the greater Number, and under it the Less, Units being under Units, Tens under Tens, then drawing a Line under them, begin with the first Figure of the Multiplier towards the right hand, and by it multiply each Figure of the Multiplicand, observing for every Ten

to carry one to the next place, then proceed to the second Figure of your Multiplier, doing as before, only you must place your Product a Figure nearer to the left hand, and so proceed to every Figure, doing as before, and removing every Product a place nearer to the left hand; then drawing a Line under them, add them as they stand, and you will have the true Product, which may better be understood by observing the work of the following Examples.

Example the First.

By one Figure.

Mul.	1728	*Multiplicand,*
by	7	*Multiplier.*
	12096	*Product.*

Having placed your Numbers as in the Margent, say 7 times 8 is 56, set down 6 and carry 5; then 7 times 2 is 14, and 5 is 19, set down 9 and carry one; then 7 times 7 is 49, and one I carried is 50, set down a Cypher and carry 5; then 7 times 1 is 7 and five I carried is 12, which set down, and the Product is 12096.

This Question is the same as if one had demanded.
In 1728 Weeks, how many Days?
Or in 1728 *Lancashire* Perches, how many Yards?
Or, in 7 Foot of Timber, how many solid Inches?
The Answer would have been alike in all.

Example the Second.

By Two Figures.

Multiply	3421	*Multiplicand.*
by	36	*Multiplier.*
	20526	
	10263	
	123156	*Product.*

First say, 6 times 1 is 6, which set down; then 6 times 2 is 12, set down 2 and carry 1; then 6 times 4 is 24 and 1 is 25, set down 5, carry 2; then 6 times 3 is 18, and 2 is 20, which set down; then beginning with the 2d Figure of the Multiplier, say 3 times 1 is 3, which set down under the second Figure from the right

 hand;

hand; then 3 times 2 is 6, which set down; then 3 times 4 is 12, set down 2, carry one; then 3 times 3 is 9, and 1 is 10, which set down and your Multiplication is finished. But now you must add the two Products as they stand, as before taught in Addition of Integers, and the Sum is the true Product, to wit, 123156. When you had multiplied by 6 instead of multiplying by 3, you might have taken half the Product of 6, setting it one place nearer the left hand, as you may see. This Question is the same as if one should ask in 3421 Yards, how many Inches?

Example the Third.

By 3 Figures.

Multiply	1642	*Multiplicand.*
by	231	*Multiplier.*
	1642	
	4926	
	3284	
	379302	*Product.*

First say, once 2 is 2, once 4 is 4, once 6 is 6, once 1 is 1; Secondly, 3 times 2 is 6, 3 times 4 is 12, set down 2, carry 1; and 3 times 6 is 18, and 1 I carried is 19, set down 9 and carry 1, then 3 times 1 is 3, and 1 is 4. Then begin with the last Figure and say, 2 times 2 is 4, and 2 times 4 is 8, and 2 times 6 is 12, go 1; Lastly, 2 times 1 is 2, and 1 is 3. These three Products placed and added as in the Example, give 379302 for the true Product.

This Question is the same as if one should ask, In 1642 Gallons of Wine, how many solid Inches?

These Examples being understood, it will be needless to explain any more; only take two or three for Practice.

Other Examples for Practice.

(I.) *Mult.*	41265		And (II.)	462725
By	1728		*By*	2007
	330120			3239075
	82530			925450
	288855			928689075 *Prod.*
	41265			
	71305920 *Prod.*			

(III.

(III.) Mul.	46725	And (IV.)		123456
By	2400	By		1000
	186900000		Prod.	123456000
	93450			
Prod.	112140000			

In the second Example, I contracted my Work by omitting the Cyphers, only keeping their places vacant.

In the third Example, I multiplied by 24, adding two Cyphers to the Product.

In the fourth Example, I added three Cyphers to the Multiplicand, for one neither Multiplies or Divides; and so of any other.

Multiplication may be performed without any Charge to the Memory, by setting down the whole Product of the Multiplication of every single Figure, whereby the carriage of the Tens will be saved; but the trouble of Addition will be the greater, as in the Work of the following Examples will be manifest.

EXAMPLE I.

Let it be required to multiply	7825
By	7
	49135
	964
	54775 *Prod.*

First, 7 times 5 is 35, which set down, then 7 times 2 is 14, which set down, 1 before 3 and 4 under it, and 7 times 8 is 56, set 5 before 1, and 6 under it. Lastly, 7 times 7 is 49, set 4 before 5, and 9 under it, as may be seen in the Work; which Numbers added as they stand, will be the true Product, which may be proved as in the common Way.

Multiply	7825
By	7
Prod.	54775

EX-

EXAMPLE II.

The Work in this is the same as the last, only it is 3 times repeated; and when the Product of any Figure will not make 10, place a Cypher in the Place, where if it had made Ten, or above, the Figure above Ten must have stood, which may be seen in the Work it self; so we will not trouble our selves, or the Learner, with any more Explication.

Multiply	4215
By	879
	31045
	689
	21035
	847
	31040
	268
	3704985

PROOF.

Multiply	4215
By	879
	37935
	29505
	33720
Product	3704985

Multiplication of divers Denominations.

Before we make an end of *Multiplication*, it will be convenient to say something concerning Multiplication of Numbers of divers Denominations. And first, When one is of divers Denominations, and the other an Integer.

EXAMPLE I.

If a Pack of Yarn cost 13 *l.* 17 *s.* 9 *d:* What will 5 Packs cost?

Begin first with the least Denomination, Multiplying by the Integer, so proceeding from one Denomination to another, till you come to the greatest; carrying still from one Denomination, the Parts belonging to the next greater.

So

So in the *Example*, I say first, 5 times 9 Pence is 45 Pence, or 3 Shillings and 9 Pence; set down 9 Pence, and carry 3 Shillings; then 5 times 7 Shillings is 35, and 3 Shillings is 38 Shillings; set down 8 Shillings and carry 3 Angels; then 5 times 1 Angel is 5, and 3 is 8 Angels; set down a Cypher, and carry 4 Pound; then going to the Pounds, work as in Integers, saying 5 times 3 is 15 and 4 is 19, set down 9 and carry 1; then 5 times 1 is 5, and one I carried is 6; which set down as in the Work, and the Answer will be found to be 69 *l.* 08 *s.* 9 *d.*

See the Work.

	l.	*s.*	*d.*
	13	17	9
			5
Answ.	69	08	9

EXAMPLE II.

If 1 C. of Tobacco cost 3 *l.* 15 *s.* 9 *d.* 1 *q.* What will 35 C. cost?

Here because it will be too tedious to Multiply by 35 at once, I Multiply by the Ratio's of 35; to wit, by 5 and 7, for 5 times 7, is 35.

Answer, 132 *l.* 11 *s.* 11 *d.* 3 *q.*

So in the *Example* I Multiply 3 *l.* 15 *s.* 9 *d.* 1 *q.* by 7, the Product is 26 *l.* 10 *s.* 4 *d.* 3 *q.* and this Product I Multiply by 5, the Product will be as in the Example, 132 *l.* 11 *s.* 11 *d.* 3 *q.* which is the Answer to the Question.

See the Work.

l.	*s.*	*d.*	*q.*
3	15	09	1
			7
26	10	04	3
			5
132	11	11	3

EXAMPLE III.

At 12 *s.* 7 *d.* 1 *q.* the Gross, What will 78 Gross of Incle cost?

Because I cannot find two Numbers, which Multiplied together, make 78, I take two which will make as near 78 as possible; to wit, 9 and 8, which Multiplied together, make 72, which wants 6 of 78. Then Multiplying the

the first Number given by 6, adding the Product to the last Product before found, gives the Answer of the Question.

See the Work.

So here I Multiply 12 s. 7 d. 1 q. by 9 first, and that Product, to wit, 5 l. 13 s. 5 d. 1 q. I Multiply by 8, which makes 45 l. 7 s. 6 d. and to this I add the Product of 12 s. 7 d. 1 q. Multiplied by 6, to wit, 3 l. 15. 7 d. 2 q. and the Sum is 49 l. 03 s. 1 d. ½ the Answer.

l.	s.	d.	q.
	12	07	1
			9
5	13	05	1
			8
45	07	06	
3	15	07	2
49	03	01	2

EXAMPLE IV.

At 6 l. 12 s. 5 d. the Bag, what will 80 Bags of Cotton cost?

See the Work.

First I Multiply by 8, and that Product by 9, which makes 72, which wants 8 of 80; and seeing the first Product was the Number given, Multiplied by 8, I add the two Products together for the Answer to the Question, which is 529 l. 13 s. 4 d.

l.	s.	d.
6	12	5
		8
52	19	4
		9
476	14	0
529	13	4

This Example might have been wrought as under, by Multiplying by 9, and that Product by 9 again, which makes 81, too much by one; wherefore if from the last Product you Subtract the first given Number, the Answer will be found as before.

Take an Example where both Numbers are of divers Denominations, but of contrary Kinds.

EX

EXAMPLE V.

At 3 *l.* 14 *s.* 5 *d.* the C. what will 36 C. 2 *q.* 14 *l.* cost?

First I Multiplied by 9, and that Product by 4, which make 36; then for the $\frac{1}{2}$ C. I took $\frac{1}{2}$ the first Number, which is the Price of one Hundred; and for the 14 *l.* I took the 4th part of the $\frac{1}{2}$ C. which 3 Numbers added together make 136 *l.* 5 *s.* 6 *d.* 0 *q.* $\frac{1}{2}$.

See the Work.

l.	*s.*	*d.*	*q.*
3	14	05	
		9	
33	09	09	
		4	
133	19	00	
1	17	02	2
	09	03	2 $\frac{1}{2}$
136	05	06	0 $\frac{1}{2}$

When both are of unlike Denominations; but of the same kind as Pounds, Shillings and Pence, by Pounds, Shillings and Pence, you must take good Notice of the following Directions.

First, Pounds Multiplied by Pounds, produce Pounds.

Secondly, Pounds Multiplied by Shillings, every 20 is one Pound, the rest Shillings.

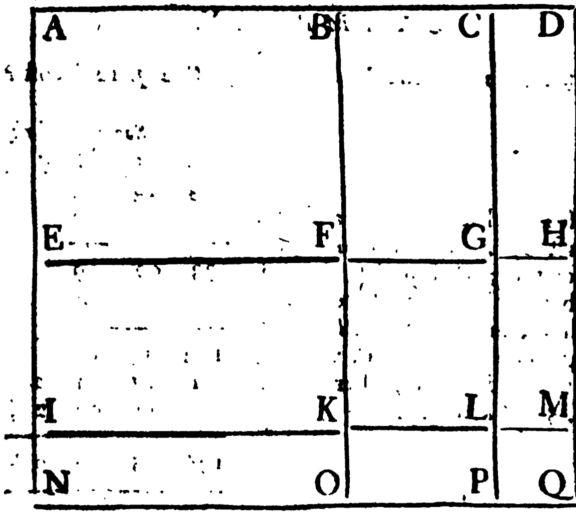
Thirdly, Pounds Multiplied by Pence, every 12 is one Shilling, the rest Pence.

Fourthly, Shillings Multiplied by Shillings, every 20 is a Shilling, every 5 is 3 Pence, and each one is 2 Farthings, and 4 tenth Parts of a Farthing.

Fifthly, Shillings Multiplied by Pence, every 5 is a Farthing, and each one 2 tenth Parts of a Farthing.

Sixthly and *Lastly*, Pence Multiplied by Pence, every 60 is a Farthing, and every 6 one tenth Part.

The Reason whereof is plain in the following *Diagram*.



Let there be two Numbers of three Denominations given, and let A F be the Square or Rectangle, made of the greatest Denomination in both Numbers, E K and B G two Rectangles, made by Multiplying the 1st Denomination by the 2d; the Product divided by an Integer of the greatest Denomination reduced into the Parts of the 2d; the Quotient shall be of the same Denomination with the Greatest, and the Remainder of the same Denomination with the 2d.

2. F L is the Square of the 2d Denomination, which being Divided by an Integer of the greatest, reduced into the Parts of the 2d; the Quotient shall be of the same Denomination with the 2d, and if there be any Remainder, it must be multiplied by a Number, which in the 3d Denomination is equal to an Integer in the 2d, the Quotient shall be of the 3d Denomination; And if there be yet a Remainder, it must be multiplied by a Number, which in the 4th Denomination is equal to an Integer in the 3d, and divided as before, the Quotient shall be of the 4th Denomination; and so forward till the Remainder cannot be reduced into lesser Terms. And thus we have done with the Square or Rectangle A C I L.

3. C H

3. C H and I O, are two Rectangles made by the Multiplication of the Summ of the greatest Denomination given, by the Summ given, which is of the 3d inferior Denomination; the Product shall be of the same Denomination with the 3d; and therefore if that Product be greater than an Integer of the 2d Denomination, reduced into the Parts of the 3d; it must be divided by a Number, which in the 3d Denomination is equal to an Integer, in the 2d; the Quotient shall be of the 2d Denomination, and the Remainder of the 3d.

4. G M, and K P are two Rectangles made by multiplying the Summ of the 2d Denomination by the 3d, and the Product being divided by one of the Integers in the greatest Denomination, reduced into the Parts of the 2d, the Quotient shall be of the same Denomination with the 3d, and the Remainder must be multiplied by a Number which in the 4th Denomination is equal to an Integer, in the 3d, the Quotient shall be of that 4th Denomination, and the remainder shall be the Numerator of a Fraction, whose Denominator is that former Divisor.

5. *Lastly*, L Q is the Square of the 2d Denomination, which must be divided, if it may be, by one Integer of the greatest Denomination, reduced into the Parts of the 3d, the Quotient shall be of the 4th Inferiour Denomination, and the Remainder shall be the Numerator of a Fraction, whose Denominator is the same Divisor.

This *Diagram* being well understood, the Multiplication of Pounds, Shillings and Pence, by Pounds, Shillings and Pence, will be easy; as may better be seen in the following Examples,

EXAMPLE.

Let it be required to Multiply 3 *l*. 5 *s*. 6 *d*. By 2 *l*. 12 *s*. 8 *d*.

First I say 2 *l.* by 3 *s.* makes 6 *l.* which set down.

Secondly 2 *l.* by 5 *s.* is 10, and 3 *l.* by 12 *s.* is 36, whose Summ is 46; which by Direction the 2d will be 2 *l.* 6 *s.* which set down.

Thirdly, 2 *l.* by 6 *d.* is 12, and 3 *l.* by 9 *d.* is 27, whose Sum is 39; which by Direction the 3d will be 3 *s.* 3 *d.*

Fourthly, 12 *s.* by 5 *s.* is 60, which by Direction the 4th will be 3 *s.* which set down.

See the Work.

l.	*s.*	*d.*	*q.*	
03	05	06		
02	12	09		
06				
02	06			
	03	03		
	03	00		
		05	03	04
				09
08	12	09	00	03

Fifthly, 12 *s* by 6 *d* is 72, and 5 *s.* by 9 *d.* is 45, whose Summ is 117, which by Direction the 5th will be 5 *d.* 3 *q.* and 4 Tenths, which set down.

Sixthly and Lastly, 6 *d.* by 9 *d.* is 54, which by Direction the 6th will be 9 Tenths, and adding all as they stand, the Summ will be the true Product; to wit, 8 *l.* 12 *s* 9 *d.* 0 *q.* 3 Tenths, as may be seen in the Work it self.

You may likewise observe by the way, that when I Multiplied by contrary Denominations, I Multiplied crosswise both ways, which in the like Case the Learner is to take Notice of.

EXAMPLE II.

Let it be required to Multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d* one Pound being taken for the Integer.

2 Shillings by 2 Shillings, makes 2 *d.* 1 *q.* 6 *Tenths*, then 2 Shillings by 6 Pence makes 12, and 2 Shillings by 6 makes 12. likewise, the Sum is 24, equal to 1 *d* 0 *q.* 8 *Tenths.* Lastly, 6 Pence by 6 Pence, makes 36, equal to 6 *Tenths*, which 3 Numbers added together, produce 3 *d.* 3 *Farthings*, for the true Product, and the Answer of the Question. Thus you see Fractions multiplied, become less in the same Proportion, as Integers by multiplying become greater.

See the Work.

s.	*d.*	
2	6	
2	6	
		q.
	2	1
	1	0
	3	3

But if it were required to multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* and making a Shilling the Integer, then the former Directions will not fit, but the Diagram holds for any.

But for this Case take the Directions following.

I. Shillings by Shillings produce Shillings.

II. Shill. by Pence, every 12 is a Shill. the rest Pence.

III. Shill. by Farthings, each one is a Farthing.

IV. Pence by Pence, every 12 is a Penny, and each 3 a Farthing.

V. Pence by Farthings, each 12 is a Farthing, and every 3 is a Quarter of a Farthing.

VI. Lastly, Farthings by Farthings, each 12 is a Quarter of a Farthing.

See the Work.

EXAMPLE III.

2 Shill. by 2 *s.* makes 4 Shill. and 2 *s.* by 6 *d.* is 12, and 2 *s.* by 6 *d.* is 12, Sum is 24, which is 2 Shill. then 6 *d.* by 6 *d.* is 36 = to 3 *d.* So the Product will be 6 Shill. and 3 *d.*

s.	d.
2	6
2	6
4	
2	0
	3
6	3

Whereby you may see the Value of your Product altereth, according as you take your Integer.

These Directions will not only fit for this, but may very well serve for the measuring of Board, Glass, &c. For seeing a Foot is divided into 12 Inches, and every Inch into four Quarters; The same Directions will fit, if instead of Shillings, Pence and Farthings, you account Feet, Inches, and Quarters.

EXAMPLE IV.

A Piece of Wainscot is 8 Foot 6 Inches and $\frac{1}{2}$ long, and 2 Foot 9 Inches $\frac{1}{4}$ broad. The Content of this Piece of Wainscot is required.

Ans.

Answ. 24 Feet, and something more, as in the Work.

These Rules will prove of excellent Use for those Persons that understand not Vulgar nor Decimal Fractions, in measuring superficial Measure.

See the Work.

f.	In.	q.
8	6	2
2	9	3
16		
7	0	
	1	0
	4	2
		3
		$0\frac{3}{4}$
24	00	$1\frac{11}{8}$ The Ans.

Contractions in Multiplication.

The foregoing Examples being well considered, are sufficient for the industrious Learner; we will here annex a Contraction or two, and conclude the Rule with some Practical Questions.

To multiply by an Unit with Cyphers, was shewn before, together with another Contraction or two, so we shall forbear those, and name some others.

1. And first, to multiply by 11 : 12 : 13, *&c.* at one Operation.

To multiply by 11, is but to set down the Multiplicand twice, the lower being removed one Place either towards the Right o Left Hand.

EXAMPLE.

Multiply 4721 by 11, the Product will be 51931.

Place your Numbers thus,

```
      4721
     4721
     -----
Prod. 51931
```

or thus

```
      4721
       4721
      -----
Prod. 51931
```

To multiply by any of the rest, is no more but to multiply by 2, 3, 4, 5, *&c.* and as you multiply, to add the Figure of the Multiplicand which stands on the right hand

EX-

EXAMPLE.

Multiply 12345 by 13.

First, I say, 3 times 5 is 15, set down 5, carry 1, and 3 times 4 is 12, and one I carried is 13, and 5 on the right hand is 18, set down 8, and carry one; then 3 times 3 is 9, and one I carried is 10,	*See the Work.* 12345 *Md.* 13 *Mr.* 160485 *Prod.*

and 4 on the right hand is 14, set down 4, carry one; then 3 times 2 is 6, and 1 I carried is 7, and 3 on the right hand is 10, set down 0, carry 1; and 3 times 1 is 3, and 1 I carried is 4, and two is 6, set down 6; and lastly annex 1, being the first Figure in your Multiplicand, and your Work is finished.

Other Examples.

Mul.	6729004 *Md.*	*Mul.*	54321 *Md.*
By	19	*By*	16 *Mr.*
	127851076		869136 *Prod.*

2. To multiply by 111, 112, 113, 114, 115, 116, &c. at one Operation.

To do which you must multiply by 1, 2, 3, 4, 5, &c. and as you multiply, add those two Figures of your Multiplicand which stand to the right hand.

EXAMPLE.

Multiply 654321, by 115; the Product is,

First, I say, 5 times 1 is 5, which set down; and 5 times 2 is 10, and 1 is 11, set down 1, carry 1; then 5 times 3 is 15, and 1 I carried is 16 and 2 on the right hand is 18, and 1	*See the Work.* 654321 *Md.* 115 *Mr.* 75246915 *Prod.*

beyond that, is 19, set down 9, and carry 1; then 5 times 4 is 20, and 1 I carried is 21, and 3 on the right hand is 24, and

and 2 beyond that is 26, set down 6 and carry 2; then 5 times 5 is 25, and 2 is 27, and 4 on the right hand is 31, and 3 is 34, set down 4, carry 3; then 5 times 6 is 30, and 3 is 33, and 5 is 38, and 4 is 42, set down 2, carry 4, which 4, I add to 6, and that to 5, makes 15, set down 5, carry 1, which 1 added to the 6, makes 7, which set down, as in the Work.

Other Examples.

Mul. 4246, by 111; and 642341, by 119.

4246	*Md.*	642341	*Md.*
111	*Mr.*	119	*Mr.*
471306	*Prod.*	76438579	*Prod.*

3. To multiply by 101, 102, 103, 104, 105, 106, &c. is no more than to multiply by 1, 2, 3, 4, 5, &c. and as you multiply, add that Figure of your Multiplicand that standeth next the right hand, except one. As you may see in the Example.

Mul.	4321	*Md.*
By	106	*Mr.*
	458026	*Prod.*

Say, 6 times 1 is 6, which set down; and 6 times 2 is 12, set down 2, carry 1; then 6 times 3 is 18, and 1 is 19, and 1 which is the next but one to the right hand, is 20, set down 0, carry 2; then 6 times 4 is 24, and 2 I carried is 26, and 2 which is next but one, is 28, set down 8, carry 2, to which add the next but one, which is 3, makes 5, which set down; to which add 4, and your Work is finished, And the Product is 458026.

Other Examples.

Mul.	427005	*Md.*	And	604150
By	101	*Mr.*	*By*	109
	43127505			65852350

Many

Many more Contractions might be added, but these being sufficient, we shall desist, and speak something concerning the Proof of the Rule.

Proof of Multiplication.

There are several Ways to prove Multiplication; but the only Proof is by Division; but that being not yet learned, we shall forbear that way for the present.

Another way mentioned by several Authors, is, by casting away the Nines, both in the Multiplicand, Multiplier, and Product. But this way being Erroneous, we shall mention it no farther.

A third way, and that which we shall use at present, is, by making Multiplication to prove it self, thus: Make that which was your Multiplicand your Multiplier, then multiplying as usual, if your Product be the same, your Work is right, else not.

EXAMPLE.

Let it be required to multiply	1234	*Md.*
By	123	*Mr.*
	3702	
	2468	
	1234	
	151782	*Prod.*

By the Work, I find the Product to be 151782.

To Prove which Mul.	123	*Md.*
By	1234	*Mr.*
	492	
	369	
	246	
	123	
	151782	*Prod.*

Here you may see the Work is contrary, but the Pro-duct the same, which is the Proof of the Work, and this of any other.

We shall here annex a Question or two to exercise Multiplication, and so conclude this Rule.

Questions in Multiplication.

I. How many Feet and Tails have 30 Thrave of Dogs when 24 Dogs make one Thrave?

See the Work.

Mul.	24
By	30
And	720
By	5

Facit 3600 Feet and Tails.

II. How many Sparrows at 10 a Penny, will buy a Yoke of Oxen of 10 *l*?

First, bring 10 *l.* into Shillings by 20, and then into Pence, by 12; and because 10 Sparrows are equal to one Penny, multiply that Product by 10, and your Work is finished; and the Answer will be 24000 Sparrows.

See the Work.

10 *l.*
20
200
12
400
200
2400
10
24000 *Sparr[ows]*

III.

III. If one Yard cost 2 Shillings and 3 Pence, what will 60 Yards cost?

Answ. 6 *l.* 15 *s.* *See the Work.*

First, Multiply 2 *s.* 3 *d.* by 8, and that Product by 7; and because 8 times 7 makes but 56, which is less than 60 by 4, therefore multiply 2 *s.* 3 *d.* by 4; add this to the last Product by 7, and it gives the Answer.

2 *s.* 3 *d.* multiplied by 4 makes

		l.	*s.*	*d.*	
Mul.			2	3	
By				8	
And			18	0	
By				7	
		6	06	0	Add
			09	0	
Facit		6	15	0	

This Method of finding the Value of any Number of Yards, Ells, Pounds, Hundreds, *&c.* at any Price *per* Yard, Ell, Pound, Hundred, *&c.* is of excellent Use for all Numbers under a Hundred, and so will be beneficial for such as Buy or Sell by retail.

But in great Numbers we shall shew you another Method in the Rule of Practice following.

Yet sometimes it may so hap that your Number, though a considerable Great Number, may be wrought by this Method, as may more plainly be seen in the following Questions.

IV. If a Pack of Yarn cost 8 *l.* 16 *s.* 5 *d.* what will 336 Packs cost? *Answ.* 2963 *l.* 16 *s.*

See the Work.

First, I multiplied by 8, and that Product by 7, for 56, and that Product by 6, for 336, for 6 times 56 is 336.

And so of many other.

l.	*s.*	*d.*
8	16	5
		8
70	11	4
		7
493	19	4
		6
2963	16	0

V. If a Hogshead of Tobacco cost 3 *l.* 7 *s.* 9 *d.* 1 *q.* what will 729 Hogsheads cost?

Answ. 2470 *l.* 4 *s.* 11 *d.* 1 *q.*

See the Work.

l.	*s.*	*d.*	*q.*
3	07	09	1
			9
30	09	11	1
			9
274	09	05	1
			9
2470	04	11	1

First, I multiplied by 9, and that product by 9 again, for 81; then because 9 times 81 is equal to 729, the Number given: I multiply that Product by 9 again, and it gives the Answer, as in the Work.

VI. How many Changes may be rung on 6 Bells?

Facit 720.

This Question is wrought by that sort of Multiplication, which some do call continued, which is nothing else but what Numbers you have given to be Multiplied this Way; you must multiply the first by the second, and that Product by the third, and that Product again by the fourth; so continuing till you have multiplied all your given Numbers, one into another.

See the Work.

1 . 2 . 3 . 4 . 5 . 6.

2

2 Changes on 2 Bells.

3

6 Changes on 3 Bells.

4

24 Changes on 4 Bells.

5

120 Changes on 5 Bells.

6

720 Changes on 6 Bells.

Take another Question in continued Multiplication.

VII. What Number is that, which divided by 1, 2, 3, 4, 5, 6, 7, 8, 9. will leave no remainder?

Ans

Answ. 362880 found by Multiplication of 1, 2, 3, 4, 5, 6, 7, 8, 9, continually one into another, the last Product is the Answer.

VIII. In 1694 Years how many Months, Weeks, Days, Hours and Minutes?

When 13 Months of 28 Days a piece make one Year, 4 Weeks make one Month, 7 Days one Week, 24 Hours one natural Day, and 60 Minutes one Hour. *Facit* 22022 Months, 88088 Weeks, 616616 Days, 14798784 Hours, and 887927040 Minutes.

The Answer is as in the Work, if you would find the Minutes in so many Years more exact, you must note that in a compleat Year are 365 Days, 5 Hours, and 49 Minutes, according to the Computation of the best Astronomers; and that is the Reason that every Fourth Year is called Leap Year, consisting of 366 Days; but the Work annexed may serve well enough for the Practice of the Rule.

See the Work.

```
     1694 Years.
       13
     ----
     5082
    1694
   ------
    22022 Months.
        4
   ------
    88088 Weeks.
        7
   ------
   616616 Days.
       24
  -------
  2466464
 1233232
 --------
 14798784 Hours.
       60
---------
887927040 Minutes.
```

IX. In 205 Miles, the Measured distance between *Manchester* and *London*, how many Furlongs, Perches, Yards, Feet, Inches, and Barley Corns?

When 8 Furlongs make a Mile, 40 Perches make a Furlong, 5 Yards and $\frac{1}{2}$ make one Perch or Rood, 3 Feet one Yard, 12 Inches one Foot, and 3 Barley Corns one Inch.

See

See the Work.

When I came to Multiply by 5 ½ I multiplied by 5 first, and for the ½, I took ½ the Number of Perches, placing it orderly under the Product I had before, and the Sum of those two were the Yards fought.

```
     205 Miles.
       8
    ----
    1640 Furlings.
      40
   -----
   65600 Perches.
     5 ½
  ------
  328000
   32800
  ------
  360800 Yards.
       3
 -------
 1082400 Feet.
      12
 -------
 2164800
1082400
--------
12988800 Inches.
       3
--------
38966400 Barley Corns.
```

X. If one Yard cost 12 Shillings, what will 142 Yards cost? *Answ.* 85 *l.* 4 *s.*

Mul.	142	
By	12	
	170	4
	l.	*s.*
Facit	85	4

Here I multiplied by 12, the contracted Way, and from the Product cutting off the last Figure, half the rest will be Pounds, and the Remainder, Shillings.

XI. At

XI. At 6 *s.* 6 *d.* a Yard, what will 142 Yards Cost?
Answ. 46 *l.* 3 *s.*

$$\begin{array}{r} 142 \\ 6\frac{1}{2} \\ \hline 852 \\ 71 \\ \hline 92|3 \\ \hline l.\quad s. \\ 46\quad 3 \end{array}$$

Here I multiplied first by 6, for 6 Shillings, and for 6 Pence, took $\frac{1}{2}$ of 142, which I add to the last Product, and from the Sum cutting off the last Figure, half of that Sum is Pounds, and the Figure cut off Shillings.

DIVISION.

BY DIVISION we discover how oft one Number is contained in another.

In *Division* are three Principal Parts to be taken Notice of.

First, The *Dividend*, or Number to be divided.

Secondly, The *Divisor*, or Number by which we divide.

Thirdly, The *Quotient*, or Number proceeding from the other two.

Sometimes by accident there is a fourth Number, called a *Remainder*.

In *Division* it holds

As the Divisor : to an Unit : :
So the Dividend : to the Quotient.

So if 4 Yards cost 32 Shillings; What will one Yard cost?

Here 4 Yards the Divisor bears such Proportion to an Unit, or one Yard, as 32 Yards, the Dividend doth bear to the Quotient; which will be the Answer to the Question.

To

To work this Question place your Numbers as underneath.

	Yd.	*Sh.*	*Yd.*	*Sh.*
If	4 :	32 : :	1	*Facit* 8

$$\begin{array}{r} 1 \\ \hline 4)\,32\ (8\ \text{Shill.} \\ 32 \\ \hline 0 \end{array}$$

Because 1 doth not Multiply, I divide 32 by 4, saying, how oft 4 in 32? Answer 8 times; which I place in my Quotient as you see.

Division is either Single or Compound.

Single Division is when the Divisor is but one Figure, and the Dividend but two at the most, as in the Question before going. And this sort of Division may either be performed by the Memory, or at most by the Table of Multiplication, by seeking your Divisor on the top of the Table, running down the same till you find the Dividend, and over against it, in the first Column, is your Quotient sought.

Compound Division is when the Dividend consisteth of many Places, and the Divisor of one or more Places.

When a Question of Compound Division is propounded to be wrought, you must proceed according to the Work of the following Rule.

First write down your Dividend, making a crooked Line at either end thereof, that on the Left-hand to contain the Divisor, and that on the Right-hand for the Quotient; and having placed your Divisor in its place, distinguish with a Point, so many Places of your Dividend towards your Left-hand, as are equal, or next exceeding your Divisor; and asking how oft your Divisor is contained in the said Summ, the Answer must be placed in your Quotient on the Right-hand the Dividend; then Multiply your Divisor by the Figure last placed in your Quotient, setting your Product under your aforesaid distinguished Summ; and drawing a Line under both, take the lower from the higher, and to the Remainder, point and bring down your next Figure of the

Divi-

Dividend, with which proceed as you did with your distinguished Number, and so on till you have pointed and brought down all the Figures of your Dividend. And Note, That as many Points as you have made in your Dividend, so many Figures will be in the Quotient; all which will be made more plain in the Work of the following Examples.

EXAMPLE I.

By one Figure.

Divide 12096
By 7

See the Work.

```
7) 12096 (1728
    ....
   7
   --
   50
   49
   ----
    19
    14
    ----
     56
     56
     ---
      0
```

Having placed the Numbers as in the Work you see, make a Point under the 2d Figure of your Dividend, and ask how oft 7 in 12; *Ans.* once; then placing 1 in your Quotient, say once 7 is 7, which placed under 12, and Subtracted from it, rest 5, to which point, and bring down the next Figure 0; then how oft 7 in 50, *Ans.* 7 times; place 7 in your Quotient, saying 7 times 7 is 49, which Subtracted from 50, rest 1, to which point, and bring down the next Figure, 9; then how oft 7 in 19, *Answ.* 2 times, set 2 in your Quotient, saying 2 times 7 is 14, which Subtract from 19, rest 5, to which point, and bring down the last Figure of your Dividend 6, asking how often 7 in 56, *Ans.* 8 times; place 8 in your Quotient, saying 8 times 7 is 56, which Subtract from 56, rest 0, and your Work is finished, and your Quotient is 1728, as in the Work.

This Question is the same, as if one should ask in 12096 Days, how many Weeks?

EXAMPLE II.

By Two Figures.

Let it be required to Divide 123156
By 36

See the following Work.

Having placed your Numbers as in the Work, make a point under 3, the third Figure of your Dividend, because you cannot have 36 in 12, the 2 first Figures thereof.

Then ask how oft 36 in 123, or which is better, how oft 3 in 12; suppose 4 times; but I cannot have 4 times 6 in 3, for you must take the first Figure of your Divisor no oftner in the first Figure, or first and second Figure of your Dividend, than you can have the 2d Figure of your Divisor, in the Remainder of the first, or first and second joined to the second or third. Suppose therefore 3 times; but can I have 3 times 6 in 33? that I can; wherefore place 3 in your Quotient, and multiplying my Divisor thereby, the Product (*viz.*) 108 I place under 123, and to the Remainder, 15, point and bring down the next Figure, 1, of your Dividend; then how oft 36 in 151, or how oft 3 in 15; *Answ.* 4 times; place 4 in your Quotient, and multiplying the Divisor thereby, your Product, to wit, 144, Subtracted from 151, leaves 7, to which point and bring down the next Figure 5; then how oft 36 in 75, or 3 in 7; *Answ.* 2 times; place 2 in your Quotient, multiplying your Divisor thereby, your Product (*viz.*) 72, brought from 75, to the Remainder 3, point and bring down your next and last Figure of your Dividend, to wit, 6, asking how oft 36 in 36, or 3 in 3; *Answ.* 1 time; place 1 in your Quotient, saying 1 time 36 is 36, which Subtracted from 36, rest 0, and your Work is finished; and the Quotient is found to be 3421.

See the Work.

```
36) 123156 (3421
      ....
    108
    ---
     151
     144
     ---
       75
       72
       --
        36
        36
        --
         0
```

This

This Question is the same, as if one should ask in 123156 Inches, how many Yards?

EXAMPLE III.

By three Figures.

Let it be required to Divide 379302
By 231

See the Work.

```
231 ) 379302 (1642
      ....
      231
      ----
      1483
      1386
      ----
       970
       924
       ----
        462
        462
        ---
          0
```

The manner of Working being the same as in the last, we shall forbear the Explication thereof; for the Operation by 2 Figures, being well understood, the Work in any other will be easy. In this Question, the Quotient you see will be 1642, and is the same as if it were asked, in 379302 solid Inches, how many Wine Gallons?

EXAMPLE IV.

Divide 746321
By 6142

See the Work.

```
6142) 746321 (121 3139/6142
      ...
      6142
      -----
      13212
      12284
      -----
        9281
        6142
        ----
        3139
```

In this Example, after your Division is finished, you see there is a Remainder of 3139, which is the Numerator of a Fraction, and the Divisor is a Denominator thereunto; and the intire Quotient is $121\frac{3139}{6142}$ as is the Work.

The Value of this Fraction, or any other in the Parts of the Integer, may be found as in the Work of the following Example.

EXAMPLE V.

Let it be required to divide a Prize worth 368424 Pounds, among 1728 Men; and let each Man's Part in Pounds, Shillings and Pence be demanded.

After the Division is finished, there is a Remainder of 360 Pound; which multiplied by 20, brings them into Shillings, to wit, 7200, which divided by the former Divisor, quotes 4 Shillings, and leaves a Remain of 288 Shillings, which multiplied by 12, brings them into 3456 Pence; and divided by the former Divisor, quotes 2 Pence, and nought remains; but if any thing had remained, it must have been multiplied by 4, for Farthings; and the like of any other. So the Quotient, or each Man's Part, will be 213 *l.* 4 *s.* 2 *d.*

See the Work.

```
                               l.  s. d.
1728) 368424 (214 . 4 . 2
          ...
        3456
        ----
         2282
         1728
         ----
          5544
          5184
          ----
           360 Remaind.
Shill. in a Pound 20
           ----
           7200 (4 Shill.
           6912
           ----
            288 Remaind.
Pence in a Shil. 12
            ----
            576
           288
           ----
           3456 (2 Pence
           3456
           ----
              0 Rem.
```

Division may be performed without charge to the Memory, by making a Tablet of your Divisor multiplied into the 9 Digits and may prove of good Use to the Learner; not only in great Numbers, but by Practising a few. This way he will attain to a good Knowledge of Division, and be enabled to work easily without such a Table.

EXAMPLE I.

Let the Dividend be 67254, and the Divisor 19. Make a Table by Duplication, or Addition, or by Multiplication. Overagainst 1 place your Divisor; for 2 double your Divisor, or first Number; for 3, add the 1st and 2d Number; the 4th is the 2d doubled; the 5th is the Summ of the 2d and 3d; the 6th is the double of the 3d; the 7th is the Summ of the 3d and 4th; the 8th is the 4th doubled; the 9th is the Summ of the 4th and 5th, otherwise the Divisor multiplied by any of the 9 Digits, gives the Number in the Table overagainst such Digit. And after this way may a Table be made for any Divisor whatever.

1	19
2	38
3	57
4	76
5	95
6	114
7	133
8	152
9	171

To work the former Question, place your Numbers as usual; then seek how far your Divisor will reach into your Dividend, which will be to two Places, which is 67; how oft 19 in 67, and by the Table seeking for that Number, or next less to it, I find it 3 times, (*viz.*) 57, which I set under 67, and subtract, and it leaves 10, to which point, and bring down the next Figure 2; then how oft 19 in 102, and by the Table I find 5 times, (*viz.*) 95, which subtracted from 102, leaves 7, to which point, and bring down 5, the next Figure of your Dividend, asking how oft 19 in 75, and by the Table I find 3 times, to wit 57, which subtract from 75, leaves 18, to which point, and bring down the next and last Figure of your Dividend 4, asking how oft 19 in 184, and by the Table I find 9 times, to wit, 171, which subtracted from 184, leaves 13 for a Remainder, and your Work is finished, and the Quotient will be $3539\frac{13}{19}$.

See the Work:

19) 67254 ($3539\frac{13}{19}$
      · · · ·
57
______
102
95
______
75
57
______
184
171
______
13 Rem.

EXAMPLE II.

Let it be required to Divide 14672865 by 6425.

See the Work.

1	6425
2	12850
3	19275
4	25700
5	32125
6	38550
7	44975
8	51400
9	57825

```
6425) 14672865 (2283
      12850
      -----
       18228
       12850
       -----
        43786
        51400
        -----
         23865
         19275
         -----
          4590
```

The making of the Table, and the working of the Example, is the ſame as before; ſo we ſhall not trouble our ſelves to Explain it, but leave it to the Conſideration of the Learner.

Diviſion of divers Denominations.

As in *Multiplication*, ſo here likewiſe in *Diviſion*, we ſhall ſay ſomething of Diviſion of Numbers of divers Denominations.

And firſt when the Dividend is a Number of divers Denominations, and the Diviſor an Integer; and for Example take the following Queſtion.

I. If a Man in 12 Months ſpend 64 *l.* 18 *s.* 9 *d.* 3 *q.* What will his Expences be for one Month?

Place your Numbers as in the Work you see done, and ask how oft 12 in 64? *Ans.* 5 times, place 5 in the Quotient, for Pounds; then 5 times 12 is 60, from 64, rest 4, to which point, and bring down 18 Shillings, and ask how oft 12 in 4 *l.* 18 *s.* or 98 Shillings? *Facit*, 8 times, which place in your Quotient for Shillings; then 8 times 12 is 96, or 4 *l.* 16 *s.* which Subtract from 4 *l.* 18 *s.* rest 2 Shillings, to which point, and bring down 9 *d.* then how oft 12 in 2 *s.* 9 *d.* or 33 Pence? *Facit*, 2 times, place 2 in your Quotient for 2 Pence, saying 2 times 12 is 24 Pence or 2 Shillings, from 2 Shill. 9 Pence, rest 9 Pence, to which point, and bring down 3 Farthings; then how oft 12 in 9 *d.* 3 *q.* or 39 *q.* *Facit*, 3 times, place 3 *q.* in your Quotient saying 3 times 12 is 36, or 9 *d.* from 9 *d.* 3 *q.* rest 3 Farthings, which is $\frac{3}{12}$ or $\frac{1}{4}$ of a Farthing.

See the Work.

```
     l.  s.  d. q.  l.  s  d. q.
12) 64 . 18 . 9 . 3 (5 . 8 . 2 . 3
60
----
 4 . 18
 4   16
-------
     2 9
     2 0
    -----
      9 3
      9 0
      ----
        3/12 or 1/4 of q.
```

So the true Quotient, or the monthly Expence, is 5 *l.* 8 *s.* 2 *d.* 3 *q.* $\frac{1}{4}$.

Take another Example.

II. Divide 4 *l.* 15 *s.* 6 *d.* among 6 Men.

See the Work.

```
   l.  s.  d.   l.  s.  d.
6) 4   15  6  (0   15  11
   4   10
   -------
       5 6
       5 6
      ----
        0
```

And so of any other.

An

An Example or 2 where both the Dividend and Divisor are Numbers of Divers Denominations.

I. Divide 31 *l.* 17 *s.* 6 *d.* 3 *q.* by 12 *l.* 12 *s.* 6 *d.*

See the Work.

l.	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*
12 .	12 .	6)	31 .	17 .	6 .	3	(2 .	10 .	6
			25 .	05 .	0				
			6	12	6	3			
			6	6	3	0			
				6	3	3			
				6	3	3			
						0			

Your Numbers being placed as in the Work, ask how oft 12 Pound 12 *s.* 6 *d.* in 31 *l.* 17 *s.* 6 *d.* or how oft 12 in 31. *Answer* 2 times, place 2 *l.* in your Quotient, and by it multiply your Divisor by the Rules in Multiplication, and the Product 25 *l.* 05 *s.* 0 *d.* Subtract from 31 *l.* 17 *s.* 6 *d.* rest 6 *l.* 12 *s.* 6 *d.* to which point and bring down 3 *q.* saying, how oft 12 in 6 *l.* 12 *s.* or 132 Shillings? *Ans.* 10 times, place 10 Shillings in your Quotient, and by it multiply your Divisor by the Rules laid down in Multiplication of divers Denominations, and the Product will be 6 *l.* 6 *s.* 3 *d.* which subtract from 6 *l.* 12 *s.* 6 *d.* 3 *q.* leaves 6 *s.* 3 *d.* 3 *q.* Lastly how oft 12 in 6 *s.* 3 *d.* or 75 Pence? *Ans.* 6 times, place 6 *d.* in your Quotient, and by it multiply your Divisor, which by the Rules before mentioned will be 6 *s.* 3 *d.* 3 *q.* which Subtract from 6 *s.* 3 *d.* 3 *q.* resteth 0, and the Quotient I find to be 2 *l.* 10 *s.* 6 *d.*

Take another Example.

II. Divide 11 *l.* 6 *s.* 8 *d.* by 3 *l.* 5 *s.*

See

See the Work.

```
l. s.   l. s. d.   l. s. d. q.
3 . 5) 11 . 6 . 8 (3 . 9 . 8 . 3
        9   15
       --------
        1   11  8
        1    9  3
       ----------
            2  5  0
            2  2  0
               3  0
```

Contractions in Division.

For the Learner's Advantage, we will here annex 2 or 3 Contractions in Division.

I. And first to divide by any of the 9 Digits, without setting down any Figures but the Quotient it self.

Examples.

To Divide by 2, is but to halve the Number, setting down the Figures of the Quotient orderly under the Dividend; so in the Example, $\frac{1}{2}$ of 7 is 3, set down 3, carry the 1 that remains, then $\frac{1}{2}$ of 16 is 8, set down 8, and $\frac{1}{2}$ of 4 is 2, and $\frac{1}{2}$ of 2 is 1; so the Quotient is 3821.

Divide 7642, by 2

Quot. $\frac{1}{2}$ 3821

To divide by 3 is to take $\frac{1}{3}$ of the Number given; so $\frac{1}{3}$ of 7 is 2, of 14 is 4, of 22 is 7, of 16 is 5, and 1 remains, which is $\frac{1}{3}$, and the Quotient is 2475 $\frac{1}{3}$.

Divide 7426, by 3

$\frac{1}{3}$ is 2475 $\frac{1}{3}$

So to Divide by 5, is to take $\frac{1}{5}$ of the Number given, so here $\frac{1}{5}$ of 6 is 1, of 17 is 3, of 24 is 4, of 45 is 9, and the Quotient is 1349, and so of any other.

Divide 6745, by 5

$\frac{1}{5}$ 1349 Quot.

After this Method may any Number be divided, if the Divisor be contained in your Multiplication Table, so $\frac{1}{12}$ of 67246 is 5603 $\frac{10}{12}$, for $\frac{1}{12}$ of 67 is 5, of 72 is 6, of 4 is 0, of 46 is 3, rest $\frac{10}{12}$.

II. *Secondly*, When your Divisor is an Unit, with any Number of Cyphers annexed to the right hand, cut from your Dividend the same Number of Places, the Remainder is the Quotient, and those out of a Decimal Fraction; so if 46575 Pound Sterling were to be divided amongst 100 Men, every Man's Part would be 465 *l.* 15 *s.* if amongst 1000 Men, every Man's Part would be 46 *l.* 11 *s.* 6 *d.* and so of any other.

III. When your Dividend and Divisor consists of Cyphers to the right hand, cut off an equal Number of Cyphers in both, with the rest proceed as by the Rules before given. So if 7962000 must be divided by 6000, cut off three Cyphers in both, $\frac{1}{6}$ Part of the remain (*viz.*) 1327 is the Quotient sought.

IV. If your Divisor have Cyphers annexed, and your Dividend none, cut off as many Figures in your Dividend as there are Cyphers in your Divisor, with the Remainder proceed as before; so if 468565 must be divided by 12000, the Quotient would be 39 $\frac{565}{12000}$.

468|565

$\frac{1}{12}$ 39 $\frac{565}{12000}$

V. If your Dividend and Divisor may be equally Divided by any Number, without leaving any Remainder, your Dividend and Divisor may be brought into less Numbers. And if your Divisor dwindle to an Unit, your last Dividend will be the Quotient sought. So if 672 were to be divided by 48, your Quotient will be 14, for seeing they are even Numbers, I halve them both as long as I can. So $\frac{1}{2}$ of 48 is 24, of 672 is 336, again $\frac{1}{2}$ of 24 is 12, of 336 is 168: Again $\frac{1}{2}$ of 12 is 6, of 168 is 84: Again $\frac{1}{2}$ of

6

6 is 3, of 84 is 42. Laftly, $\frac{1}{3}$ of 3 is 1, of 42 is 14, the Quotient fought, becaufe my Divifor is become an Unit.

See the Work.

The Higher Num. are all Divifors	48	24	12	6	3	1
The Lower Numbers are Divid.	672	336	168	84	42	14

Proof of Divifion.

Multiplication and Divifion mutually Prove each other: For in Multiplication, if you divide your Product by your Multiplier, the Quotient will be your Multiplicand: Likewife in Divifion, if you multiply your Quotient by your Divifor, the Product will be your Dividend.

Example in both.

```
Multiply 41725
By         632
      --------
         83450
        125175
        250350
      --------
632) 26370200  (  41725 Proof of the Multiplication.
     .....          632
      2528        -------
     ------        83450
       1090       125175
        632       250350
     ------      --------
       4582      26370200 Proof of the Division.
       4424
     ------
        1580      And fo of any other.
        1264
     ------
        3160
        3160
     ------
           0
```

QUEST. I.

I. Divide 46462 Yards among 246 Men.

See the Work.

```
246) 46462 (188 Yards
       ...
     246
     ----
     2186
     1968
     ----
      2182
      1968
      ----
       214
         4
      ----
       856 (3 Quarters.
       738
      ----
       118
         4.
      ----
       472 (1 Nail.
       246
      ----
       226 Remain.
```

Every Man must have 188 Yards, 3 Quarters and one Nail, together with $\frac{226}{246}$ Parts of a Nail.

QUEST. II.

II. If a C. of Tobacco or 112 *l.* cost 2 *l.* 11 *s.* 4 *d.* What will one Pound Weight Cost? *Answ.* 5 *d.* $\frac{1}{2}$.

See the following Work.

First

$$\begin{array}{l} \begin{array}{rrr} l. & s. & d. \\ 2 & 11 & 4 \end{array} \\ \underline{20} \\ 51 \\ \underline{12} \\ 112)\ 616\ (5 \text{ Pence.} \\ \underline{560} \\ 56 \\ \underline{4} \\ 224\ (2 \text{ Farth.} \\ \underline{224} \\ 0 \end{array}$$

First, I brought 2 *l.* 11 *s.* 4 *d.* into Pence, and divided by 112, gives 5 *d.* in the Quotient, the Remainder multiplied by 4, for Farthings, and divided by 112 gives 2 Farthings.

QUEST. III.

III. A Square Acre contains 160 Perches; Now the side of a Field is 12 Perches, how much on the other side will go to make an Acre? *Answ.* 13 Perches and $\frac{1}{3}$.

See the Work. 160

$\frac{1}{12}$ 13 $\frac{1}{3}$

QUEST. IV.

IV. A Captain and 160 Soldiers gain a Prize worth 362 Pound, of which the Captain had $\frac{1}{5}$ for his Share, the rest was divided equally among the Soldiers? What was each Man's Part?

Answ. { The Captain's Share { 72 *l.* 8 *s.*
Each Soldier's Share } 1 *l.* 16 *s.* $\frac{1}{5}$

See the following Work.

The

l.		
The Prize 362		
$\frac{1}{5}$ 72	s. 8	The Captain's Share.
289	12	
20		
160) 5792 (36 Shill. and $\frac{32}{160}$ or $\frac{1}{5}$ each Soldier's Share.		
480		
992		
960		
32		

And thus much shall suffice for Division.

REDUCTION.

BY REDUCTION, we Change Money, Weight, Measure, &c. out of one Denomination into another, whereby we know how many of one Denomination, are equal to so many of the other.

We shall (though contrary to other Authors) divide Reduction into three Parts.

First, *Reduction* by *Multiplication*.
Secondly, *Reduction* by *Division*.
Thirdly, *Reduction* by *Multiplication and Division*.

Of these in their Order.

Reduction by *Multiplication*, is when we bring a greater Denomination into a less, as Pounds into Shillings, Pence into Farthings, Yards into Quarters or Nails, &c.

See the Work of the following Questions.

R

Reduction by Multiplication.

QUEST. I.

Reduce 36 *l.* 7 *s.* 9 *d.* 1 *q.* into Farthings.

First, I multiplied by 20, and as I multiplied took in the 7 Shillings, and brought the Pounds into Shillings, those Shillings, to wit, 727, I multiplied by 12, bringing in the 9 *d.* *Facit*, 8733 Pence, which multiplied by 4, and adding 1 to the Product, brings them all into Farthings, and your Work is finished.

```
l.  s. d. q.
36 . 7 . 9 . 1
  20
-----
 727 Shillings.
  12
-----
1463
727
------
8733 Pence.
    4
------
34933 Farthings.
```

QUEST. II.

In 672 Yards, how many Quarters and Nails?

See the Work.

```
  672 Yards.
    4
-----
 2688 Quarters.
    4
-----
10752 Nails.
```

QUEST. III.

Reduce 21 C. 2 *q.* 11 *l.* into Ounces.
Answ. 38704 Ounces.

See the Work.

See the Work.

```
C.  q.  l.
21  2  11
 4
----
 86 Quarters.
 28
----
 689
173
-----
2419 Pounds.
  16
-----
14514
2419
-----
38704 Ounces.
```

QUEST. IV.

Reduce 72467 Nobles into Groats. *Answ.* 1449340 Groats.

See the Work.

```
 72467 Nobles.
    20
-------
1449340 Groats.
```

QUEST. V.

In 6 Baggs of Pepper, weighing each 3 C. 2 *q.* 11 Pound, how many Pound? *Answ.* 2418.

See

See the following Work.

				Or thus.		
C.	q.	lb.		C.	q.	lb.
3	2	11		3	2	11
		6		4		
21	2	10		14		
4				28		
86 Quarters.				113		
28				29		
688				403		
173				6		
2418				2418 Pounds.		

QUEST. VI.

In 142 Hogsheads how many Gallons and Pints?

See the Work.

63 Gallons make a Hogshead, and [8] Pints a Gallon.

142 Hogsheads.
63

426
852

8946 Gallons.
8

71568 Pints.

Reduction by Division.

[R]*eduction* by *Division*, is when we bring a less Deno[mina]tion into a Greater, as Farthings into Pence, Shillings [into] Pounds, and Nails into Quarters of Yards, *&c.* [In] this part of the Rule, we will use the Converse of [the] Questions we had in the former Part of the Rule, [whi]ch will be as a Proof of the Work.

QUEST. I.

In 34933 Farthings, How many Pounds?

Instead of dividing the Farthings by 4, I took $\frac{1}{4}$ part, and so $\frac{1}{12}$ of that, to bring them into Shillings, and from the Shillings I cut off the last Figure, and took half the rest, instead of dividing by 20; and the *Answer* is 36 *l.* 7 *s.* 9 *d.* 1 *q.*

	34933	Farthings
$\frac{1}{4}$	8733.	1 Farthing
$\frac{1}{12}$	72|7.	9 Pence
$\frac{1}{20}$	36.	7 Shillings

QUEST. II.

In 10752 Nails, how many Yards and Quarters?

See the Work.

10752 Nails

Ans. 672 Yards — $\frac{1}{4}$ 2688 Quarters

$\frac{1}{4}$ 672

QUEST. III.

In 38704 Ounces, How many Hundred, Quarters and Pounds?

See the Work.

```
16) 38704 (2419 Pounds
    32
    --
     67
     64
     --
      30
      16
      --
      144
      144
      ---
        0

28) 2419 (86
    224      ¼  21 C. 2 q. 11
    ---
     179
     168
     ---
      11
```

Answ. 21 C. 2 q. 11 *l.*

QUE

QUEST. IV.

Reduce 1449340 Groats into Nobles.

Facit 72467 Nobles.

144934.0 Groats.
$\frac{1}{2}$ 72467 Nobles.

QUEST. V.

Six equal Bags of Pepper weigh 2418 Pound, the weight of one Bag is demanded.

2418
$\frac{1}{6}$ 403
Answ. 3. C. 2. q. 11 l.

28) 403 (14 Quarters
28
123 3 C. 2 q. 11 l.
112
11 Pound.

QUEST. VI.

In 71568 Pints; How many Hogsheads? *Answ.* 142.

71568 Pints.
$\frac{1}{8}$ 8946 Gall.

63) 8946 (142
63
264
252
126
126
0

Reduction by Multiplication and Division.

Under this Head may be comprised the Method of exchanging Coins, Weights and Measures, that have not that immediate reference to one another, as those spoken of before; and is of excellent Use to most Persons, as well Merchants as meaner Chapmen.

We shall make all plain in the Work of the following Examples.

QUEST. I.

In 672 Ninepences, how many 13 Pence ½ Pennies?

In this or the like Question, I consider how many of one Coin are equal to any Number of the other; and I find that 3 Ninepences are equal to two Thirteenpence half Pennies. And which to make my Multiplicator, I consider whether the Answer, or the Number sought, will be greater or less than the Number given; and accordingly I make the greater, or less, my Multiplicator.

So here I consider, That in 672 Ninepences, there are less Thirteenpence half Pennies; so the less Number, to wit 2, is my Multiplicand, and 3 my Divisor.

```
     672          Answer        448 : 13½
       2
    ----
3)  1344 (448
    12           Or thus        672
    --            its double   1344
     14              ½ is       448 the Answer.
     12
     --
      24
      24
      --
       0
```

The

The Method how these Divisors and Multiplicators are found in this and the following Questions, is thus; bring both your Coins, Weights, Measures, *&c.* into their least equal known Parts, which may be brought into lesser, by dividing them both by some known Divisor; or by taking $\frac{1}{2}$ or $\frac{1}{3}$, *&c.* as oft as you can.

So in 9 *d.* are 36 Farthings,
And in 13 $\frac{1}{2}$, are 54 Farthings.

Say $\frac{1}{2}$ of 36 is 18, of 54 is 27; $\frac{1}{3}$ of 18 is 6, of 27 is 9; $\frac{1}{3}$ of 6 is 2, of 9 is 3; so are 2 and 3 the Numbers sought.

Then $\frac{36}{54} \Big| \frac{18}{27} \Big| \frac{6}{9} \Big| \frac{2}{3}$

The other Numbers are Multiplicators and Divisors, as well as these two, but these being the least, are most fit for Use.

QUEST. II.

In 100 Thirteenpence Halfpennies, how many Ninepences?

This is but the Converse of the last, and by the Work you may see the Answer is 150.

$$\begin{array}{r} 100 \\ 3 \\ \hline 300 \\ \hline \end{array}$$

$\frac{1}{2}$ 150 Ninepences.

QUEST. III.

In 672 *English* Ells, how many Yards *English*? *Ans.* 840.

$$\begin{array}{r} 672 \\ 5 \\ \hline 3360 \\ \hline \end{array}$$

$\frac{1}{4}$ 840 the Answer.

QUEST.

QUEST. IV.

In 672 Ells *English*, how many Ells *Flemish*? When 1 Yard $\frac{1}{4}$ is an Ell *English*, and $\frac{3}{4}$ of a Yard an Ell *Flemish*. *Answer* 1120.

QUEST. V.

In 642 Nobles, how many Crowns? *Answer* 856.

```
      642
        4
   ------
   3) 2568 (856
      ...
      24
   ------
       16
       15
   ------
        18
        18
   ------
         0
```

QUEST. VI.

In 856 Crowns, how many Nobles? *Answer* 642. The Converse of the last.

QUEST. VII.

In 672 little Hundred, how many great Hundred? *Answer* 600.

A little Hundred is 100; a great Hundred is 112.

```
112) 67200 (600
      .
     672
    -----
       0
```

QUEST.

QUEST. VIII.

In 600 Great Hundred, how many Little Hundred? *Anſ.* 672, the Converſe of the laſt.

QUEST. IX.

In 672 Guineas at 1 *l.* 1 *s.* 8 *d. per* Piece, How many Pounds Sterling? *Anſ.* 728.

See the Work.

$$
\begin{array}{r}
672 \\
13 \\
\hline
2016 \\
672 \\
\hline
12)\ 8736\ (\ 728 \\
84 \\
\hline
33 \\
24 \\
\hline
96 \\
96 \\
\hline
0
\end{array}
\qquad
\begin{array}{lr}
\text{Or thus} & 672 \\
\tfrac{1}{12} & 56 \\
\hline
\text{The Sum} & 728
\end{array}
$$

$$
\begin{array}{l r|r}
\text{Pence in a Pound} & 24|0 & 12 \\
\hline
\text{Pence in a Guinea} & 26|0 & 13
\end{array}
$$

In the Second Operation, becauſe 1 *s.* 8 *d.* is the $\frac{1}{12}$ of a Pound, I take $\frac{1}{12}$ of 672, which added to 672, gives the *Anſwer* 728, as before.

QUEST. X.

In 728 Pounds, How many Guineas? *Anſ.* 672. The Converſe of the laſt.

QUEST. XI.

A Merchant at *Amſterdam* is indebted to a Merchant at *London* in 642 Pound, and would pay it in *Spaniſh Guilders* at

at 2 Shillings *per* Piece, How many must the *English* Merchant receive ? *Ans.* 6420.

Two Shillings being the tenth Part of a Pound, I only add a Cypher to the Pounds, and it gives the *Answer*, to wit, 6420.

QUEST. XII.

In 672 *Spanish Guilders*, How many *French Pistoles*, at 17 Shillings 6 Pence *per* Piece ?

Ans. 76 $\frac{18}{35}$, or 76 *Pistoles* and 14 Shillings *Sterling*.

See the Work.

```
       672
         4
       ----                 24 | 8 | 4
35)  2688 (76              210 |70 |35
       ..
      245
      ---
       238
       210
       ---
        28
```

The Golden Rule ; or Rule of Three.

IT is called, *The Golden Rule*, for the Excellency thereof; sometimes it is called, *The Rule of Three*, or, *Rule of Proportion*, because there are always three Numbers given to find a fourth, which must bear such Proportion to the third, as the second doth bear to the first.

The chiefest Difficulty lies in stating your Question, which that you may do, observe the following Rule.

The Question First and Third are of the same,
The Middle Number hath another Name,
And which to make the Third you cannot miss,
The unknown Quantity it always is.

EX-

EXAMPLE.

If 16 Yards cost 5 Pound, What will 144 Yards Cost?

Yd. l. Yd.
Thus Stated, if 16 : 5 :: 144 :

Here you see the first and the third Number is Yards, the middle Number is Pounds, and because I wanted the Price of 144 Yards, I put it in the last or third Place.

This being understood, we will give you such another Rule, for the working of any such Question, which is this that follows:

If the Fourth the Second must exceed, then see
By the Great Extream it multiplied be;
But if it must be less than Second, aim
To multiply it by the less Extream.

EXAMPLE.

If 13 Packs cost 326 Pounds, What will 39 Packs cost? *Answ.* 978 *l.*

```
      P.   l.    P.
   If 13 : 326 :: 39:
              39
          -------
            2934
            978
          -------
   13)  12714 (978 l. the Answer.
          117
          -------
           101
            91
          -------
            104
            104
          -------
              0
```

L Ha-

Having stated your Question, as before, it may be easily seen, that the 4th Number will exceed the 2d, for 39 Packs must needs cost more than 13 Packs; wherefore I multiply the 2d, or middle Number by the Greater of the Two Extreams, viz. 39, then must the less Extream, to wit, 13, be my Divisor.

So multiplying 326 by 39, the Product 12714, I divide by 13, and the Quotient is 978 Pound, the Answer.

QUEST. II.

If 64 Yards of Broad Cloth cost 38 Pound 8 Shillings, what will 5 Yards of the same Cloth cost? Answ. 3 l.

Because your Numbers ought to be of one Denomination, before any Work can be done, you must reduce 38 Pounds 8 Shillings into Shillings, then State and Work your Question as underneath.

```
   Yd.   s.    Yd.        l.  s.
If 64 : 768 :: 5 :        38  8
             5            20
       ------            ----
64)   3840 (60 Shill.    768 Shill.
            [or 3 l. the Answ.
      384
      ---
       00
```

Here I multiplied the middle Number by the less Extream, because the fourth must be less than the second, the Reason is evident.

Note likewise, That your fourth Number must be of the same Denomination with your second, so here your second Number being Shillings, your fourth Number or the Answer to the Question is Shillings; likewise to wit, 60 Shillings, which divided by 20, gives 3 Pound, the Answer to the Question.

QUEST. III.

If 6 Yards of Holland cost 3 Pound 12 Shillings and 6 Pence, What will 64 ½ cost.

Before

Before you state your Question, bring the first and third Number into Quarters, and your second into Pence, as underneath.

```
Yd.          l.  s.  d.          64 1/4
6            3   12  6             4
4            20                  ------
----         ----------          257 Quart.
24 Quar.     72
             12
             ----
             150
             72
             ----
             870 Pence.
```

Then state and work your Question as in the following Work.

```
  Qr.  d.    Qr.                   l.   s.   d.  q.
If 24 : 870 :: 257 :         Ans. 38 . 16 . 4 . 1
       257
   ---------
       6090
      4350
     1740
   ---------
24) 223590  (9316 Pence.
       ....  1/12 77|6=4  Pence.
     216              l.   s.   d.  q.
    -----      1/20  38 . 16 . 4 . 1 the Answer.
       75
       72
      ----
        39
        24
       ----
        159
        144
      15/24 = 5/8 or 1 Farthing.
```

The fourth Number being Pence, I reduce them into Shillings by taking $\frac{1}{12}$ Part, which makes 776 *s.* and 4 Pence, and 776 Shillings Divided by 20, which is done by cutting off the last Figure, and taking $\frac{1}{2}$ of the Rest.

QUEST. IV.

If for 3 Weeks Diet I pay 11 Shillings and 3 Pence, What is that a Year? or which is the same, if 21 Days require 11 *s.* 3 *d.* What will 365 Days require?
Ans. 9 *l.* 15 *s.* 6 *d.* $\frac{3}{7}$.

See the Work.

```
      D.   d.     D.                      s. d.
  If 21 : 135 :: 365 :                    11 3
              135                         12
             ----                         --
             1825                         135
             1095
             365
             ----
     21 ) 49275(2346 d.
          42          12) 195 = 6 d.
          --          l.  s.  d.
          72          9 . 15 . 6 3/7 the Answ.
          63
          --
          97
          84
          --
          135
          126
          ---
          9/21 or 3/7
```

QUEST. V.

How many Yards of Velvet at 13 *s.* 4 *d.* the Yard, will 136 *l.* 12 *s.* Buy? *Ans.* 204 Yards, and $\frac{9}{10}$ Parts.

See the following Work.

13 *s.* 4 *d.*

yds. d. l. s.
13 : 4 136 . 12
12 20

130 2732
13 12

160 5464
2732

32784

Then say,

pen. *yd.* *pence*
If 160 : 1 :: 32784?

160) 32784 (204 $\frac{144}{160}$
320

784
640

144	36	18	9
160	40	20	10

QUEST. VI.

A Soldier's weekly Pay which is 3 Shillings and 5 Pence, is forborn for 3 Years 9 Months and 10 Days; What is then due? *Anſ.* 33 *l.* 2 *s.* 4 *d.* $\frac{1}{7}$.

See the Work.

s. *d.*
3 5
12

41 Pence.

3 *Y* : 9 *M* :: 10 *Days*,
365 28

1095 252
252
10

1357 Days.

Then say,

D. *d.* *Days.*
If 7 : 41 :: 1357
41

1357
5428

55637

$\frac{1}{7}$ = 7948 $\frac{1}{7}$

Pence.
7948 $\frac{1}{7}$
$\frac{1}{12}$ 662|2 4 *d.*
Anſ. 33 *l.* 2 *s.* 4 *d.* $\frac{1}{7}$

QUEST.

QUEST. VII.

If 20 Men do a Piece of Work in 60 Days, In how many Days will 30 Men do the same Work? *Ans.* 40 Days.

See the Work.

M.	D.	M.
20 :	60 : :	30

```
        20
30) 1200 (40
    120
    ---
     00
```

This Question, and some that follow, are by most Authors esteemed as Questions of *The Rule of Three Inverse*, but we will not confound the Learner with such needless Differences; for we shall make no Distinction between *The Rule of Three Direct*, and *Inverse*. The Rule you have for working of your Question being sufficient in all Cases: For here I consider that the 4th Number sought will be less than the 2d, because 30 Men will needs require less time than 20 Men; wherefore I multiply the middle Number by the less Extream, and divide by the greater, and the *Answer* is, as in the Work.

QUEST. VIII.

How many Yards of Stuff $\frac{3}{4}$ of a Yard broad, will line a Cloak containing 5 Yards $\frac{1}{2}$ in length, and is 1 *Yd.* $\frac{1}{4}$ broad?

See the Work.

```
   Yd. Qr.  Qr.
If  5 : 22 : : 3
           5
   ------------ Qr.
   3) 110 (36 2/3 or 9 Yards and 2/3.
        9
      ---
       20
       18
      ---
        2
       ---
        3
```

QUEST.

QUEST. IX.

If 360 Men be in Garrison, and have Provision for 6 Months; but hearing of no Relief till the end of 9 Months, how many Men must be turned out, that the Provision may last so much longer. *Answer* 120 Men.

Mo. Men. Mo.
Say if 6 : 360 : : 9.
6

9) 2160 (240 Men to be retained, and the remainder to 360, (*viz.*) 120 must be turned out.
18

36
36

00

QUEST. X.

If a Traveller go 160 Mile in 7 Days, when the Day is 16 Hours long; how many Days will he go the same Journey, when the Day is 12 Hours long? *Answer* in 9 Days and 4 Hours.

See the Work.
Ho. Da. Ho.
Say if 16 : 7 : : 12
16

12) 112 ($9\frac{1}{3}$
108

$\frac{4}{12}$ or $\frac{1}{3}$

But many times you may have a Question of the *Rule of Three* proposed, that may require some Preparation before you can state your Question, either by *Addition*, *Subtraction*, *Multiplication*, or *Division*, &c. As may be seen in the Examples following.

QUEST.

QUEST. XI.

A Merchant at *London* buys 64 Tunn of *French* Wine for 460 *l.* the Freight thereof from *France* to *London* cost 220 *l.* for Loading and Unloading 10 *l.* for Custom 15 *l.* the Charge of the Cellar 8 *l.* and would gain 250 *l.* by the Bargain.

A Gentleman comes and demands the Price of 24 Tunn of the said Wine.

The Question is, what he must give? *Answ.* 361 *l.* 2 *s.* 6 *d.*

By *Addition* find the Total Summ of the Freight, with all the Expences and Gain; which is 963 Pound.

Tunn. *l.* *Tunn.*

Then say if 64 : 963 :: 24 :

460	24
220	3852
10	1926
15	64) 23112 (361 *l.* 2 *s.* 6 *d.*
8	192
250	391
96	384
	72
	64
	$\frac{8}{64}$ or $\frac{1}{8}$

QUEST. XII.

If 60 Gallons of Water do in 1 Hour's time fall into a Cistern, containing 200 Gallons; and by a Pipe in the same Cistern there runs out 45 Gallons in the Hour; in how many Hours will it be filled? *Answer* in 13 Hours and 20 Minutes.

Find

Find how much it fills more than it empties, by Subtraction, which is 15 Gallons.

60 Filling Cock	*Gall.* *H.* *Gall.*
45 Emptying Cock	Then say if 15 : 1 :: 200
15 Difference	15) 200 (13 $\frac{1}{3}$

15

50
45

$\frac{5}{15}$ or $\frac{1}{3}$

QUEST. XIII.

A *Butcher* sends his Man with 216 Pound to a Fair, to buy Cattle; Oxen at 11 *l.* Cows at 40 *s.* the piece, Colts at 1 *l.* 5 *s.* the piece. Hogs at 1 *l.* 15 *s. per* piece, and of each a like Number. How many of each must he buy? *Answer* 13 of each sort, and he would have 8 *l.* remaining.

Bring the Price of each sort of Cattle into Shillings, and by *Addition* find the Summ of those Shillings, which will be 320 *s.* and 216 *l.* is 4320 *s.* as you may see.

	11 *l.*	the Price of an Ox, equal to	220 Shill.
216	2	the Price of a Cow, equal to	40
20		the Price of a Colt, equal to	25
		the Price of a Hog, equal to	35
Shill. 4320			320

Sh. *Cat.*
Then say if 320 : 1 :: 4320
320) 4320 (13 *Facit*

320

1120
960

160

QUEST. XIV.

Two Persons, *A* and *B* depart from one Place, and both go one Road; but *A* goes 3 Days before *B*, and Travels 30 Miles a Day; *B* follows after, and Travels 50 Miles a Day: How many Miles, and in how many Days Travel will *B* overtake *A*?

First find by *Subtraction* how much *B* exceeds *A* daily; then by *Multiplication* find how many Miles *A* hath Travelled at *B*'s setting out; as under.

	30 *A*'s daily Travel
B 50	3
A 30	
20 Excess	90 Miles which *A* hath Travelled at *B*'s setting out.

M. D. M.
Then say if 20 : 1 : : 90

He will overtake him at the end of 4 Days and a half; and will have Travelled 225 Miles. 9.0 4.½ the Answer.

QUEST. XV.

In 460 Pounds, how many Shillings, Sixpences, Fourpences, Twopences, and Pence, of each a like Number, may there be found therein? *Answer* 4416.

Then say if 25 : 1 : : 110400

d.	l. 460	25) 110400 (4416
12	20	100
6	9200	104
4	12	100
2	18400	40
1	9200	25
25	110400	150
		150
		0

The Proof is easie

The

The Golden Rule; or *Rule of Three repeated.*

The aforegoing Questions being well understood and considered, are sufficient for the Understanding both how to state and work any Question in the *Rule of Three*: In this Place therefore, we will treat of the *Golden Rule Compound*; or, *Rule of Three repeated.*

We shall in the Practice thereof work the Examples given, by so many single *Rules of Three* as the Question admits of; it being most consentaneous to Reason, most Intelligible to the young Learner, and many times as quick; besides a great many Questions not admitting of any other way of Work, whereby the Rules before given, without troubling the Learner with any more, will be sufficient thereunto; yet here and there, for the Reader's Satisfaction, we shall name the common way also.

QUEST. I.

If the Carriage of 20 Packs from *Manchester* to *London*, which is 136 Miles, cost 16 Pound; What will the Carriage of 12 Packs from *Manchester* to *Chester* cost, being 28 Miles?

See the following Work.

First I say, if 20 Packs cost 16 Pound, what will 12 Packs cost? *Answer* 9 Pound, 12 Shillings.

```
P.   l.   P.
20 : 16 : : 12
     12
     ---
     32
     16
     ---
20) 192 (9 l. 12 s.
    180
    ---
     12
```

 Then

Then say again, if 136 Mile cost 9 *l.* 12 *s.* or 192 Shillings, what will 28 Mile cost? *Answer* 1 *l.* 19 *s.* 6 *d.* $\frac{6}{17}$.

```
 M.   Sh.   M.
136 : 192 :: 28
         28
       ------
       1536
       384
       ------
136)  5376  (39 Sh.
             1 l. 19 s. 6d. 6/17
      408
      ------
      1296
      1224
      ------
        72
        12
      ------
       144
        72
      ------
       864 (6 d.
       816
      ------
  48         6
 ----   or  ----
 136         17
```

If you would work this Question at one Operation by the Rules commonly given, you must take Notice there are five Numbers given to find a sixth, in Proportion thereto.

Which Numbers must be so placed as the three first may contain a Supposition, and the two last a Demand; which that you may place right, let the first Term be of the same Denomination with the fourth, the second of the same Denomination with the fifth, and the third with the Term required.

So in the foregoing Question the Numbers will be disposed in the following Order.

P. M. l. P. M. P. l. P.
If 20 . 136 : 16 : : 12 . 28. Or thus, If 20 : 16 : : 12
M. M.
136 28

Then will your two first Numbers multiplied together be the first Number in the single *Rule of Three*, the third will be the second, and the two last multiplied together, will be the third, then will your Numbers stand thus, as you see.

If 2720 : 16 : : 336

136
20
2720

28
12
56
28
336

336
16
2016
336
2720) 5376(1*l.*
2720
2656
20
53120 (19*sh.*
2720
25920
24480 The *Answer* is 1 *l.* 19 *s.* 6 *d.* $\frac{6}{17}$ as before.
1440
12
2880
1440
17280 (6 *d.*
16320
$\frac{96}{272} \Big| \frac{0}{0} \Big| \frac{48}{136} \Big| \frac{24}{68} \Big| \frac{12}{34} \Big| \frac{6}{17}$

QUEST.

QUEST. II.

If 20 Pound gain 16 Pound in 15 Months; what Sum of Money will gain 24 Pound in 3 Months.

First say, If 16 Pound come from 20 Pound, what will 24 Pound come from? *Answer* 30 Pound.

```
     l.   l.    l.
If 16 : 20 :: 24
              20
          -------
     16) 480 (30 l.
         48
         ---
          00
```

Then I say again, if 15 Months require 30 *l.* what will 3 Months require? *Answer* 150 *l.* the Number sought.

If 15 Months require 30 Pound, 3 Months will require more Money, because it is less time: Therefore I multiply by the greater Extream, and divide by the less; and the *Answer* will be found to be 150 Pound, as in the Work.

```
     M.   l.   l.
If 15 : 30 :: 3
          15
        ----
     3) 450 (150 l.
        3
        ---
        15
        15
        ---
         0
```

This Question may be otherwise stated, yet wrought by two Operations, as before; for you may say, If 15 Months come from 20 *l.* what will 3 Months from more Money, because less time? *Answer* 100 *l.*

```
15 : 20 :: 3
     15
    ----
3) 300 ( 100
    ..
   3
  ---
   000
```

Say

Say again, If 16 Pound come from 100 Pound, what will 24 come from? *Answer* 150, the Answer as before.

```
l.    l.     l.
16 : 100 : : 24
16) 2400 (150
    16
    --
     80
     80
     --
      0
```

But if this Question were to be wrought at one Operation, you must place your Number as before directed; and seeing the Operation before was after such Manner, as the one Operation was Direct and the other Inverse, the Fashion of your Work will be altered, for you must Multiply them cross-wise, *viz.* the first Number of the first rank, by the second Number of the third rank; and the latter Term of the first rank, by the first Term of the last rank, setting the Products under their own Multiplicators; so will the Product standing under the first rank, be your first Number in a *Single Rule of Three direct*; and the Product under the last rank will be your third Number, and your middle Number the second.

See the Work.

```
   l. M.  l.    l. M.
If 16 . 15 : 20 : : 24 . 3
       3              15
      --              ---
      48              120
                       24
                      ---
                      360
```

Then

Then if 48 : 20 : : 360
20

48) 7200 (150*l.* The Anſwer as before.

48

240
240

00

The Golden Rule Compound.

I have been the more large in the foregoing Queſtions; not only to ſhew the variety of the Work, but that I might be the ſhorter in the following Examples: In moſt of which I ſhall only ſtate the Queſtion, and give the Anſwer; leaving the reſt to the Exerciſe of the Learner.

QUEST. III.

What is the Intereſt of 672 *l.* for 7 Years, at Six *per Cent.* Simple Intereſt? *Anſwer* 282 *l.* 4 *s.* 9 *d.* 2 *q.*

l. *l.*
First I ſay, If 100 : 6 : : 672
6

40.32 Intereſt for one Year.

Secondly if 1 : 40.32 : : 7
7

l. 282.24
20

s. 4.80
12

160
80

d. 9.60
4

q. 2.40

QUEST.

QUEST. IV.

What is the Interest of 21 *l.* for 5 Months and 11 Days, or 151 Days, at 6 *per Cent.* Simple Interest? *Ans.* 10 *s.* 5 *d.*

```
      l.   l.   l.                    D.   l.     D.
First, If 100 : 6 :: 21     Secondly, If 365 : 1.26 :: 151
                     6                          151
           ---------                      ---------
           Facit 1.26                        126
                                            630
                                           126
                                      ---------
                               365) 190.260 (.521
                                    1825
                                    ------
                                     776
                                     730
                                    ------
                                     460
```

QUEST. V.

If 12 Rood of Ditching be wrought by 2 Men in 6 Days, How many Rood shall be wrought by 8 Men in 24 Days? *Ans.* 192 Rood.

```
         M.  R.           R.
1st, If 2 : 12 :: 8 : Facit 48
         D.  R.  D.        R.
2d, If 6 : 48 :: 24 : Facit 192
```

QUEST. VI.

If 20 Dogs, for 30 Groats
Go 40 Weeks to Grass,
How many Hounds for 60 Crowns,
May Winter in that Place?
Ans. 2000 Dogs.

The Operation.

```
      C.  D.     C.             M.   D.    M.
   If 2 : 20 : : 60     Then if 40 : 600 : : 12
             60                      40
       ----------              ----------
   2) 1200 (600            12) 24000 (2000
       12
       --
        0
```

This Question is the same as if one should say, if 20 Dogs for 2 Crowns be kept 40 Weeks, how many Dogs will 60 Crowns keep the Remainder of the Year, or Winter Quarter, which is 12 Weeks?

QUEST. VII.

If 12 Men build a Wall 30 Foot long, and 6 Foot high, and 3 Foot thick in 15 Days, In how many Days will 60 Men make a Wall 300 Foot long, 8 Foot high, and 6 Foot thick? *Ans.* In 80 Days.

```
                  F.l.   D.    F.l.   D.
First, say, If 30 : 15 : : 300 : 150
                  F.     D.    F.H.
        Then, If 6 : 150 : : 8 : 200
                  F.t.   D.
       Again, If 3 : 200 : : 6 : 400
                  M.     D.    D.
      Lastly, If 12 : 400 : : 60 : 80
```

QUEST. VIII.

If 35 Ells of *Vienna* make 24 at *Lyons*, and 3 Ells at *Lyons* make 5 Ells at *Antwerp*, and 100 Ells at *Antwerp* 125 at *Frankfort*: How many Ells at *Frankfort* make 42 at *Vienna*? *Ans.* 60 Ells at *Frankfort*.

First,

	Ant.		Frank.		Ant.		Frank.
First, If	100	:	125	::	5	:	6 ¼
	Ly.		Frank.		Ly.		Frank.
2. If	3	:	6 ¼	::	24	:	50
	Vien.		Frank.		Vien.		Frank.
3. If	35	:	50	::	42	:	60

(O R,)

Vien.	Ly.	Ly.	Ant.	Ant.	Frank.	Vien.
35	24	3	5	100	125	42

Then you may see the first and 7th Term are of one Denomination, the 2d and 3d of another, the 4th and 5th of another, and the 6th and 8th of another; wherefore Multiply the 2d, 4th, 6th and 7th continually for your Dividend, and the first, 3d, and 5th multiplied together, will be your Divisor, and the Quotient will be the Answer to the Question, (*viz*) 60 at *Frankfort*.

```
  24            35
   5             3
-----          -----
  120          10500 Divisor.
  125
-----
  600
 240
120
-----          105|00) 6300|00 (60
15000                  630
   42                  -----
-----                   00
 30000
60000
-----
630000 Dividend.
```

By the way, we may observe, that in the working the Questions in the *Rule of Three* aforegoing, we have continually used Multiplication before Division, though it be not out of any Necessity; but seeing Fractions are not yet Learned, and in dividing there is often a Remainder, we

 do

do it to avoid the Trouble of having a Fraction to multiply, or else we may as well divide the middle Number by that Number which is the Dividend, and multiply that Quotient by the other Extream, the Product is the *Answer* to the Question, as in the other way.

EXAMPLE.

If 13 Packs of Wool cost 65 Pound, What will 142 Packs of the same Wool cost? *Ans.* 710 *l.*

P. l. P.		142
Say, if 13 : 65 : : 142		5
13) 65 (5		——
65		710
——		
0		

Here I divided 65 by 13, the Quotient 5 I multiplied 142 by, and the Product, to wit, 710 Pound is the *Answer.*

Contractions in the Rule of Three.

This being considered, you may oftentimes contract your Work, in *Questions of the Rule of Three.*

And first, if at any time you have a Question given in *The Rule of Three Repeated,* and the First and Second Number in every Operation be the same, you may contract your Work by dividing the second Number by the first, and the Quotient will be a common Multiplicator, by which you must multiply all your third Numbers, and those Products will be the Numbers sought. And thus will all your Divisions be saved, except one, and that commonly a small one.

EXAMPLE.

A, B, C, and *D,* have 100 Pound *Sterling* to be divided among them, in such sort, that as oft as *A* hath 3 Pound, *B* must have 5 Pound, and as oft as *B* hath 5 Pound, *C* must have 7 Pound, and as oft as *C* hath 7 Pound, *D* must have 10 Pound, What must each have?

Add

Add the Proportions into one Sum, and say,

3		*l.*	*l.*		*l.*
5	If	25 :	100 : :		3
7		25 :	100 : :		5
10		25 :	100 : :		7
——		25 :	100 : :		10
25					

Or Shorter thus, if *l.* 25 : *l.* 100 : : { *l.* 3 : 5 : 7 : 10 :

25) 100 (4 — 100 — 0

4 × 3 = 12 for *A*. — 4 × 5 = 20 for *B*.

A 12
B 20
C 28
D 40
——
100 for Proof.

4 × 7 = 28 for *C*. — 4 × 10 = 40 for *D*.

In this Example you may see there are 3 Divisions saved, and that Division you have, but a small one; and therefore will prove of excellent Use in the Rule of *Fellowship* and *Alligation*, where the first and second Number in every Operation is commonly the same; so hereafter, we will call this Method the contracted way, in *Fellowship*.

Sometimes a Question of the *Rule of Three* may be performed by *Multiplication* only.

Sometimes by *Division* only.

Sometimes by *two Multiplications*.

Sometimes by *two Divisions*.

And, Sometimes by a *Smaller Multiplication* and *Division* than your given Numbers allow of.

If

If at any time your Divisor, with either of the other Numbers, may be severally divided by some common Measure, without leaving any Remainder, and your Divisor come to be an Unit, the *Answer* will be given by Multiplication only; as in the following Example.

If 6 Gross of Incle cost 15 *l.* What will 48 cost? *Ans.* 120 *l.*

Gr.		*l.*		*Gr.*
6	:	15	::	48
$\frac{1}{6}$ 1	:	15	::	8
		8		

Ans. 120 *l.*

Here $\frac{1}{6}$ of 6 is 1, of 48 is 8, and 8 times 15 is 120 Pounds, the *Answer*.

But if either of the other Numbers come to be an Unit, the Work may be performed by Division only; as in the following Example.

If 18 Gross cost 12 *l.* What will 6 Gross cost? *Ans.* 4 *l.*

Gr.		*l.*		*Gr.*
18	:	12	::	6
$\frac{1}{6}$ 3	:	12	::	1

3) 12 (4
   12
   ——
   0

Here I take $\frac{1}{6}$ of 18 which is 3, and a 6th part of 6 is 1; then divide 12 by 3, gives 4 Pound, the *Answer* to the Question.

If your Divisor be exactly contained in both your other Numbers, the Question may be answered by two Multiplications; as in the following Example following.

If

If 3 Groſs coſt 9 *l.* What will 12 Groſs coſt? *Anſ.* 36 *l.*

```
Gr.  l.   Gr.
 3 : 9 :: 12
 3 : 3 :: 4
     4
    ---
    12
     3
    ---
    36
```

Here the 3d Part of 9 is 3, of 12 is 4; multiply 3 by 4 gives 12, and that by 3 gives 36; the *Anſwer*.

If you would uſe two Diviſions, divide your Diviſor by your ſecond Number, and the 3d by that Quotient, which gives the *Anſwer*.

EXAMPLE.

If 9 give 3, What will 45 give? *Anſ.* 15.

```
3) 9 (3) 45 (15
   9     3
   --    --
   0     15
         15
         --
          0
```

So dividing 9 by 3, quotes 3, and by that dividing 45, quotes 15, the *Anſwer*.

Sometimes you muſt uſe both Multiplication and Diviſion, yet but ſmall ones.

EX-

EXAMPLE.

If 48 Yards of Linnen Cloth cost 3 *l.* 12 *s.* What will 112 Yards cost? *Anſ.* 8 *l.* 8 *s.*

Yd.	*s.*	*Yd.*
48 :	72 ::	112
24 :	72 ::	56
12 :	72 ::	28
6 :	72 ::	14
3 :	72 ::	7

l.	*s.*
3	12
20	
60	
12	
72	

```
    72
     7
   ———
3) 504 (168
   3      8 l. 8 s.
   ——
   20
   18
   ——
    24
    24
   ——
     0
```

You might have contracted your Work more ſtill, by taking ¼ of the firſt and laſt, at firſt, and then ½ of that; which would have been the ſame; as you may ſee.

Proof of the Golden Rule.

We will ſhew you how to prove the *Rule of Three*, and ſo conclude this Rule.

If 4 Numbers be proportional, the Product of the two Means, is equal to the Product of the two Extreams.

Hence to prove your Work, multiply the 4th Number found, by the firſt Number, and if that Product be equal to the Product of the 2d by the 3d, the Work is right, elſe not.

So if 8 *Yds.* coſt 16 *l.* What will 45 *Yds.* coſt? *Anſ.* 90 *l.*

8 *Y.*

```
Y.  l.   Y.
8 : 16 : : 45
     45
    ----
     80
     64
    ----
8) 720 ( 90
    72
    ----
     00
```

Then the 4 proportional Numbers will be,

```
8 : 16 :: 45 : 90
   45          8
  ----       -----
   80        720 the Prod. of the 1 and 4.
  64
  ----
  720 the Product of the 2 and 3.
```

You see the Product of the first and 4th is equal to the Product of the 2d and 3d, which shews your Work to be right.

Hence, if of 4 Numbers, the first be to the 2d as the 3d is to the 4th, those 4 Numbers shall be Proportional.

But if your 3d Number be less than the first, and require more; or more, and require less, then the Product of your first and second will be equal to the Product of your third and fourth.

EXAMPLE.

If 12 Men do a Piece of Work in 16 Days, In how many Days will 24 Men do the same Piece of Work.

	M.		D.		M.
If	12	:	16	: :	24

```
      16
      12
     ---
      32
     16
     ---
24) 192 (8 Days the Answer.
    192
    ---
      0
```

Then the 4 Numbers will be

M.		D.		M.		D.
12	:	16	: :	24	:	8
16				8		
72				192		
12						
192						

Here you may see the Product of the first by the 2d is equal to the Product of the 3d and 4th; which shews the Work to be right.

The Rule of PRACTICE.

THIS Rule is only a Contraction of the *Golden Rule* aforegoing; for when the Price or Value of one Yard, Ell, Hundred, *&c.* is given, and the Price or Value of any other Quantity of Yards, Ells, Hundreds, *&c.* were required; the first Number being always an Unit, the Question may more quickly be wrought by the Aliquot Parts of a Shilling or Pound, as the Nature of the Question requires. It will be convenient to treat first of such Questions as may be wrought by the Aliquot Parts of a Shilling; wherefore it will be necessary to annex a Table for that purpose.

A

A Table of the Aliquot Parts of a Shilling.

	d.		th.	
	1		one 12	
	$1\frac{1}{2}$		one 8	
For	2	Take	one 6	Part.
	3		one 4	
	4		one 3	
	6		one $\frac{1}{2}$	

The Use of this Table is Easy, for you may see that for 1 Penny you must take $\frac{1}{12}$ Part, for 1 Penny Half-penny, one 8th Part, for 2 Pence one 6th Part; and so of any other.

Example I.

At 1 Penny the Yard, what will 672 Yards cost?

$\frac{1}{12}$ 5|6 Shillings.

Ans. 2 *l.* 16 *s.*

Here I said, $\frac{1}{12}$ of 67 is 5, of 72 is 6, then cutting off the last Figure, $\frac{1}{2}$ the rest is Pounds, and the Answer is, 2 *l.* 16 *s.*

II.

At 1 Penny $\frac{1}{2}$ the Yard, what will 672 Yards cost?

$\frac{1}{8}$ 8|4

$\frac{1}{2}$ 4 *l.* 4 *s.* the *Ans.*

In this Example I took $\frac{1}{8}$ Part, and from that cutting off the last Figure and taking $\frac{1}{2}$, the rest gives the Answer, to wit, 4 *l.* 4 *s.*

III.

At 2 Pence the Yard, What will 672 Yards cost?

$\frac{1}{6}$ 11|2

Ans. 5 *l.* 12 *s.*

Here I took a 6th Part, cutting off as before, and the Answer is 5 *l.* 12 *s.*

IV.

As 3 Pence the Yard, What will 672 Yards cost?

$\frac{1}{4}$ | 168

Ans. 8 l. 8 s.

Here I took $\frac{1}{4}$ part which made 168 s. equal to 8 l. 8 s.

V.

As 4 Pence the Yard, What will 672 Yards cost?

$\frac{1}{3}$ | 224

Ans. 11 l. 4 s.

Here I took one third Part, which made 224 Shil. equal to 11 l. 4 s. the Answer.

VI.

As 6 Pence the Yard, What will 672 Yards cost?

$\frac{1}{2}$ | 336

Ans. 16 l. 16 s.

Here I took $\frac{1}{2}$ for 6 Pence, which made 336 s. equal to 16 l. 16 s. the Answer.

After this Method may any Questions consisting of the Aliquot Parts of a Shilling be resolved.

But if your Question consist of un-aliquot Parts, as 5 d. [illegible] then it must be divided into Aliquot Parts, as 5 d. into 3 d. and 2 d. 7 d. into 4 d. and [illegible] 3 d. or 3 Three-pences [illegible] as in the Examples following may be seen.

I.

As 5 Pence the Yard, What will 672 Yards cost?

$\frac{1}{4}$ | 168
$\frac{1}{6}$ | 112

280

Ans. 14 l.

First, I took $\frac{1}{4}$ of 672 for 3 d. makes 168 Shillings, then I took $\frac{1}{6}$ of 672 for 2 d. makes 112 Shillings: which two Numbers added together, make 280 Shillings, equal to 14 Pounds, the Answer.

Here

Here I took, first ⅓ for 4 *d.* then ¼ for 3 *d.* which added together, makes 392 Shillings = to 19 *l.* 12 *s.* the Answer.

If I had taken, first ½ for 6 *d.* then $\frac{1}{12}$ for 1 *d.* or ⅙ of 6 *d.* for one Penny, it had been the same; as you may try at your Leisure.

II.

At 7 *d.* the Yard, What will 672 Yards cost?

⅓	224
¼	168
	39|2

Anſ. 19 *l.* 12 *s.*

Here for 6 *d.* I took ½ of the Number given, and for 3 *d.* ¼ of the said Number, or ½ of 6 *d.* for 3 *d.* both being the same; as you may see, and the *Answer* will be 5 *l.* 8 *s.* *See the Work.*

III.

At 9 Pence the Yard, What will 144 Yards cost.

½	72
¼	36 = ½ of 72
	10|8

Anſ. 5 *l.* 8 *s.*

Here first, I took one half for 6 *d.* and one half of that for 3 *d.* and ½ of 3 *d.* for 1 *d.* ½, and ½ of 1 *d.* ½ for 3 *q.* which is just 11 *d.* 1 *q.* which Sums added together, and divided by 20, as before, give 35 *l.* 5 *s.* the *Answer.*

IV.

At 11 *d.* 1 *q.* the Yard, What will 752 Yds. cost?

½	376
	188
	94
	47
	70|5

Anſ. 35 *l.* 5 *s.*

We will now proceed to Questions that consist of a Shilling, and some Number of Pence and Farthings beside, that the Learner may understand all Varieties.

IV.

At 3 Pence the Yard, What will 672 Yards cost?

$\frac{1}{4}$	16|8
Anſ.	8 *l.* 8 *s.*

Here I took $\frac{1}{4}$ part which made 168 *s.* equal to 8 *l.* 8 *s.*

V.

At 4 Pence the Yard, What will 672 Yards cost?

$\frac{1}{3}$	22|4
Anſ.	11 *l.* 4 *s.*

Here I took one third Part, which made 224 Shill. equal to 11 *l.* 4 *s.* the Anſwer.

VI.

At 6 Pence the Yard, What will 672 Yards cost?

$\frac{1}{2}$	33|6
Anſ.	16 *l.* 16 *s.*

Here I took $\frac{1}{2}$ for 6 Pence, which made 336 *s.* equal to 16 *l.* 16 *s.* the Anſwer.

After this Method may any Queſtion conſiſting of the Aliquot Parts of a Shilling be reſolved.

But if your Queſtion conſiſt of un-aliquot Parts, as 5 *d.* 7 *d.* 8 *d.* 9 *d.* 11 *d.* 13 *d.* &c. then it muſt be divided into Aliquot Parts, as 5 *d.* into 3 *d.* and 2 *d.* 7 *d.* into 4 *d.* and 3 *d.* or 6 *d.* and 1 *d.* 9 *d.* into 6 *d.* and 3 *d.* or 3 Three-pences, and ſo of any other, as in the Examples following may be ſeen.

I.

At 5 Pence the Yard, What will 672 Yards coſt?

$\frac{1}{4}$	168
$\frac{1}{6}$	112
	28|0
Anſ.	14 *l.*

Firſt, I took $\frac{1}{4}$ of 672 for 3 *d.* makes 168 Shillings, then I took $\frac{1}{6}$ of 672 for 2 *d.* makes 112 Shillings; which two Numbers added together, make 280 Shillings, equal to 14 Pounds; the *Anſwer.*

Here

Here I took, first $\frac{1}{3}$ for 4 *d.* then $\frac{1}{4}$ for 3 *d.* which added together, makes 392 Shillings = to 19 *l.* 12 *s.* the Answer.

If I had taken, first $\frac{1}{2}$ for 6 *d.* then $\frac{1}{12}$ for 1 *d.* or $\frac{1}{6}$ of 6 *d.* for one Penny, it had been the same; as you may try at your Leisure.

II.

At 7 *d.* the Yard, What will 672 Yards cost?

$\frac{1}{3}$ 224
$\frac{1}{4}$ 168

39|2

Ans. 19 *l.* 12 *s.*

III.

At 9 Pence the Yard, What will 144 Yards cost.

$\frac{1}{2}$ 72
$\frac{1}{4}$ 36 = $\frac{1}{2}$ of 72

10|8

Ans. 5 *l.* 8 *s.*

Here for 6 *d.* I took $\frac{1}{2}$ of the Number given, and for 3 *d.* $\frac{1}{4}$ of the said Number, or $\frac{1}{2}$ of 6 *d.* for 3 *d.* both being the same; as you may see, and the *Answer* will be 5 *l.* 8 *s.* *See the Work.*

IV.

At 11 *d.* 1 *q.* the Yard, What will 752 Yds. cost?

$\frac{1}{2}$ 376
188
94
47

70|5

Ans. 35 *l.* 5 *s.*

Here first, I took one half for 6 *d.* and one half of that for 3 *d.* and $\frac{1}{3}$ of 3 *d.* for 1 *d.* $\frac{1}{4}$, and $\frac{1}{2}$ of 1 *d.* $\frac{1}{2}$ for 3 *q.* which is just 11 *d.* 1 *q.* which Sums added together, and divided by 20, as before, give 35 *l.* 5 *s.* the *Answer.*

We will now proceed to Questions that consist of a Shilling, and some Number of Pence and Farthings beside, that the Learner may understand all Varieties.

In

I.

At 13 *d.* ½ the Yard, What will 144 Yards cost?

⅛	18
	16|2

Ans. 8 *l.* 2 *s.*

In this Question I let the Number given stand for a Shilling, and only take an ⅛ Part for 1 *d.* ½, and adding them together gives 168 Shillings, equal to 8 *l.* 2 *s.* the *Answer.*

II.

At 16 *d.* ½ the Yard, What will 143 Yards cost?

35	9	
17	10	2
19|6	7	2

Ans. 9 *l.* 16 *s.* 7 *d.* 2 *q.*

It is evident, it will cost 143 Shillings and 143. 4 *d.* ½ wherefore I let it stand for a Shilling, and taking ¼ Part for 3 *d.* and ½ of 3 *d.* for 3 halfpence, adding all together, makes 196 Shillings and 7 *d.* ½, equal to 9 *l.* 16 *s.* 7 *d.* ½, the *Answer.*

III.

At 22 *d.* ½ the Yard, What will 144 Yards cost?

72
36
18
27|0

Ans. 13 *l.* 10 *s.*

Here I let it stand for 1 *s.* then taking ½ for 6 *d.* and ½ of 6 *d.* for 3 *d.* and ½ of 3 *d.* for 1 *d.* ½, the Sum of all which is 270 Shillings, equal to 13 *l.* 10 *s.* the *Answer.*

Here follow some more Questions of divers Natures, for the further Exercise of the Learner.

I.

At 3 *d.* Farthing ½ Farthing ¼ Farth. the Yard, What will 144 Yards cost?

¼	36	
	3	
	1	6
	0	9
	4|1	3

Ans. 2 *l.* 1 *s.* 3 *d.*

For 3 *d.* I took ¼ Part, and for a Farthing $\frac{1}{12}$ of that, and for ½ a Farthing, I took ½ of the Farthing, and for the Quarter of the Farthing, ½ of the ½ Farthing, which added together, makes 41 *s.* and 3 *d.* equal to 2 *l.* 1 *s.* 3 *d.*

II.

At 6 *d.* 1 *q.* a Yard, What will 41 Yards cost?

```
 10   3
 10   3
 00  10   1
 ----------
 2|1   4   1
```

Anſ. 1 *l.* 1 *s.* 4 *d.* 1 *q.*

Here I took for 3 *d.* twice, and for one Farthing, $\frac{1}{12}$ Part of 3 *d.* which added, makes 21 *s.* 4 *d.* 1 *q.* or, 1 *l.* 1 *s.* 4 *d.* 1 *q.*

III.

At 3 Farthings the Ell, what will 6725 Ells cost?

```
              d.
 1/8  840     7 1/2
 ----------------
 1/2  42|0  3   3
```

Anſ. 21 *l.* 00 *s.* 3 *d.* 3 *q.*

Here I took $\frac{1}{8}$ Part for 1 *d.* $\frac{1}{2}$, then $\frac{1}{2}$ of that would ſtand for 3 Farthings, and the *Anſwer* will be 21 *l.* 00 *s.* 3 *d.* 3 *q.*

IV.

At 3 *d.* $\frac{1}{2}$ the Ell, What will 728 Yards cost?

```
        4
      ----
      2912
 1/5   582 2/5
 ------------
       97   d.
       72   9   q.
            3   0. 2/5
            3   0. 2/5
```

Anſ. 8 *l.* 10 *s.* 3 *d.* 0 *q.* $\frac{4}{5}$

Firſt, I reduced the Yards into Ells, *Facit* 582 $\frac{2}{5}$ Ells; then for 2 *d.* I took a 6th Part, and for the 1 *d.* $\frac{1}{2}$, $\frac{1}{8}$ Part: Laſtly, For the $\frac{2}{5}$, I took $\frac{1}{5}$ of the Price of one Ell, twice; and the *Anſwer* will be, as in the Work, *viz.* 8 *l.* 10 *s.* 3 *d.* 0 *q.* $\frac{4}{5}$.

We ſhall now proceed to Queſtions relating to *Aliquot Parts of a Pound*; to which purpoſe take the following Table.

Aliquot

Aliquot Parts of a Pound.

	Sh.	Pence			th.
For	1	0	Take	one	20
	1	8		one	12
	2	0		one	10
	2	6		one	8
	3	4		one	6
	4	0		one	5
	5	0		one	4
	6	8		one	3
	10	0		one	$\frac{1}{2}$

Examples follow.

(1.)

At 1 s. Yard, what 14|4 Yard?

$\frac{1}{20}$

Answer 7 *l.* 4 *s.*

(2.)

At 1 s. 8 *d.* the Yard, what 144 Yard?

$\frac{1}{12}$

Facit 12 *l.* the Answer.

(3.)

At 2 s. Yard, what 67|2 Yard?

$\frac{1}{10}$ part is 67 $\frac{2}{10}$

Answer 67 *l.* 4 *s.*

(4.)

At 2 s. 6 *d.* what 172 Yard?

$\frac{1}{8}$ is 21 $\frac{4}{8}$

Answer 21 *l.* 10 *s.*

(5.)

At 3 *s.* 4 *d.* the Yd. what 751 Yard?

$\frac{1}{6}$ part is 125 $\frac{1}{6}$

Answer 125 *l.* 3 *s.* 4 *d.*

(6.)

At 4 s. Yard, what 176 Yard?

$\frac{1}{5}$ part is 35 $\frac{1}{5}$

Answer 35 *l.* 4 *s.*

(7.)

At 5 *s.* Yard, what 725 Yard?

$\frac{1}{4}$ part is 181 $\frac{1}{4}$

Answer 181 *l.* 5 *s.*

(8.)

At 6 *s.* 8 *d.* the Yd. what 17[illegible] Yard?

$\frac{1}{3}$ part is 58[illegible]

Answer 58 *l.* 13 *s.* 4 *d.*

(9.)

At 10 *s.* Yard, what 144 Yard?

$\frac{1}{2}$ part is 72

Answer 72 *l.*

If your Question consist not of Aliquot Parts, divide it into such, the Sum of which will be the Answer to the Question; as in the following Examples may more fully appear.

(1.)

At 3 s. *Yd.* what will 144 *Yds.* cost?			
$\frac{1}{10}$ for 2 Shill. is	14 *l.*	8 *s.*	
$\frac{1}{20}$ for 1 Shill. or $\frac{1}{2}$ of 2 *s.* for 1 *s.* } is	7	4	
The Sum is the *Ans.*	21 *l.*	12 *s.*	

(2.)

At 7 *s.* Yard, what will 144 Yards cost?			
$\frac{1}{4}$ part for 5 Shillings is	36		
$\frac{1}{10}$ for 2 Shillings	14	8	
The Sum is the *Ans.*	50	8	

(3.)

At 15 *s.* 6 *d.* *C.* what 721 *C.* cost?			
$\frac{1}{2}$ for 10 Shill. is	360	10	
$\frac{1}{2}$ of 10 *s.* for 5 *s.*	180	5	
$\frac{1}{10}$ of 5 *s.* for 6 *d.* is	18	0	6
The Sum is the *Ans.*	558	15	6

(4.)

At 11 *s.* 4 *d.* the *Gr.* what 150 *Gross* cost?			
$\frac{1}{2}$ for 10 Shill. is	75		
$\frac{1}{10}$ of 10 *s.* for 1 *s.* is	7	10	
$\frac{1}{3}$ of 1 *s.* for 4 *d.* is	2	10	
The Sum is the *Ans.*	85 *l.*		

(5.)

At 2 *s.* 4 *d.* $\frac{1}{2}$ *Pound*, what 141. *Pound* cost?			
$\frac{1}{10}$ for 2 *s.* is	14	2	0
$\frac{1}{6}$ of 2 *s.* for 4 *d.* is	2	7	0
$\frac{1}{8}$ of 4 *d.* for 2 *q.* is	0	5	$10\frac{1}{2}$
Sum is the *Ans.*	16	14	$10\frac{1}{2}$

(6.)

At 17 *s.* 6 *d.* the *B.* what 375 *Bundles* cost?			
$\frac{1}{2}$ for 10 *s.* is	187	10	0
$\frac{1}{2}$ of 10 *s.* for 5 *s.* is	93	15	0
$\frac{1}{2}$ of 5 *s.* for 2 *s.* 6 *d.* is	46	17	6
The Sum is the *Ans.*	328	02	6

If your Question consist of Shillings and Pence, as in the last, you may multiply by the Number of Shillings, and take the correspondent Aliquot Parts for the Pence, according to the first Table; and from the Sum, cutting off the last Figure, and taking half the rest, the Answer will be the same as in the foregoing Method; and in some particular Cases may be more Convenient, and oftentimes more Easie.

EXAMPLES.

(1.)

At 7 s. 1 d. the *Ell*, what will 144 *Ells* cost?

Multiply by 7

Product 1008

$\frac{1}{12}$ of 144 is 12 for 1 Penny.

Sum is 102|0

$\frac{1}{20}$ 51 *l.* the Answer.

(2.)

At 17 s. 4d. *Yd.* what 172 *Yard*?

Multiply by 17

Product 2924

$\frac{1}{3}$ of 172 for 4 *d.* is 57 4

2981|1 4

Answer 149 *l.* 1 s. 4 *d.*

(3.)

At 9 s. 9 *d.* what 141 *Yards*?

Multiply by 9

Product is 1269

$\frac{1}{2}$ of 141 for 6 *d.* is 70 6

$\frac{1}{2}$ of that for 3 *d.* is 35 3

Sum is 137|4 9

Answer 68 *l.* 14 s. 9 *d.*

(4.)

At 1 *l.* 14 *d.* 9 *d.* $\frac{1}{2}$ the *Yard*, what will 144 *Yds.* cost?

Multiply by 34

576

432

Product is 4896

$\frac{1}{2}$ of 144 for 6 *d.* is 72

$\frac{1}{2}$ of last for 3 *d.* is 36

$\frac{1}{6}$ of that for 2 *q.* is 6

Sum is 501|0

Answer 250 *l.* 10 s.

If your Question consists of Shillings only, you may contract your Work thus. If your Shillings be even, multiply your Number given by half the Number of Shillings; the first Figure to the Right-hand in your Product, is a double Number of Shillings; and in your Operation, ought to be set apart in the place of Shillings, the rest is Pounds.

(1.)
At 16 *s.* the Yard, what will 672 Yards cost?

Multiply	672
By ½ of 16 (*viz.*)	8
Pr. with Sh. apart is	537 12
Answer	537 *l.* 12 *s.*

(2.)
At 6 *s.* the Yard, what will 172 Yards cost?

	172
Multiply by	3
Pr. with Sh. apart is	51 12
Answer	51 *l.* 12 *s.*

(3.)
At 12 *s.* the Yard, what will 172 Yards cost?

	172
Multiply by	6
Pr. with Sh. apart is	103 4
Answer	103 *l.* 4 *s.*

(4.)
At 14 *s.* the Yard, what will 125 Yards cost?

	125
Multiply by	7
Pr. with Sh. apart is	87 10
Answer	87 *l.* 10 *s.*

Or you may multiply as usual; and when you have finished your Operation, cut off your last Figure, doubling it for Shillings, and making the rest Pounds.

Examples follow.

(1.)
At 12 *s.* the Yard, what will 144 Yards cost?

	144
Multiply by	6
	86|4
Answer	86 *l.* 8 *s.*

(2.)
At 8 *s.* the Bundle, what will 172 Bundles cost?

	172
Multiply by	4
	68|8
Answer	68 *l.* 16 *s.*

But if your Number of Shillings be odd, work for the greatest even Number of Shillings therein, and for the odd two Shillings take the 20th part of the given Number; those Results added together give the Answer.

EXAMPLE.

(1.)

At 17 *s.* the Yard, what will 172 Yards cost?

	172	
Mult. for 16 *s.* by	8	
Product	137	12
$\frac{1}{20}$ for 1 *s.* is	8	12
Sum and Answer the former way.	146 *l.*	4 *s.*

(2.)

At 19 *s.* the Yard, what will 144 Yards cost?

	144
Mult. for 18 *s.* by	9
Product	129.6
$\frac{1}{20}$ for 1 *s.* is	7.2
Sum	136.8
Answer	136 *l.* 16 *s.*

If your Question consists of Pounds, Shillings and Pence; for the Pounds multiply, and for the Shillings and Pence work by the former Rules.

EXAMPLE.

(1.)

At 2 *l.* 17 *s.* 5 *d.* the Hundred, what will 144 Hund. cost?

	144	
Multiply by	2	
Product	288	
$\frac{1}{2}$ of 144 for 10 *s.* is	72	
$\frac{1}{2}$ of that for 5 *s.* is	36	
$\frac{1}{10}$ of 144 for 2 *s.* is	14	8
$\frac{1}{12}$ of 5 *s.* for 5 *d.* is	3	0
The Sum	413	8
Answer	413 *l.* 8 *s.*	

(2.)

If a Pack of Cotton cost 11 *l.* 11 *s.* 11 *d.* what will 111 Packs cost?

	111
Same one Fig. nearer	111
Product for 11 *l.*	1221
$\frac{1}{10}$ for 11 *s.* is	61 1
$\frac{1}{12}$ of 11 *s.* for 11 *d.* is	5 1 9
	1287 2 9
Answer	1287 *l.* 2 *s.* 9 *d.*

If the Price of one be given, and the Price of any other Number be required, together with $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, or any other Part, you must work as before for the Integral Part, and for the Fractional Part take such part of the given Price; the total Sum of which Parts will be the Answer to the Question, as in the following Examples may appear.

EXAM-

EXAMPLE.

(1.)

At 4 *l.* 16 *s.* 3 *d.* the Hundred, what will 34 *C.* ¼ cost?

	34 ¼
For 4 *l.* Multiply by	4
Product	136
½ of 34 for 10 *s.* is	17
½ of 10 *s.* for 5 *s.* is	8 10 *s.*
¼ of 5 *s.* for 1 *s.* 3 *d.*	2 2 6
½ of the given Price	2 8 1½
½ of that	1 4 0¾
The Sum	167 4 8¼
Answer	167 *l.* 4 *s.* 8 *d.* ¼

(2.)

At 3 *l.* 17 *s.* 6 *d.* the Hundred, what will 144 *C.* 2 *q.* 21 *lib.* cost?

	144 02 21
For 3 *l.* Mult. by	3
Product	432
¼ of 3 *l.* for 15 *s.*	108
⅙ of that for 2 *s.* 6 *d.*	18
½ of the given Price	1 18 9
¼ of that for 14 *lib.*	9 8½
½ of that for 7 *lib.*	4 10¼
Sum	560 13 3¾
Answer	560 *l.* 13 *s.* 3 *d.* 3 *q.*

But if the Fractional Parts cannot conveniently be taken, the quickest, easiest and best way is performed by the *Decimal Rule of Practice* following.

The Doctrine of VULGAR FRACTIONS.

Notation of Vulgar Fractions.

WHAT a *Vulgar Fraction* is, was shewed in the *Introduction*, and so needs no Repetition.

A Vulgar Fraction is either Single or Compound.

A Single Vulgar Fraction hath only one Numerator, and one Denominator, and is either *Proper* or *Improper*.

A Proper single Fraction hath its Numerator always less than its Denominator; as $\frac{1}{4}$, $\frac{11}{15}$, $\frac{35}{79}$, &c.

An Improper single Fraction is when the Numerator is greater than its Denominator, as $\frac{4}{3}$, $\frac{15}{12}$, $\frac{79}{35}$, &c.

A com-

A compound Vulgar Fraction is such as hath more Nume tors and Denominators than one, as $\frac{3}{4}$ of $\frac{4}{5}$, and is easily know by having this Word *of*, placed betwixt them; so $\frac{2}{3}$ of $\frac{3}{4}$ of is a compound Fraction: The Formation is easy; for 7 Pen will be $\frac{7}{12}$ of $\frac{1}{20}$ of a Pound, and three Farthings is $\frac{3}{4}$ of $\frac{1}{12}$ $\frac{1}{20}$ of a Pound.

From hence proceeds another Number called a mixt Nu ber, and consisteth of two Parts, the one Whole, the other B ken; so 3 Yards and 3 Quarters is express'd in a mix'd Nu ber thus $3\frac{3}{4}$; others are $11\frac{7}{19}$, $144\frac{21}{348}$, *&c.*

Things commonly express'd by Fractions, or broken Nu bers, are the Parts of Coins, Weight, Measure, Time, *&c.* Shillings, Pence or Farthings are Fractions in respect of Pound; so Quarters, Pounds and Ounces are Fractions, in spect of a Hundred.

And thus much for *Notation.*

Reduction of Vulgar Fractions.

Because *Addition* and *Subtraction* of *Vulgar Fractions* canno well be performed without the knowledge of *Reduction*, we will first treat of it, and then of the rest in Order.

By *Reduction* we bring Fractions into their least equivalent Parts.

And into common Denominators.

Or into one Denomination.

By *Reduction* we find the Value of any Fraction in the known Parts of the Integer.

Reduce whole or mixt Numbers into improper Fractions, *& contra.*

As likewise compound Fractions into single.

Of these in their Order.

I. To bring Fractions into their least equivalent Parts, may be performed several ways; a general Rule for which is either of these that follow.

First, Divide the Denominator by the Numerator, and the Divisor by the Remainder, if any be; thus doing till you find nothing to remain; your last Divisor is the greatest common Measure sought; or Divide the Denominator by the Numerator, and likewise by the Remainders as long as there is any; the last Divisor is your greatest common Measure fought, as before.

By which dividing your Numerator and Denominator, reduceth your given Fraction into its least Parts.

Note, If your last Divisor be an Unit, the Fraction is in its least Terms already.

EXAM-

EXAMPLE.

Let us find the greatest common Measure of $\frac{555}{629}$. Here I divide 629 by 555, remains 74; by which dividing 555, rest 37; by which dividing 74, nothing remains: So is 37 my last Divisor, the common Measure sought.

See the Work.

The first Way.

```
555) 629 (1
     555
     ---
      74) 555 (7
          518
          ---
           37) 74 (2
               74
               --
                0
```

Facit 37 for the greatest common Measure.

The second Way.

```
555) 629 (1
     555
     ---
      74) 629 (8
          592
          ---
           37) 629 (17
               37
               ---
               259
               259
               ---
                 0
```

Facit 37 as before.

Then if *you* divide 555 and 629 severally by 37, the two Quotients will be 15 and 17, which placed, Fractional-wise, thus, $\frac{15}{17}$, will be equal in Value to the former Fraction, but in its least Terms.

So the greatest common Measure of $\frac{637}{819}$ will be found to be 91; by which dividing both the Numerator and Denominator, reduceth the Fraction into its least Parts, to wit $\frac{7}{9}$, and so of any other.

But Fractions may more quickly be abbreviated, if you can descry any Number that will evenly divide both your Numbers, without leaving any Remainder, which in all even Numbers may be done, by halving both as often as you can if; your Numbers

Numbers end with 5, or a Cypher, it may be done by taking $\frac{1}{5}$ part, or one tenth part; and so in many other.

So $\frac{144}{240}$ by halving, will be abbreviated into $\frac{9}{15}$, and $\frac{9}{15}$ by taking $\frac{1}{3}$, will become $\frac{3}{5}$, the least Parts required, as you may see in the Work.

$$\frac{144}{240} \Big| \frac{72}{120} \Big| \frac{36}{60} \Big| \frac{18}{30} \Big| \frac{9}{15} \Big| \frac{3}{5}.$$

II. When several Fractions are given to be reduced into other equivalent Fractions, having a common Denominator, use this Rule.

Multiply every Numerator into each Denominator continually, except its own, which shall be new Numerators; then multiply all the Denominators one into another, for a common Denominator, and your Work is finish'd.

EXAMPLE.

Let $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$ be reduc'd into other Fractions, which shall have one common Denominator.

Multiply 1, 3, and 4 together, *Facit* 12; and 2, 2 and 4, *Facit* 16; and 3, 3 and 2, *Facit* 18; so have you three new Numerators. Next multiply 2, 3 and 4 into one another, *Facit* 24, for a common Denominator to the former Numerators.

So $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$ will be equal to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

Reduce $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{11}$, and $\frac{24}{75}$, into a common Denominator, and you will find $\frac{22275}{29700}$ equal to $\frac{3}{4}$, $\frac{16500}{29700}$ equal $\frac{5}{9}$, $\frac{18900}{29700}$ equal to $\frac{7}{11}$, and $\frac{9504}{29700}$ equal to $\frac{24}{75}$, and thus of any other.

III. Fractions of divers Denominations may be brought into one Denomination, by involving the less into the Parts of the greater, whereby it will become a compound Fraction.

EXAMPLE.

So if $\frac{3}{4}$ of a Shilling, and $\frac{4}{7}$ of a Pound must be brought into the Fraction of a Pound, you may observe that $\frac{3}{4}$ of a Shilling is $\frac{3}{4}$ of $\frac{1}{20}$ of a Pound, because one Shilling is one twentieth of a Pound; which compound Fraction, when reduced by one of the following Rules will be $\frac{3}{80}$ of a Pound; so have you both in one Denomination, as was required.

So $\frac{3}{7}$ of an Ounce reduced into the Fraction of a *C.* weight, will be $\frac{3}{7}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of a *C.* weight, equal to $\frac{3}{784}$ *C.* and so of any other.

IV. To

IV. To find the Value of any vulgar Fraction in the known Parts of the Integer; do thus:

Multiply the Numerator of the Fraction given, by the known Parts of the next inferior Denomination; which Product divided by the Denominator, quotes the Parts of that Denomination fought; the Remainder, if any, multiplied by the Parts of the next inferior Denomination, and divided as before, gives the Parts of the next Denomination; and thus muſt you do, till you have it brought into the leaſt known Parts.

EXAMPLE.

What is $\frac{133}{480}$ of a Pound Sterling? *Anſw.* 5 *s.* 6 *d.* 2 *q.*

```
                       133 Numerator
                        20 Shillings in a Pound
                      ----
Denominator = 480)    2660 (5 Shillings
                      2400
                      ----
                       260 Remain
                        12 Pence in a Shilling
                      ----
                       520
                       260
                      ----
Denominator = 480)    3120 (6 Pence
                      2880
                      ----
                       240
                         4 Farthings in a Penny
                      ----
Denominator = 480)     960 (2 Farthings
                       960
                      ----
                         0
```

After the ſame manner the Value of $\frac{74}{195}$ of a Hundred will be 1 Quarter, 14 Pound, and 8 $\frac{8}{195}$ Ounces.

V. To reduce whole or mixt Numbers into improper Fractions, do thus.

If your Number given be an Integer, it is but making an Unit Denominator thereunto; ſo 7 reduced into an improper Fraction, will be $\frac{7}{1}$.

If your Denominator be fixed, the Product of it, and your Integer given will be the Numerator thereunto.

So if 7 were to be reduced into an improper Fraction, whose Denominator should be 11, the improper Fraction answering, would be $\frac{77}{11}$, and so in any other.

But if it be a mixt Number, then multiply the Integral Part of your mixt Number, by the Denominator of your Fractional Part, and to the Product add the Numerator of the said Fractional Part, the Sum will be the Numerator to the former Denominator.

So $2\frac{3}{5}$ will be $\frac{13}{5}$, and $7\frac{11}{17}$ will be $\frac{130}{17}$, &c.

On the contrary, if you would reduce any improper Fraction into its equivalent whole or mixt Number, do thus:

Divide your Numerator by the Denominator, the Quotient is the whole or integral Part, and the Remainder, if any, is Numerator to the former Denominator.

So if $\frac{95}{19}$ were reduced, it would be a whole Number, (*viz.*) 5: And if $\frac{45}{13}$ were reduced, it would be a mixt Number, to wit, $3\frac{6}{13}$, and so of any other.

VI. To reduce a compound Fraction into a single Fraction.

Multiply all the Numerators one into another for a new Numerator, and the Denominators one into another for a new Denominator, so have you the single Fraction sought.

So if $\frac{3}{4}$ of $\frac{5}{9}$ were reduced into a single Fraction, it would be $\frac{15}{36}$; and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{4}$ would be $\frac{6}{24}$, or $\frac{1}{4}$.

And thus much shall suffice for Reduction.

Addition in Vulgar Fractions.

Addition of Fractions (after they are reduced or abbreviated, if occasion be) is very easy, and consisteth only in adding together their Numerators, the total of which is the Numerator to the given Denominator, and is the Sum of the Fractions sought. And this happens either in Fractions with Fractions, whole Numbers with Fractions, mixt with Fractions, mixt Numbers with mixt, or mixt with Integers.

I. *First* in Fractions with Fractions.

EXAMPLE.

Add $\frac{2}{7}$ to $\frac{3}{7}$ the Sum is $\frac{5}{7}$, and the Sum of $\frac{6}{11}$, $\frac{1}{11}$ and $\frac{4}{11}$ is $\frac{11}{11}$, or 1.

So if $\frac{2}{13}$ and $\frac{5}{9}$ were to be added, their Sum would be found to be $\frac{83}{117}$ for $\frac{2}{13}$ and $\frac{5}{9}$, will be reduced into $\frac{18}{117}$, and $\frac{65}{117}$, and then by Addition will be $\frac{83}{117}$.

And

And if $\frac{3}{5}$ of a Pound were to be added to $\frac{3}{7}$ of a Shilling, the Sum will be found to be $\frac{87}{140}$.

First $\frac{3}{7}$ of a Shilling, reduced into the Fraction of a Pound, will be $\frac{3}{140}$. Secondly, $\frac{3}{5}$ and $\frac{3}{140}$ will be reduced into $\frac{420}{700}$, and $\frac{15}{700}$; and by Addition, Thirdly, the Sum is $\frac{435}{700}$; and Fourthly by abbreviation, into $\frac{87}{140}$.

II. In whole Numbers with Fractions.

EXAMPLES.

Add 7 and $\frac{3}{5}$ together, the Sum will be $\frac{18}{5}$.

So if 5, 17, $\frac{5}{12}$ and $\frac{3}{7}$ of $\frac{9}{13}$ were to be added, the Sum would be $\frac{24803}{1092}$.

First, $\frac{3}{7}$ of $\frac{9}{13}$ will be reduced into this single Fraction $\frac{27}{91}$.

Secondly, by Reduction $\frac{5}{12}$ and $\frac{27}{91}$ will become $\frac{455}{1092}$, and $\frac{324}{1092}$.

Thirdly, by Addition the Sum of those two is $\frac{779}{1092}$.

Fourthly, by Addition, 5 and 17 make 22.

Lastly, 22 added to $\frac{779}{1092}$ is $\frac{24803}{1092}$.

And 11 *l.* and $\frac{5}{6}$ of a Shilling added together, is $\frac{265}{24}$.

III. In Mixt Numbers with Integers.

EXAMPLES.

Add 7 and $5\frac{3}{5}$ together, the Sum is $12\frac{3}{5}$, or $\frac{63}{5}$.

So if 3, 9, $2\frac{11}{15}$, and $5\frac{2}{7}$ were added, the Sum would be $\frac{2102}{105}$.

For $2\frac{11}{15}$, and $5\frac{2}{7}$ will become $\frac{41}{15}$ and $\frac{37}{7}$, and these two again will become $\frac{287}{105}$ and $\frac{555}{105}$; and these added to the Sum of 3 and 9, (*viz.*) 12, become $12\frac{842}{105}$, or $\frac{2102}{105}$.

IV. In mixt Numbers with mixt.

EXAMPLES.

Add $2\frac{3}{5}$ to $7\frac{5}{9}$, the Sum will be $\frac{457}{45}$; for $2\frac{3}{5}$ and $7\frac{5}{9}$ will be reduced to $\frac{117}{45}$ and $\frac{340}{45}$, and by Addition into $\frac{457}{45}$.

And if $5\frac{4}{9}$ were to be added to $4\frac{3}{9}$, the Sum will be $9\frac{7}{9}$; these being of like Bases, are very easy, being perform'd without any Reduction, by Addition only.

V. Fifthly and lastly, in mixt Numbers and Fractions.

EXAMPLES.

Add $\frac{3}{5}$ and $7\frac{2}{5}$ into one Sum, *Facit* 8: for $\frac{3}{5}$, and $\frac{2}{5}$ is $\frac{5}{5}$, or 1, and 7 and 1 is 8.

So the Sum of $2\frac{5}{7}$, $13\frac{2}{9}$, $\frac{3}{11}$ and $\frac{21}{25}$ will be $\frac{295378}{17325}$, which will be equal to $17\frac{853}{17325}$.

And thus of any other.

As in *Addition* we took the Sum of Numerators, after the Work of *Reduction* (if any) was perform'd; so in *Subtraction* (after such Work if need be) we must take the Difference of the Numerators, observing all the Cases as in *Addition.* Of which in their Order.

I. *First*, Where both are Fractions.

EXAMPLES.

So if the Difference of $\frac{2}{7}$ and $\frac{3}{7}$ were required it would, by Subtracting the less Numerator from the greater, be found to be $\frac{1}{7}$.

Again, If the Difference betwixt $\frac{2}{13}$ and $\frac{5}{9}$ was sought, these two Fractions, because of unequal Bases by *Reduction*, would become $\frac{18}{117}$, and $\frac{65}{117}$; and then by subtracting the less Numerator from the greater, the difference sought will be $\frac{47}{117}$.

II. *Secondly*, Where one is an Integer, and the other a Fraction.

EXAMPLES.

If the Difference betwixt 7 and $\frac{3}{5}$ were sought, it would be $6\frac{2}{5}$, for 1 from 7, rest 6; which 1 reduced into a Fraction, whose Denominator is 5, is $\frac{5}{5}$, then $\frac{3}{5}$ from $\frac{5}{5}$ rest $\frac{2}{5}$, in all $6\frac{2}{5}$; and the Difference of 11 Pound and $\frac{1}{24}$ of a Shilling, will be 10 *l.* $\frac{23}{24}$.

III. *Thirdly*, Where one is an Integer, and the other a mixt Number.

EXAMPLE.

From 7 let us subtract 2 and $\frac{4}{5}$, the Remainder will be $4\frac{1}{5}$. So if from 13 we subtract $11\frac{3}{7}$, the Remainder is $1\frac{4}{7}$.

IV. *Fourthly*, Where both are mixt Numbers.

EXAMPLE.

From $16\frac{2}{3}$, subtract $11\frac{9}{13}$; the Remainder is $\frac{194}{39}$, or $4\frac{38}{39}$. And from 13 Pound and $\frac{5}{18}$ subtract $8\frac{7}{11}$ Shillings, rest $12\frac{542}{572}$ of a Pound.

V. *Fifthly* and *Lastly*, Where one is a mixt Number, and the other a Fraction.

EXAMPLE.

From $7\frac{2}{5}$, subtract $\frac{3}{5}$, the Remainder will be $6\frac{4}{5}$.

From $16\frac{9}{11}$, subtract $\frac{21}{95}$, the Remainder is $\frac{17344}{1045} = 16\frac{624}{1045}$; for $16\frac{9}{11}$ will be reduced into $\frac{185}{11}$ then $\frac{185}{11}$ and $\frac{21}{95}$, will become $\frac{17575}{1045}$ and $\frac{231}{1045}$, which by subtraction becomes $\frac{17344}{1045}$, or $16\frac{624}{1045}$.

Here it may be observed, That if one cannot distinguish the greater of two Fractions, by reducing them both into equal Bases, the greater or lesser is easily known.

Multipli-

In multiplication of vulgar Fractions, reduce mixt Numbers into improper Fractions; whole Numbers, like Fractions, and compound Fractions into single, abbreviating where occasion is; then the Rule is: Multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator, which Numerator and Denominator is the Product sought.

EXAMPLES.

I. Let us multiply $\frac{4}{13}$ by $\frac{2}{9}$ the Product will be $\frac{8}{117}$ for 2 times 4 is 8, and 9 times 13 is 117, which placed Fractional-wise is the Product sought.

And if it were required to multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* as the Fraction of a Pound, 2 *s.* 6 *d.* being $\frac{1}{8}$ of a Pound; multiply $\frac{1}{8}$ by $\frac{1}{8}$. *Facit* $\frac{1}{64}$ of a Pound, equal to 0 *l.* 0 *s.* 3 *d.* 3 *q.* by which it is evident, that multiplication of Fractions decreases the Value in the same Proportion, as whole Numbers increase it, as is intimated further in Multiplication of Decimals.

So $\frac{1}{2}$ multiplied by $\frac{1}{2}$ becomes $\frac{1}{4}$. See this demonstrated in Mr. *Leybourn*'s *Cursus Mathematicus*, Pag. 38.

II. If one be an Integer and the other a Fraction; As if we would multiply $\frac{2}{3}$ by 7, the Product will be $\frac{14}{3}$, or $2\frac{4}{3}$, for 7 made like a Fraction is $\frac{7}{1}$; then as before.

So if $\frac{5}{18}$ were to be multiplied by 12, the Product would be $\frac{10}{3}$, or $3\frac{1}{3}$; for $\frac{5}{18}$ and $\frac{12}{1}$, may, by abbreviating the cross Terms, 12 and 18, be brought into $\frac{2}{3}$ and $\frac{5}{1}$; and by Multiplication, into $\frac{10}{3}$, or $3\frac{1}{3}$.

III. If both be mixt Numbers, as if $2\frac{3}{5}$ must be multiplied by $5\frac{3}{7}$, the Product would be $\frac{494}{35}$, or $14\frac{4}{35}$.

So if 21*l.* 16*s.* and 9*d.* were to be multiplied by 3*l.* 12*s.* 6*d.* the Product would be $\frac{50663}{640}$ equal to 79 *l.* $\frac{103}{640}$; for first, 21 *l.* 16 *s.* 9 *d.* would be made $21\frac{67}{80}$, and 3 *l.* 12 *s.* 6 *d.* would be $3\frac{5}{8}$, and those two again would become $\frac{1747}{80}$ and $\frac{29}{8}$, and then by Multiplication would be $\frac{50663}{640}$ *l.* or 79 *l.* $\frac{103}{640}$.

IV. If you would take the Parts of any Fraction or mixt Number, it is easily done by Multiplication: Thus if you would take $\frac{1}{3}$ of $\frac{3}{4}$, the same would be $\frac{1}{4}$; for $\frac{1}{3}$ multiplied by $\frac{3}{4}$ produceth $\frac{3}{12}$, or $\frac{1}{4}$, the Part sought; So $\frac{11}{13}$ of $15\frac{3}{7}$ will be $13\frac{5}{91}$, which is nothing but the Product of one by the other.

Division in Vulgar Fractions.

In *Division of Vulgar Fractions* as in *Multiplication*, we must reduce mixt Numbers into improper Fractions; whole Numbers like Fractions, and compound Fractions into single, abbreviating where may be needful; and then the Rule will be to multiply the Denominator of the Divisor, by the Numerator of the Dividend, for the Numerator of the Quotient; and the Numerator of the Divisor by the Denominator of the Dividend,

for

for the Denominator of the Quotient, and your Work is finished; or invert the Divisor, then work as in Multiplication.

EXAMPLES.

I. Let it be required to divide $\frac{8}{117}$ by $\frac{4}{13}$, the Quotient will be found to be $\frac{2}{9}$; for 13 times 8 is 104, for a Numerator, and 4 times 117 is 468, for a Denominator; which Fraction abbreviated by 4, becomes $\frac{26}{117}$, and that again by 13, becomes $\frac{2}{9}$, as in the Work.

The Work.

$$\frac{4}{13})\ \frac{8}{117}\ (\frac{104}{468} \Big| \frac{26}{117} \Big| \frac{2}{9}$$

Divide $\frac{36}{114}$ by $\frac{6}{19}$ the Quotient will be $\frac{684}{684}$ equal to 1 Integer, by which it doth appear the Fractions were equal one to the other; and had been the same as if I had divided $\frac{6}{19}$ by $\frac{6}{19}$; for any Fraction divided by it self, quotes Unity.

$$\frac{6}{19})\ \frac{36}{114}\ (\frac{684}{684} = 1$$

II. If one be an Integer and the other a Fraction, as if we would divide $\frac{2}{5}$ by 7, the Quotient would be $\frac{2}{35}$.

$$\frac{7}{1})\ \frac{2}{5}\ (\frac{2}{35}.$$

But if you must divide 7 by $\frac{2}{5}$, the Quotient will be $\frac{35}{2}$ or $17\frac{1}{2}$.

$$\frac{2}{5})\ \frac{7}{1}\ (\frac{35}{2} = 17\frac{1}{2}$$

III. If both be mixt Numbers, or one a Fraction and the other a mixt Number, as if $5\frac{3}{7}$ must be divided by $2\frac{3}{5}$, the Quotient would be $\frac{190}{91}$, or $2\frac{8}{91}$, for $2\frac{3}{5}$ would by Reduction become $\frac{13}{5}$, and $5\frac{3}{7}$ would be $\frac{38}{7}$, which would quote $\frac{190}{91}$, or $2\frac{8}{91}$.

See the Work.

$$\frac{13}{5})\ \frac{38}{7}\ (\frac{190}{91} = 2\frac{8}{91}.$$

Divide $\frac{21}{37}$ by $5\frac{7}{11}$, the Quotient would be $\frac{231}{2294}$.

You may Note, If a Fraction be divided by a whole Number, the Denominator multiplied by that Number, the Product is the new Denominator, and the Numerator the same as before.

The Rule of Three in Vulgar Fractions.

In the *Rule of Three*, or *Golden Rule in Vulgar Fractions*, if any of your Terms be Integers, mixt or compound Fractions, they must be reduced, as hath been before shewn; then stating your Question, as shewn in the *Golden Rule* aforegoing, and multiplying and dividing, as in *Multiplication* and *Division of Vulgar Fractions*; your Work is finished, and the Quotient gives your Answer.

EXAMPLE.

If $\frac{2}{3}$ of a *Yard* cost $\frac{5}{8}$ of a Pound, what will $\frac{11}{17}$ of a *Yard* cost?

Thus Stated.

Yd. l. Yd.

$$\text{If } \frac{2}{3} : \frac{5}{8} :: \frac{11}{17} \qquad \begin{array}{r} 11 \\ 5 \\ \hline 55 \\ \hline 136 \end{array} \qquad \begin{array}{r} 17 \\ 8 \\ \hline 136 \end{array}$$

$$\frac{2}{3})\ \frac{55}{136}\ (\frac{165}{272} \text{ equal to } 12\ s.\ 1\ d.\ 2\ q.\ \frac{6}{17}.$$

II. If

If $\frac{3}{4}$ of a Yard of Velvet cost 7 Shill. and 3 Pence, what will 9 Yards and $\frac{1}{3}$ cost?

Your Numbers reduced and stated as afore taught, appear as in the Work.

$$\begin{array}{cccc} & Yd. & l. & Yd. \\ \text{If} & \frac{3}{4} : & \frac{87}{240} :: & \frac{28}{3} \end{array}$$

Contracted thus; If $\frac{3}{1} : \frac{29}{15} :: \frac{7}{1}$. *Facit* $\frac{203}{45}$ $l. = 4l.$ $10s.$ $2d.$ $2q.$ $\frac{2}{3}$

In this Question, seeing the Numerators of the two last Terms, and their altern Denominators, may be severally abbreviated; one, (*viz.*) the Number of the last Term, and the Denominator of the 2d by 4, and the number of the 2d Term and Denominator of the last, by 3; the contracted Terms of which are $\frac{29}{60}$ for the 2d Term, and $\frac{7}{1}$ for the 3d or last Term, then the Work will stand thus; If $\frac{3}{4} : \frac{29}{60} :: \frac{7}{1}$. And seeing again the Denominator of the last Term is an Unit, and the Denominator of the two first Terms may be abbreviated by 4, after which the 3 Terms offer themselves thus; If $\frac{3}{1} : \frac{29}{15} :: \frac{7}{1}$. And the 4th Term is easily found by multiplying the Numerators of the two last Terms together, for the Numerator of the Quotient; and the Numerator of the 1st Term by the Denominator of the 2d, for the Denominator of the said Quotient, so the 4th Term sought will be $\frac{203}{45}$, equal to $4l.$ $\frac{23}{45}$, or $4l.$ $10s.$ $2d.$ $2q.$ $\frac{2}{3}$, as in the Work.

III. If $\frac{3}{4}$ of a Pound of Flax cost 8 Pence, what will 1 Pound cost? *Facit* $10d.$ $\frac{2}{3}$.

The Work.

$$\begin{array}{ccc} & lb. & d. \\ \text{If} & \frac{3}{4} : & \frac{8}{1} :: \frac{1}{1} \end{array}$$

$$\frac{3}{4}) \ \frac{8}{1} \ (\frac{32}{3} = 10\,d.\ \frac{2}{3}.$$

If either of the Extreams be a Fraction and the other not, as here, reduce it to a like Denomination, cancel the Denominators, and work as in Integers. So if $3\,q. : 8\,d. :: 4\,q.$ *Facit* $10\,d.$ $\frac{2}{3}$ as before.

IV. If 3 Men do a piece of Work in $4\frac{1}{2}$ Hours, in how many Hours shall 10 Men do the same Work? *Facit* 1 Hour 21 Min.

The Work.

$$\begin{array}{cccc} & M. & H. & M. \\ \text{If} & \frac{3}{1} : & \frac{9}{2} :: & \frac{10}{1} \end{array}$$

$$\frac{10}{1}) \ \frac{27}{2} \ (\frac{27}{20} \text{ equal to 1 Hour and } \frac{7}{20}.$$

In this Question the last Term was my Divisor, because more Men require less time.

V. If the Penny white Loaf weigh 7 Ounces, when a Bushel of Wheat cost $5\,s.$ $6\,d.$ what is the Bushel worth, when the Penny white Loaf weighs but 2 Ounces and $\frac{1}{2}$. *Ans.* $15\,s.$ $\frac{2}{5}$.

$$\begin{array}{cccc} & l. & s. & l. \\ \text{Say if} & \frac{7}{12} : & \frac{11}{2} :: & \frac{5}{24} \end{array}$$

$$\frac{5}{24}) \ \frac{77}{24} \ (\frac{77}{5} = 15\,s.\ \frac{2}{5}, \text{ the Answer.}$$

Seeing

Seeing the Denominators of the Dividend and Divisor are both the same, throw them away, the Numerator of the Dividend is the Numerator of the Quotient, and the Numerator of the Divisor, Denominator thereto.

Double Rule of Three in Vulgar Fractions.

Take a Question or two in *Double Golden Rule in Vulgar Fractions*, and so finish *Vulgar Fractions.*

Question I. If 13 *l.* 6 *s.* 8 *d.* in $\frac{3}{4}$ of a Year gain 1 *l.* $\frac{1}{12}$, what will 50 *l.* gain in 5 Months.

$$\text{First I say if } \overset{l.}{\tfrac{40}{3}} : \overset{l.}{\tfrac{13}{12}} :: \overset{l.}{\tfrac{50}{1}}. \ \textit{Facit } 4\ l.\ 1\ s.\ 3\ d.$$

$$\tfrac{40}{3})\ \tfrac{650}{12}\ (\tfrac{65}{16} = 4\tfrac{1}{16}.$$

$$\text{Say again, If } \overset{Y.}{\tfrac{3}{4}} : \overset{l.}{\tfrac{65}{16}} :: \overset{Y.}{\tfrac{5}{12}}. \ \textit{Facit } 2\ l.\ 5\ s.\ 1\ d.\ 2\ q.\ \tfrac{2}{3}.$$

$$\tfrac{3}{4})\ \tfrac{325}{192}\ (\tfrac{325}{144} = 2\ l.\ \tfrac{37}{144}.$$

Question II. If 50 Pound in 5 Months gain 2 *l.* 5 *s.* 1 *d.* 2 *q.* $\frac{2}{3}$ or 2 *l.* $\frac{37}{144}$, what time will 13 *l.* 6 *s.* 8 *d.* or 13 *l.* $\frac{1}{3}$ require to gain 1 *l.* 1 *s.* 3 *d.* or 1 *l.* $\frac{1}{12}$?

$$\text{First say if } \overset{l.}{\tfrac{50}{1}} : \overset{yr.}{\tfrac{5}{12}} :: \overset{l.}{\tfrac{40}{3}}, \ \textit{Facit } \overset{yr.}{\tfrac{25}{16}}, \text{ or 1 Year and } \tfrac{9}{16}.$$

Say again, If $\frac{325}{144} : \frac{25}{16} :: \frac{13}{12}$, *Facit* $\frac{468}{624}$, or $\frac{3}{4}$ of a Year, or 9 Months.

Note, The former Proportion was Inverse, and the 2d was Direct. This shall suffice for the *Golden Rule in Fractions.*

Questions to exercise Vulgar Fractions.

Question I. The difference of two Numbers is $21\frac{75}{97}$, the lesser is $17\frac{3}{7}$, what is the greater? *Answer* $39\frac{137}{679}$, found by *Addition.*

Question II. There is in 3 Bags 56 *l.* $\frac{3}{5}$; in the first Bag 12 Pound and $\frac{5}{8}$; in the 2d, 21 $\frac{7}{12}$, what is the 3d Bag? *Answer* 22 $\frac{47}{120}$, found by *Addition* and *Subtraction.*

Question III. What Number added to $11\frac{5}{7}$ will produce 36 $\frac{1}{11}$? *Answer* $24\frac{29}{77}$, found by *Subtraction.*

Question IV. What is $\frac{7}{11}$ of $\frac{19}{37}$? *Answer* $\frac{133}{407}$, found by *Multiplication.*

V. What Number multiplied by $\frac{3}{7}$ produceth $11\frac{9}{17}$? *Answer* $26\frac{46}{51}$, found by *Division.*

Arithmetical

Arithmetical Progression.

PROGRESSION consisteth of two Parts, *Arithmetical* and *Geometrical*.

Arithmetical Progression, is when a Rank of Numbers above Two, Increase or Decrease equally, by the continual Addition or Subtraction of some equal Number.

So 1, 3, 5, 7, 9, 11, and 42, 35, 28, 21, 14, 7, are two Ranks of Numbers in *Arithmetical Progression*; the first increasing by the continual Addition of Two, and the second decreasing by the continual Subtraction of Seven; and so of any other.

In *Arithmetical Progression* these Five things are to be considered.

(1.) The first Term commonly the least Term.
(2.) The last Term commonly the greatest.
(3.) The Number of Terms.
(4.) The equal Difference or common Excess.
(5.) The Sum of all the Terms or total Aggregate.

Any Three of these Five being given, the other two may be found, which will admit of 20 Propositions, as may be seen in Mr. *Oughtred's Clavis Mathematica*, Chap. 19. Prob. 4. Either in the *Latin* or Late *English* Translation: But we shall not concern our selves with them all, but only such as may be of common Use.

But in the first place we will lay down some *Theorems*, for the better Understanding of what follows after.

THEOREM I.

Any Term of an *Arithmetical Progression*, contains the first (that is the least) Term, together with the Product of the common Excess and Number of Terms before it.

So in this *Arithmetical Progression*, 2, 5, 8, 11, 14, 17, The Term 17 is equal to the first Term Two, added to the

Product of 5, the preceding number of Terms by 3, the common Excess.

Hence may arise this *Corollary.*

That if the Common Excess be multiplied by the Number of Terms *Minus* Unity, and to the Product the least Term be added, the Sum is equal to the greatest.

THEOREM II.

If Three Numbers be in *Arithmetical Progression*; the double of the Mean is equal to the Sum of the Extreams.

So 2, 4, 6, are Three Numbers in *Arithmetical Progression*, and the double of the Mean 4, is equal to the Sum of the two Extreams 2 and 6.

THEOREM III.

If Four Numbers are in *Arithmetical Progression*, the Sum of the two Means is equal to the Sum of the two Extreams.

So 7, 11, 15, 19, are Four Numbers in *Arithmetical Progression*, and the Sum of the Two Means, 11 and 15, is equal to the Sum of the two Extreams 7, and 19.

THEOREM IV.

In any *Arithmetical Progression*, any Term doubled is equal to the Sum of any other two Terms equally Distant:

EXAMPLE.

3, 8, 12, 18, (23) 28, 33, 38, 43.

In the annexed *Arithmetical Progression*, the double of 23 is equal to the Sum of 3, and 43, or of 8 and 38, or of 13, and 33, or of 18, and 28, all Numbers which are equally Distant.

THEOREM V.

In any *Arithmetical Progression*, the Sum of any two Terms, is equal to the Sum of any other two Terms of like Distance from them.

EXAMPLE.

8, 11, 14, 17, 20, 23, 26, 29.

In the annexed *Progression*, the Sum of 14 and 23, is equal to the Sum of 8 and 29, or of 11 and 26, or of 17 and 20; all being alike Distant.

THEO-

THEOREM VI.

1. In any *Arithmetical Progression* whatsoever, if the Sum of the greatest and least Terms, be multiplied by the Number of Terms, and the Product divided by 2, the Quotient is equal to the Sum of all the Terms.

2. Or if the Sum of the greatest and least, be multiplied by ½ the Number of Terms, the Product is equal to the Sum of all the Terms.

3. Or if the half Sum of the greatest and least Terms, be multiplied by the Number of Terms, the Product is equal to the Sum of all the Terms.

4. Or the middle Number (when the Progression is odd) multiplied by the Number of Terms, gives the Sum of all the Terms.

EXAMPLE.

3, 6, 9, 12, 15, 18, 21.

(1.)	(2.)	(3.)	(4.)
21	21	12 ½ Sum.	12
3	3	7	7
24	24	84 Sum.	84 Sum.
7	3.5		
168	120	Every way the same.	
½ 84 Sum.	72		
	84|0 Sum.		

THEOREM VII.

In a Progression of natural Numbers, as 1, 2, 3, 4, &c. if the last Term be multiplied by the next greater, one half of the Product is equal to the Sum of the whole Progression.

1, 2, 3, 4, 5, 6, 7.

So the Product of 7 by the next greater 8, gives 56, the half of which is 28, which is the Sum of the whole Progression.

THEOREM VIII.

In a natural Progression of odd Numbers, as 1, 3, 5, 7, &c. the Sum of the whole is equal the Square of the Number of Terms.

1, 3, 5, 7, 9, 11, 13.

The Number of Terms 7 squared is 49, the Sum of the whole.

THEOREM IX.

In a *Natural Progression* of even Numbers, the Sum of the whole, is equal to the Product of the Number of Terms, by the Number of Terms *Plus Unity.*

2, 4, 6, 8, 10, 12.

Here the Number of Terms is 6, which multiplied by 7, gives 42, equal the Sum of the whole.

THEOREM X.

In any *Arithmetical Progression* whatsoever, if from the greatest Term the least be taken, the Remainder divided by the common Excess, and to the Quotient adding Unity, you have the Number of Terms.

2, 4, 6, 8, 10, 12, 14.

From 14 subtracting 2, rest 12, divided by the common Excess 2, gives 6, to which add Unity, makes 7, equal to the Number of Terms.

THEOREM XI.

In any *Arithmetical Progression* whatsoever, if from the last Term the first Term be subtracted, and the Remainder divided by the Number of Terms *Minus Unity*, the Quotient is the common Excess.

3, 5, 7, 9, 11, 13.

From 13 subtracting 3, rest 10, which divided by 5, one less than the Number of Terms quotes 2, the common Excess.

Let these *Theorems* suffice, we will now return to where we left, in having any 3 of the 5 given, to find the other 2

PROP

PROP. I.

The first or least Term, the last or greatest Term, and the Number of Terms being given, to find the common Excess.

Or the first, second and third given, to find the fourth.

RULE.

From the second subtract the first, the Remainder, divided by the third *Minus Unity*, quotes the fourth.

By *Theorem* the first and the Corollary.

EXAMPLE.

A Man had 12 Sons, the youngest was 3 Years old, and the elder was 58, they increased in *Arithmetical Progression*, What was the common Difference of their Ages?

The 2d 58
The 1st 3

The 3d $-1=11$) 55 (5
55

0

Answer, They increased by five Years.

PROP. II.

The First, Second, and Third given, to find the Fifth.

RULE.

Multiply the $\frac{1}{2}$ Sum of the first and second, by the third, the Product is the fifth.

By *Theorem* the 6th, and third Way.

EXAMPLE.

A Man buys 17 Yards of Kersey in *Arithmetical Progression*, for the first Yard he paid two Shillings, or 24 Pence, and for the last Yard ten Shillings, or 120 Pence: What did the whole amount to?

The

The first Term	24
The last Term	120
Sum =	144
$\frac{1}{2}$ Sum	72
Number of Terms	17
	504
	72
	1224 the *Answer* in Pence.
	$\frac{1}{12}$ 10|2
Ans. 5 *l.* 2 *s.*	5 *l.* 2 *s.*

PROP. III.

The First, Second, and Fourth given, to find out the Third.

RULE.

From the second subtract the first, the Remainder divided by the fourth, the Quotient *Plus Unity*, is equal to the third: By *Theorem* the first and the *Corollary*.

EXAMPLE.

A Man going a Journey, his first Day's Travel was five Miles, his last Day's Travel was 35 Miles, he increased his Journey every Day three Miles, How many Days did he travel? *Ans.* He travell'd 11 Days.

The second	35
The first	5

3) 30 (10 + 1 = 11
3
00
00
0

PROP.

PROP. IV.

The Second, Third and Fourth given, to find the First.

RULE.

Multiply the fourth by the third *Minus Unity*, the Product subtracted from the second leaves the first.

EXAMPLE.

A Man in 6 Days went to *London* from *Manchester*, every Day's Journey was greater than the Day before by four Miles, his last Day's Journey was 40 Miles; What was the first? *Answ.* 20 Miles.

The 4th is 4
The 3d—1 is 5

The Product 20 which subtract from the 2d 40, leaves 20 the first Day's Journey.

40
20

20 the *Answer.*

PROP. V.

The First, Third and Fourth given, to find out the Fifth.

RULE.

From the Product of the third into the fourth, subtract the fourth, and to the Remainder add the double of the first, $\frac{1}{2}$ the Product of that Sum multiplied by the third, gives the fifth.

EXAMPLE.

A 100 Eggs are placed in a right Line a Yard distant one from another, and the first a Yard distant from a Basket: It is required to know how far one must go, before he bring the Eggs one by one into the Basket without breaking any?

The

The third	100	*Sir* Jonas Moore *makes the Distance run but* 10000 *Yds which is too little by* 100 *Yards.* Moore's Arith. p. 324. Last Edition.
The fourth	2	
	200	
The fourth	2	
Rest	198	
The doub. of the 1st.	4	
Multip. by the 3d =	20200	
	10100	

Answ. 10100 Yards, or 5 Miles and $\frac{3}{4}$ wanting 20 Yards.

PROP. VI.

The Second, Third and Fifth given, to find the First.

RULE.

Divide the fifth by the third, and from the Quotient subtract $\frac{1}{2}$ the Product of the fourth into the third, *Minus Unity*: The Remainder is the first.

EXAMPLE.

A Man is to receive 300 Pound at 12 several Payments, each Payment to exceed the former by four Pound, he is willing to bestow the first Payment, on any one that can tell him what it is.

What must the Arithmetician have for his Pains?

$$12)\ 300\ (25 \qquad 11$$
$$22 \qquad 4$$
$$3 \qquad 44$$
$$\tfrac{1}{2}\ 22$$

Ans. Three Pounds are the Workman's Wages.

Many more Propositions might have been added, but the foregoing are sufficient in most Cases; wherefore, we will begin with *Geometrical Progression.*

Geo-

Geometrical PROGRESSION.

GEometrical Progression is when a Rank of Numbers above two, Increase or Decrease by an equal *Ratio*; that is, by the continual Multiplication or Division of some equal Number.

So 2, 4, 8, 16, 32, 64, and 1215, 405, 135, 45, 15, 5, are two Ranks of Numbers in Geometrical Progression, the first ascending or increasing, by continually multiplying the foregoing Term or Number by 2, or by a double *Ratio*. And the second descending or decreasing by continually dividing the preceding Term by 3, or in a triple *Ratio*.

In any Geometrical Progression, the same things are to be considered as in Arithmetical Progression: As first, the first Term commonly the least. Secondly, the last Term commonly the greatest. Thirdly, the Number of Terms. Fourthly, the *Ratio* or Common Excess. Fifthly, the Total Sum of all the Terms.

But before we mention any Propositions, we will annex some Theorems, as preparatory thereunto.

THEOREM I.

If three Numbers be in Geometrical Progression, the Square of the Mean or Middle Number, is equal to the Product of the two Extreams.

EXAMPLE.

3, 9, 27, are three Numbers in Geometrical Progression, and the Square of 9, the Mean, is equal to the Product of 27 by 3, the two Extreams; and so in others.

THEOREM II.

If four Numbers be in Geometrical Progression, the Product or *Rectangle* of the two Means is equal to the Product of the two Extreams.

EXAMPLE.

3, 15, 75, 375, are four Numbers in Geometrical Progression, and the Product of the two Means, (*viz.*) of 15 by 15, is equal to the Product of 375 by 3.

S   This

This will likewise hold, if the four Numbers be discontinued, as in these four Numbers following, 6, 12 : : 18, 36, for the Product of 36 by 6, is equal to the Product of 18 by 12. And hence proceeds that excellent Rule in Arithmetick called, *The Rule of Proportion, Rule of Three*; or *Golden Rule*.

THEOREM III.

If any Term of an Arithmetical Progression be squared, it will be equal to the Product of any other two Terms of like Distance from that Term either way.

EXAMPLE.

3, 6, 12, 24, (48,) 96, 192, 384, 768.

In the annexed Geometrical Progression, the Square of 48 is equal to the Product of 768 by 3, or of 384 by 6, or of 192 by 12, or of 96 by 24; all being Terms equally distant.

THEOREM IV.

In any Geometrical Progression whatsoever, the Product of the two Extreams, is equal to the Product of any other two immediate Terms of like Distance from both.

EXAMPLE.

5, 20, 80, 320, 1280, 5120.

So in this Geometrical Progression, the Product of the two Extreams 5120 by 5, is equal to the Product of 1280 by 20, or of 320 by 80, all being a like Distance from both.

THEOREM V.

Any Geometrical Progression may be continued *ad Infinitum* upwards, and ascending by Multiplication, and downward or descending by Division, the *Ratio* or common Excess being given, that being your Multiplicator upwards, and your Divisor downwards; notwithstanding oftentimes, the Terms will not continue Integral Numbers, neither in the ascending or descending Part thereof, as hereafter declared.

EXAMPLE.

&c. $\frac{32}{9}$, $\frac{16}{3}$, 8, 12, 18, 27, $\frac{81}{2}$, $\frac{243}{4}$, &c.

So in the annexed Progression, if 8, 12, 18, were to be continued infinitely forward and backward, the common Excess

Excess being 1 $\frac{1}{2}$ or $\frac{3}{2}$, suppose forward 8 first, I multiply 18 by $\frac{3}{2}$, gives 27 for the next Term, and 27 multiplied by $\frac{3}{2}$ gives $\frac{81}{2}$ for the next Term, and here the Integral Parts or Terms cease, and multiplying $\frac{81}{2}$ by $\frac{3}{2}$ gives $\frac{243}{4}$, and so as far as you please: Then in the descending Part, if I divide 8 by $\frac{3}{2}$ the Quotient is $\frac{16}{3}$ for the next descending Term, and that by $\frac{3}{2}$ gives $\frac{32}{9}$, and so on as far as you please.

THEOREM VI.

Any Geometrical Progression where the *Ratio* is *Multiple* (that is, where the greater Term is exactly measured by the less) may be continued upwards, *ad infinitum* in integral Numbers, but downwards sometimes not so far as Unity.

EXAMPLE I.

1, 2, 4, 8, 16, 32, 64, &c.

In the annexed Progression the *Ratio* or Common Excess being two, by which multiplying any Term, as 8 gives 16, and that by 2 produceth 32, and that by 2 gives 64, and so *ad infinitum* in Integral Numbers; and in descending, it will come down as far as Unity, for 8 divided by 2 quotes 4, and that by 2 gives 2, and 2 by 2 gives 1, then Integers cease,

EXAMPLE II,

$\frac{1}{2}$ 3, 6, 12, 24, 48, 96, &c.

But in this Progression though the Terms may be continued upward *ad infinitum*, as in the last, yet it will not descend so far as Unity without a Fraction, because 3 cannot be divided by the *Ratio*, which is 2 without a Remainder.

THEOREM VII.

In any Geometrical Progression, if the *Ratio* be not *Multiple*, the same can neither be continued upward *ad infinitum* in Integral Numbers, nor downwards so far as Unity.

$\frac{81}{4}$, 27, 36, 48, 64, $\frac{256}{3}$.

In the annexed Progression where the *Ratio* is ^{3}, the Terms quickly become mixt Numbers, both in the ascending and descending Part thereof; for seeing 64 and 27 cannot be multiplied or divided evenly by $\frac{3}{4}$, the Integral Terms cease.

THEOREM VIII.

In any Geometrical Progression, if the Extreams be Prime Numbers one to another, the same *Progression* can be continued no farther, either upwards or downwards in Integral Numbers; so in the last Example, supposing 27 and 64 to be the Extream Terms, and they being Prime one to another, therefore they can be continued no farther either way in Integral Numbers.

Numbers are said to be Prime one to another, when only Unity is their Common Measure.

THEOREM IX.

In any *Geometrical Progression* proceeding from Unity, the second Term (the first Term not being computed) the 4th, 6th, and 8th Term, and all the following Terms whose Exponents may be divided by 2, are Square Numbers; The 3d, 6th, 9th, and all the following Terms, whose Exponents may be divided by 3, are Cube Numbers. The 6th, 12th, 18th, and the following Terms, whose Exponents may be divided by 6, are both Square and Cube Numbers. The 5th, 7th, 11th, 13th, and all the following Terms, whose Exponents are Prime Numbers, are neither Square nor Cube Numbers.

Exponents are a Series of Natural Numbers proceeding from Unity, shewing the Places of the Terms of the Progression.

EXAMPLE.

Numbers are said to be Prime which Unity only measureth.

0	1	2	3	4	5	6	7	8
1	2	4	8	16	32	64	128	256

This Example needs no Explication.

Note, That in this and some following Theorems, whether proceeding from Unity or not, it being Commodious, we have annexed their Indices or Exponents, placing a Cypher over the first Term of the Progression, whereby it may be seen, how far any Term is distant from Unity, or from the first Term if not Unity.

THE-

THEOREM X.

In any Geometrical Progression, proceeding from Unity, if any Term be squared or multiplied by it self, it will produce another Term of the same Progression doubly distant from Unity.

EXAMPLE.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
1 . 2 : 4 . 8 . 16 . 32 . 64 . 128 . 256

So in this Progression the Square of 8, the 3d Term, is equal to 64, which is the 6th Term, or doubly distant from the 1st, or Unity.

THEOREM XI.

In any Geometrical Progression proceeding from Unity, the Rectangle of any two Terms, is equal to that Term of the same Progression, signified by the Sum of the others Exponents.

EXAMPLE.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7
1 . 3 . 9 . 27 . 81 . 243 . 729 . 2187

In this Progression, the Product of the 3d and 4th Term, (*viz.*) of 81 by 27, or of the 5th and 2d Term, (*viz.*) of 243 by 9, is equal to the 7th Term of the same Progression, which is 2187, because the Sum of either of their Exponents makes 7.

THEOREM XII.

In any Geometrical Progression, not proceeding from Unity, if any Term be squared or multiplied by it self, and the Product divided by the first or least Term, the Quotient gives a Term doubly distant from the first.

EXAMPLE.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768

In the annexed Progression, if the 4th Term, (*viz.*) 48 be squared and divided by the first Term 3, the Quote is 768, which is the 8th Term, and doubly distant from the first.

THE-

THEOREM XIII.

In any Geometrical Progression not proceeding from Unity, if any two Terms be multiplied together, and the Product divided by the least or first Term, the Quotient will be equal that Term signified by the Sum of the other Exponents.

EXAMPLE.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7
2 . 6 . 18 . 54 . 162 . 486 . 1458 . 4374

In this Progression if the 2d and the 5th be multiplied together, and the Product divided by the least Term, the Quotient will be equal to the 7th Term, because the Sum of their Exponents make 7. *Note*, These 4 last *Theorems* are useful in finding any following Term of a Geometrical Progression, without producing all the immediate Terms.

THEOREM XIV.

In any finite Geometrical Progression, where the Ratio is double, the Difference of the greatest and least Term, is equal to the Sum of all the Terms, except the greatest.

EXAMPLE.

3 . 6 . 12 . 24 . 48 . 96 . 192

In this Progression, if from the greatest Term 192 we take the least Term 3, the Remainder 189 is the Sum of all, excepting the greatest.

THEOREM XV.

In any finite Geometrical Progression it holds,
As the Ratio, or common Excess Minus Unity :
Is to Unity : :
So the Difference of the greatest and least Term :
To the Sum of all, except the greatest.

EXAMPLE.

3 . 9 . 27 . 81 . 243 . 729 . 2187

So in the annex'd Progression, I say,
As the Ratio Minus Unity, (*viz.*) 2 :
Is to Unity, (*viz.*) 1 : :
So 2184 the Difference of the greatest and least :
To 1092 the Sum of all the rest.

COROL-

COROLLARY.

Hence it follows, That if the Ratio of any Geometrical Progression be double, the Difference of the greatest and least Term is equal to all the rest; if the Ratio be triple, the Excess or Difference is double the Sum of all the rest. If quadruple, triple. If quintuple, quadruple; and so on.

THEOREM XVI.

In any finite Geometrical Progression it holds,
As the Difference of the two greatest Terms:
Is to the greatest::
So the greatest Minus the least:
To the Total Sum of all, excepting the least.

EXAMPLE.

5 . 10 . 20 . 40 . 80 . 160

In this Progression, As 80 : to 160 : : So 155 : to 310, is the Sum of all but the least.

THEOREM. XVII.

In any Geometrical Progression whatsoever, Decreasing and Continued *ad Infinitum*, it holds,
As the common Difference Minus Unity:
Is to Unity::
So the first or greatest Term:
To the Sum of all the following Terms, *in Infinitum.*

EXAMPLE.

162 . 54 . 18 . 6 . 2 . $\frac{2}{3}$. $\frac{2}{9}$. $\frac{2}{27}$. $\frac{2}{81}$. $\frac{2}{243}$ &c.

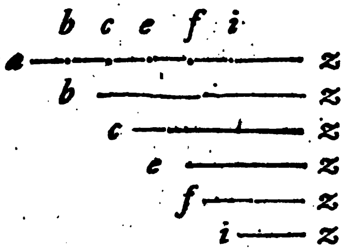
Let the first or greatest Term of an infinite decreasing Progression be 162, and let the Ratio be triple; then will the Terms descend, as in the *Example*: For 162 divided by 3, gives 54; and 54 by 3, quotes 18, and so on, as in the Table, and still farther, *ad Infinitum*. And it will follow,

That as 2: (*viz.* the Ratio Minus Unity) is to Unity, or 1 : : So is 162: the greatest or first Term to 81, which is the Sum of all the remaining Terms *ad Infinitum*. This appears plain by the 15th *Theorem*. Wherefore in any Geometrical Progression descending in any given Proportion, *ad*

ad infinitum, the least Term vanisheth; and therefore it holds as in this *Theorem*.

This may appear strange to many, how it should be possible to give the Summ of an infinite Progression in Numbers; whereas if the Work were actually begun, and the Terms continued, it would after a Thousand Years Labour; and after Thousands of Millions of Terms, be never nearer finishing. And yet that the Sum of this infinite Progression should so easily be found, it appear'd to me at first as a Notion (if I may so speak) almost Divine; but that it may be performed, take the following Demonstration.

Let there be several continual Proportionals, as *a z*, *b z*, *e z*, &c. all which transfer into the first *a z*, then will *a b*, *b c*, *c e*, *e f*, &c. be proportional Differences, which together with the last Quantity, *i z*, are equal to the first *a z*; because if that Proportional Number be continued downward *ad Infinitum*; the last Quantity, as said before, vanisheth. Therefore the infinite proportional Differences, are equal to the whole Line, *a z*; farther because it holds, that as *a z* to *b z*, so *b z* to *c z*, and so on; and



by Division, as *a b* to *b z*, so *b c* to *c z*. And by Conversion, as *a b* the first Difference to *a z* the first Quantity: So *b c* the second Difference to *b z* the second Quantity, and so on; Therefore as *a b* the first Difference to *a z* the first Quantity; So all the Differences to all the Quantities, that is to the whole Sum of all the infinite Quantities, which was to be demonstrated.

Hence may arise this *Corollary*.

That the first Term of an infinite descending Geometrical Progression, where the Ratio is double, is equal to the Sum of all the rest *ad Infinitum*.

But if the Ratio be triple, the first Term is double the Sum of all the rest; in a quadruple Progression, triple, in a quintuple one, quadruple, and so on. Hence we may demonstrate Unity not to be the beginning of Numbers.

THE

THEOREM. XVIII.

In any Geometrical Progression continued downward, *ad Infinitum*, it will be, as the Difference of the two first, or greatest Terms, is to the second Term : : So the first, or greatest Term to the Sum of all the rest *ad Infinitum*. So in the last Example the Difference of the two first Terms is 108, the second Term is 54, the first 162.

Wherefore as 108 : to 54 : : So is 162 : to 81, the Sum of all the rest, *ad Infinitum*.

Wherefore the Difference of the two first Terms, the first Term, and the Sum of the infinite Terms are Continual Proportionals, as was demonstrated in the last. Hence may arise this *Corollary*.

That when the two first, or greatest Terms differ only by Unity; the Square of the first Term is equal to the Sum of all the rest, *ad Infinitum*.

Many more *Theorems* might be laid down, but these are sufficient; we will only annex a *Proposition* or two, and so conclude both *Arithmetical* and *Geometrical Progression*.

PROP. I.

In any Geometrical Progression proceeding from Unity, the Ratio being known, how to find any remote Term without producing all the intermediate Terms.

RULE.

Find a few of the leading Terms, over which place their Exponents; then by *Theorem* 10. multiply the last found Term by it self, which will produce a Term double thereto. And this last multiplied by it self, produceth another Term doubly distant again: Thus do, till either you have the Term sought, or one that falls a little short; if so, multiply the Term last found, by that Term answering the difference of the Exponent of the last found Term, and that sought, this last Product, is the Term required, by *Theorem* 11.

EXAMPLE.

A Country Gentleman going to a Fair to buy Oxen, meets with a crafty Youth, who had a Company of very good Oxen, in Number 23. The Gentleman demanding the Price, was answered, He should have them for 16

Pound the piece, one with another. The Gentleman bids him 15 Pound *per* Piece, and take all. The young Spark tells him it would not be taken : But says he, If you will give me what the last Ox will come to, by doubling the whole Number by a Farthing, you shall have all ; to which the Gentleman Assents. The Question is, What the Gentleman paid for the Oxen ?

4 or 5 of the first Terms are easily got, as thus,

0 . 1 . 2 . 3 . 4 . 5 *Exponents.*
1 . 2 . 4 . 8 . 16 . 32 *Terms.*

Note, You need only to find that Term which will answer the Exponent 22, which will be the 23d Term ; because the Exponents are less by one than the Terms ; for in this Method we account not the first Term, which the Learner is desired carefully to observe.

So if I multiply the 5th Term, 32, by it self, it gives the 10th Term 1024, by *Theorem* 10. which multiplied again by it self gives 1048576, which is the 20 Term from the first ; but taking the first into the Number, is the 21st Term ; and seeing I want two Terms more, I multiply this last Product by the Term under the Exponent 2, which is 4, which gives 4194304, the last Term, and the Price of the Oxen in Farthings, which makes 4369 *l.* 1 *s.* 4 *d.* a great rate to pay for so many Oxen.

PROP. II.

In any Geometrical Progression not proceeding from Unity, the Ratio being known, and the first Term, to find any remote Term, without producing all the intermediate Terms.

RULE.

Find a few of the leading Terms, as in the last, and multiply the last by it self, and divide the Product by the first, or leading Term, the Quote gives a Term doubly distant from the 1st, by *Theorem* 12 ; and this again multiplied by it self, and divided by the first Term, gives a Term doubly distant from the last, by the same *Theor.* Thus do until either you have the Term sought, or one that falls a little short ; if so, multiply the last Term found, by that Term answering to the difference of their Exponents ; and this Product divided

ded by the first, or leading Term, quotes the Term required, by *Theorem* 13.

EXAMPLE.

A Nobleman dying left Ten Sons; to whom, and to his Executor he bequeathed his Estate, in Manner and Form following (*viz.*) *Imprimis*, To his Executor, in seeing his Will performed, he left 1024 Crowns; the youngest Son was to have as many, and half as many as the Executor; and so every Son to exceed the next younger, by the equal Ratio of 1 $\frac{1}{2}$. The Question is, what the eldest Son's Portion is?

Calculate 5 or 6 of the first Terms, as here we have found five.

Executor	Youngest	Next	Next	Next	Next
1024 .	1536 .	2304 .	3456 .	5184 .	7776
0 .	1 .	2 .	3 .	4 .	5 .

Then multiplying the 5th, 7776 by it self, it will produce 60466176, and this divided by 1024, the first Term, quotes the 10th Term, or what the eldest Son must have.

Here the Ratio being half triple, the difference of the greatest and least, is half double the Sum of all the rest, excepting the greatest. By *Theorem* 15.

If the whole Estate had been demanded, it may be found by *Theorem* 15, to be 175099 Crowns.

PROP. III.

First Number, Common Excess, and number of Places given, to find the the total Sum of all the Places.

RULE.

Find the last Term, as in the last Proposition; then from the greatest Term subtract the least, the Remainder divided by the common Excess Minus Unity, quotes the Sum of all, excepting the greatest, by *Theorem* 15, to which adding the greatest, gives the Sum of the whole.

Otherwise, or in other Words thus; the Difference of the greatest and least Terms divided by the Excess Minus Unity, the Quotient multiplied by the Excess, and to the Product adding the first Number, the Sums are equal the Total.

Or according to *Corollary* in *Theorem* 15. it holds, That if the Ratio of your Progression be double, the Difference of the greatest and least added to the greatest, gives the total Sum.

If the Ratio be triple, $\frac{1}{2}$ the Difference added to the greatest is the total. If the Ratio be quadruple $\frac{1}{3}$ of the Difference added to the greatest, is equal the total Sum of the rest. And so on.

EXAMPLE.

A Merchant having a soft young Man to his Son, Covetous enough, but scarce able to keep a Shop-Book, was minded to Purchase for him some considerable Lands in the Country; and bid him enquire out some handsome Estate that would be sold, and he would buy it for him: The young Man overjoy'd at the News, runs to an Inn, where he heard divers Country-Gentlemen lodg'd; and in all hast ask'd them if any of them would sell their Estate; most of them were very Angry, and near beating of him; but one of them being a facetious Gentleman, resolv'd to put a Trick upon him; and told him, That he had a neat Hall, with a goodly Park and Mannor, on the Bank of a pleasant River, and a great number of sufficient Tenants; all which, with the Royalty of a fair Market-Town, and the Patronage of a Parish-Church belonging thereto, should be his, upon Condition he would lay him down one Penny on the Threshold of the Porch-Door belonging to the Hall, Two-pence at the next Door, Four-pence at the 3d Door, and so on doubling, till he had gone thro' all the Doors, which were 64 in all. I'll have it, saith the young Man, and here's a Piece in Earnest; and in all hast tells his Father what a Purchase he had made, wishing him to give him a Hundred Pound, for that he thought could not but abundantly Satisfy. Thou Calf, quoth his Father, the King of *Spain's* Revenues would not pay what thou hast promised, if they were sold at 20 Years Value, much less can my Estate pay for thy Purchase, for it will not bring thee past

the

the 24th Threshold. The best is, the Gentleman knows thee not; and if he did, he could get no Advantage of one that has nought; but I'll warrant thee, he is making Merry with a Fool's Earnest. Now I desire to know what the Sum to be laid down on the 24th Threshold was, and what the whole, which he promised would have come to?

First, the Sum to be laid down on the 24th Threshold, by *Prop.* 1. will be found to be 8388608 Pence. And by this *Proposition* the Sum of the whole unto the 24th Threshold will be found to be 16777215 Pence, equal to 69905 *l.* 1 *s.* 3 *d.* which the Father must be worth, else he could not bring him over the 24th Threshold.

Secondly, the number to be laid down on the 64th Threshold, by the said first *Proposition*, is 9223372036854775808 Pence; and by this *Proposition*, the Sum of the whole, which the young Man should have given for the Purchase, will be 18446744073709551615 Pence, equal to 76861433640456465 Pound, 1 Shilling and 6 Pence; by which it may appear the Gentleman spoke within Compass, for this Sum would purchase the Yearly Rent of 3843071682022823 *l.* 5 *s.* 0 *d.* $\frac{1}{4}$, which is a great deal more than the King of *Spain*'s Revenues are worth: For supposing his Revenues were worth One Hundred Millions *per Annum* (which I think no Potentate of the Earth is worth) it would be no more considerable to the Sum last mentioned, than a Red-Herring of an Ounce Weight would be to the loading of 20 Ships of 50 Tunn Burden a piece, which may be thus demonstrated; for allowing 20 Hundred to the Tunn, the whole number of Ounces, equal to the Burthen of so many Ships of such Capacity, will be 35840000; and this number of Ounces multiplied by One Hundred Million, is only 3584000000000000, which is less than the aforegoing number by 259071682022823, which is a number large enough to load a great many more Ships.

EXAMPLE II.

What will a Horse cost by trebling the Nails in his Shoes (which are 32) with a Farthing?

Answer 965114681693 *l.* 13 *s.* 4 *d.*

See

See the Work.

Nails		The 8th Nail = 2187
1 =	1	Multiply by 2187
2 =	3	
3 =	9	15309
4 =	27	17496
5 =	81	2187
6 =	243	4374
7 =	729	
8 =	2187	4782969

Trebled is 14348907 = the 16th Nail
Multiply by the same 14348907

100442349
129140163
114791256
57395628
43046721
57395628
14348907

205891132094649
Trebled 617673396283947 = the 32 Nail

And the whole Sum will be 926510094425920 Farthings.

EXAMPLE III.

A Gentleman having a Coat and Wast-Coat with 12 Dozen of Silver Plate-Buttons. A Corn-man seeing it, and fansying it, demands of the Gentleman the Price thereof; who answered, If he would double every Button with a Barley-Corn, proceeding from the first gradually to the last, it should be his. To which the Baker Assents.

I demand the number of Barley-Corns, together with the Worth and Weight of the same?

Observe

Observe the following Work.

Buttons 1 = 1	the 9th But. = 256	18th But. = 131072
2 = 2	256	131072
3 = 4		
4 = 8	1536	262144
5 = 16	1280	917504
6 = 32	512	131072
7 = 64		393216
8 = 128	65536	131072
9 = 256	131072	
		17179869184
	The 36th Button =	34359738368
		34359738368
		274877906944
		206158430208
		103079215104
		274877906944
		103079215104
		240518168576
		309237645312
		171798691840
		103079215104
		137438953472
		103079215104
		1180591620717411303424
	The 72d Button =	2361183241434822606848

Which last Number must be multiplied by it self, and then by the common Excess, and so you will have what the last Button will amount to.

See

See the rest of the Work.

The 72 Button = 2361183241434822606848
2361183241434822606848

18889465931478580854784
9444732965739290427392
18889465931478580854784
14167099448608935641088
14167099448608935641088
4722366482869645213696
4722366482869645213696
18889465931478580854784
9444732965739290427392
7083549724304467820544
9444732965739290427392
2361183241434822606848
9444732965739290427392
4722366482869645213696
7083549724304467820544
18889465931478580854784
2361183241434822606848
2361183241434822606848
14167099448608935641088
7083549724304467820544
4722366482869645213696

5575186299632655785383929568162090376495104 Button
11150372599265311570767859136324180752990208 = 144

22300745198530623141535718272648361505980415 T. Sum

Which last Number is the exact Quantity of Barley, which the whole 12 Dozen of Buttons will amount to. Now for the Worth.

An Ounce *Averdupoise* hath been exactly weighed, and found to contain 681 Grains of Barley; therefore a Pound *Averdupoise* would contain 10896 Grains: And seeing a Bushel of the same Barley did weigh 50 Pound, the Grains in a Bushel will be 544800. Wherefore dividing the whole number of Barley-Corns by 544800, the number of Bushels wi[ll]

will be as here 4093383011477720840957363853327613-09665, and above $\frac{3}{4}$, and esteeming Barley at 2 Shillings the Bushel, the Value of the whole Quantity of Barley will be 4093383011477720840957363853327613 0966 *l.* 11 *s.* and 11 *d.* $\frac{1}{2}$, which in Words at length is, Four Millions of Millions of Millions of Millions of Millions of Millions, Ninety three thousand three hundred eighty three Millions of Millions of Millions of Millions of Millions, Eleven thousand four hundred seventy seven Millions of Millions of Millions of Millions, Seven hundred twenty thousand eight hundred fourty Millions of Millions of Millions, Nine hundred fifty seven thousand three hundred sixty three Millions of Millions, Eight hundred fifty three thousand two hundred seventy six Millions, one hundred and thirty thousand nine hundred sixty six Pound, Eleven Shillings, and Eleven Pence half-Penny. Which Sum is so vastly great, that if the whole Globe of the Earth and Sea, with whatsoever is contained on or therein, were converted into solid Gold, and coined into Guineas of equal quantity with those we now have, and to be valued at 30 *s.* *per* Piece; a hundred of such Guineas would come as near purchasing all the Land upon the Face of the whole Earth, as the said Quantity of Guineas would purchase all that Barley, which may seem as a Paradox, yet may easily be demonstrated to be true.

For suppose every Degree of the Meridian Circle answer to 80 English Miles upon the Earth, which Supposition is too much; none yet having accounted above 73; and Mr. *Norwood* by Experiment found only 69, and something above $\frac{1}{2}$ to answer to a Degree on the Earth; but supposing 80 Miles, that we may not take too little, the Circumference of the Earth in Miles is 28800, and in Inches is 1824768000, and the Solidity is 106565851563063758-35315840 Inches; and computing Guineas at one Pound ten Shillings *per* Piece, and to weigh five Penny Weight nine Grains, as they ought to do, a solid Inch of Gold would be worth 55 *l.* 7 *s.* but according to the account concerning the Value of Gold given by Sir *Jonas Moore* in his *Mathematical Compendium*, p. 16. a solid Inch of Angel Gold (which is the best) will be worth 33 *l.* 16 *s.* 4 *d.* by which we may see how Guineas are advanced above the worth, but taking them according to the greater Rate, the worth of the whole Globe of the Earth, converted into such

Gold, will be 5898702283522221145586952830 Pound 19 Shillings and 6 Pence.

And according to the former Computation the square Miles on the Face of the whole Earth will be 26401904614, one third of which being allowed for Seas, the Remainder will be 176012697.6 square Miles: And seeing a square Mile contains 640 square Acres, the Number of square Acres on the Face of the Earth will be 112648126464, and valuing an Acre at 20 Shillings, which is too much, accounting one with another, the worth will be 225296252928o Pounds, which may near as soon be purchased with a 100 Guineas, as the Barley before named with the whole Quantity of the said Gold.

Nay, if we suppose the Earth and Seas and all contained therein were converted into fine Sand, the Number of Grains of Sand would far come short of the aforesaid Number of Barley Corns, so that the Bulk of Barley exceeds some Millions of our Earth we live upon, if 'twere possible to be brought into one place.

And, Lastly, If the weight be considered, seeing a Bushel weighs 50 Pound, the weight of the whole will be 2046693 15057288604204786819266380654832594 Pound, 12 Ounces, 3 Drams $\frac{1}{2}$. All this may seem impossible to any but an Accountant, who is the best Judge of the great and almost incredible Power of Numbers.

In the last place we will annex a Table of *Geometrical Progression* fitted to the last Question, whereby any Question of *Geometrical Progression* proceeding from Unity, and in a *Duple Ratio*, may be resolved by Inspection if the Number of Terms exceed not 144.

The TABLE follows.

A Table

A Table of Geometrical Progression, proceeding from Unity, and continued to 144 Places, the *Ratio* or common Excess being Two.

1	1	36	34359738368
2	2	37	68719476736
3	4	38	137438953472
4	8	39	274877906944
5	16	40	549755813888
6	32	41	1099511627776
7	64	42	2199023255552
8	128	43	4398046511104
9	256	44	8796093022208
10	512	45	17592186044416
11	1024	46	35184372088832
12	2048	47	70368744177664
13	4096	48	140737488355328
14	8192	49	281474976710656
15	16384	50	562949953421312
16	32768	51	1125899906842624
17	65536	52	2251799813685248
18	131072	53	4503599627370496
19	262144	54	9007199254740992
20	524288	55	18014398509481984
21	1048576	56	36028797018963968
22	2097152	57	72057594037927936
23	4194304	58	144115188075855872
24	8388608	59	288230376151711744
25	16777216	60	576460752303423488
26	33554432	61	1152921504606846976
27	67108864	62	2305843009213693952
28	134217728	63	4611686018427387904
29	268435456	64	9223372036854775808
30	536870912	65	18446744073709551616
31	1073741824	66	36893488147419103232
32	2147483648	67	73786976294838206464
33	4294967296	68	147573952589676412928
34	8589934592	69	295147905179352825856
35	17179869184	70	590295810358705651712

71	1180591620717411303424
72	2361183241434822606848
73	4722366482869645213696
74	9444732965739290427392
75	18889465931478580854784
76	37778931862957161709568
77	75557863725914323419136
78	151115727451828646838272
79	302231454903657293676544
80	604462909807314587353088
81	1208925819614629174706176
82	2417851639229258349412352
83	4835703278458516698824704
84	9671406556917033397649408
85	19342813113834066795298816
86	38685626227668133590597632
87	77371252455336267181195264
88	154742504910672534362390528
89	309485009821345068724781056
90	618970019642690137449562112
91	1237940039285380274899124224
92	2475880078570760549798248448
93	4951760157141521099596496896
94	9903520314283042199192993792
95	19807040628566084398385987584
96	39614081257132168796771975168
97	79228162514264337593543950336
98	158456325028528675187087900672
99	316912650057057350374175801344
100	633825300114114700748351602688
101	1267650600228229401496703205376
102	2535301200456458802993406410752
103	5070602400912917605986812821504
104	10141204801825835211973625643008
105	20282409603651670423947251286016
106	40564819207303340847894502572032
107	81129638414606681695789005144064
108	162259276829213363391578010288128
109	324518553658426726783156020576256
110	649037107316853453566312041152512

111	1298074214633706907132624082305024
112	2596148429267413814265248164610048
113	5192296858534827628530496329220096
114	10384593717069655257060992658440192
115	20769187434139310514121985316880384
116	41538374868278621028243970633760768
117	83076749736557242056487941267521536
118	166153499473114484112975882535043072
119	332306998946228968225951765070086144
120	664613997892457936451903530140172288
121	1329227995784915872903807060280344576
122	2658455991569831745807614120560689152
123	5316911983139663491615228241121378304
124	10633823966279326983230456482242756608
125	21267647932558653966460912964485513216
126	42535295865117307932921825928971026432
127	85070591730234615865843651857942052864
128	1701411834604692317316873037158841[illegible]28
129	340282366920938463463374607431768211456
130	680564733841876926926749214863536422912
131	1361129467683753853853498429727072845824
132	2722258935367507707706996859454145691648
133	5444517870735015415413993718908291283296
134	10889035741470030830827987437816582766592
135	21778071482940061661655974875633165533184
136	43556142965880123323311949751266331066368
137	87112285931760246646623899502532662132736
138	174224571863520493293247799005065324265472
139	348449143727040986586495598010130648530944
140	696898287454081973172991196020261297061888
141	1393796574908163946345982392040522594123776
142	2787593149816327892691964784081045188247552
143	5575186299632655785383929568162090376495104
144	11150372599265311570767859136324180752990208

Take an Example or Two in the Uſe of the aforegoing Table.

Q U E S T. I.

What will 20 Pieces of Cloth coſt by doubling every Piece by a Farthing ? And ſuppoſing every Piece to contain 50 Yards;

Yards, and each Yard worth 1 *l.* 1 *s.* 8 *d.* what will the Difference be in the Price?

RULE.

Subtract Unity from the 21st Number, gives 1048575 Farthings, which reduced, is 1092 *l.* 5 *s.* 3 *d.* 3 *q.*

And multiplying 20 by 50 gives 1000, the Number of Yards, which at 1 *l.* 1 *s.* 8 *d.* the Yard, will amount to 1083 *l.* 6 *s.* 8 *d.* which subtracted from the former Number, gives 8 *l.* 18 *s.* 7 *d.* 3 *q.* the Difference sought.

QUEST. II.

A certain Man whose Daughter was Married on *New Years-Day*, gave her Husband towards her Portion One Shilling, promising to double it on the first Day of every Month for one whole Year; I demand what was her Portion?

Subtract an Unit from the 13th Number, being one Number more than the Number of Months, and the Remainder is 4095 Shillings, or 204 *l.* 15 *s.* the *Answer*. And thus of any other.

Or, Suppose the Progression be *Quadruple* (supposing the Progression proceeds from Unity) the last Number may be found by the Table by subtracting an Unit from the double Number of Terms given.

EXAMPLE.

What will 9 Packs of Broad Cloth cost by *Quadrupling* every Pack by one Shilling?

From the double Number of Terms subtracting an Unit, leaves 17, and the 17th Number in the Table is 65536, the Number of Shillings the last Pack will cost, and $\frac{1}{3}$ of the Difference of the last and first Term added to the last, gives 87381 Shillings, or 4369 Pounds, One Shilling, for the Price of the whole, by *Theorem* the 15th.

More Examples and more Uses might be shewn, but let these suffice.

The

The *Combination, Election, Permutation* and *Composition* of *Numbers* or *Quantities.*

Combination of Numbers is how oft a less number of Quantities may be taken out of a great Number of Quantities, without considering their Places.

As if 10 Letters of the Alphabet, *a . b . c . d . e . f . g . h. i. k* were given, and it were required to know how many Combinations of 2 Letters, as *ab . ac . ad, &c.* may be taken in the said 10 Letters; or how many Combinations of 3 Letters, as *abc . abd . abe, &c.* may be found in the same Letters.

EXAMPLE.

How many Combinations of two Letters in 8, (*viz.*) *a . b . c . d . e . f . g . h?*

First, It is easily seen *A* will combine with all the following Letters, *b . c . d . e . f . g . h*, from whence will arise 7 Combinations, to wit, *ab . ac . ad . ae . af . ag . ah.*

Secondly, *b* will be combined with all following it self, (but not with *a*, for *ba* is all one as *ab*) as *c . d . e . f . g . h*, whence will arise 6 Combinations, *viz. bc . bd . be . bf . bg . bh*; and so every Letter will combine with those following it self, as may be seen at large in the following *Table.*

In

In all 28 Combinations, which is the Anſwer.

ab . ac . ad . ae . af . ag . ah
bc . bd . be . bf . bg . bh
cd . ce . cf . cg . ch
de . df . dg . dh
ef . eg . eh
fg . fh
gh

And if to every Binary already found, be added its following Letters, it will produce the Ternaries, or all the Combinations of 3 Letters, as in the following Synopſis is evident.

abc . abd . abe . abf . abg . abh
acd . ace . acf . acg . ach
ade . adf . adg . adh
aef . aeg . aeh
afg . afh
agh

In all 56 Combinations.

bcd . bce . bcf . bcg . bch
bde . bdf . bdg . bdh
bef . beg . beh
bfg . bfh
bgh

Again, If to every Ternary already found, be added its following Letters, it will produce all the Combinations on 4 Letters in 8. And ſo of any other.

cde . cdf . cdg . cdh
cef . ceg . ceh
cfg . cfh
cgh

def . deg . deh
dfg . dfh
dgh

efg . efh
egh

fgh

But becauſe this may ſeem tedious in large Numbers, we have here exhibited another Method, whereby to find the Combinations in any given Numbers or Quantities with much Eaſe.

THUS,

Having placed the given Number of Quantities by it ſelf, decreaſe it gradually by an Unit ſo often as there are Quantities in the Combination, placing them one after another with a ſign of Multiplication betwixt them; which Num-

bers

ers must be multiplied into one another for a Dividend: then placing an Unit with the like Number of Places, decreasing by Unity with the Sign of Multiplication, betwixt each multiply them continually for a Divisor, and the Quotient will be the Number of Combinations sought.

EXAMPLE.

How many Combinations of 5 Letters in 10? *Facit.* 252.

$5 \times 4 \times 3 \times 2 \times 1$) $10 \times 9 \times 8 \times 7 \times 6$ (252

The Product of the Divisor is 120.

The Product of the Dividend is 30240.

And the Quotient will be 252.

Which will be the Number of Combinations of 5 Letters in 10.

EXAMPLE II.

A Country Farmer going to a Fair, makes a Bargain with a Moorlander for 50 Sheep, which were to be chosen out of 100; but he thinking him long in chusing them, tells the Farmer, that if he would give him a Farthing for every Parcel of 50 Sheep, which may be taken out of the said 100, he should have the whole Hundred, to which the Farmer assents. The Question is what they will cost?

If you Work according to the Rule last laid down, the Number of Combinations of 50 in 100 will be 100891306544874079257172497256, which Number of Farthings reduced into Pounds, Shillings, and Pence, will be 634284443175771658928888017 Pounds, 19 Shillings, 6 Pence; which Sum is so great, that if any Man were able to pay a Hundred Thousand Pound a Day, he would be above Sixty thousand Millions of Millions of Years in paying it: Such a Vanity may be concluded for want of Judgment.

Note, In any given Number of Quantities, the Number of Combinations increase gradually, till you come about the Mean Numbers, and so decrease gradually again. So in 8 Quantities, there are more Combinations of 3 and 5, than of 2 and 6, and more of 2 and 6, than of 1 and 7, as may be seen in the following Table.

Note farther, That if the Number of Quantities be even, $\frac{1}{2}$ the Number of Places shews the greatest Number of Combinations that can be made in those Quantities.

So if the Number of Quantities be 8, the $\frac{1}{2}$ of which is four, shews the greatest Number of Combinations in these Quantities will be of 4 in 8, as in the Table.

1		
2		
3		
4	In 8 =	
5		
6		2
7		

But if the Number of Quantities be odd, then those two Numbers which are next together, and whose Sum is equal to the given Number of Quantities, shew the greatest Number of Combinations; so of 7 Quantities the greatest Number of Combinations will be of 3 and 4 Quantities in 7, and are equal as in the Table.

1		
2		21
3	In 7 =	3
4		35
5		2
6		

Of Election of Quantities.

By *Election of Quantities* is meant, any Number of Quantities given, how many several ways I may take them without Respect had to their Places, as *A. B. C.* may be taken 7 Ways, *viz. a b c. ab. ac. bc* and *abc*.

The Election of Quantities may easily be found out by the Geometrical Table of Progression aforegoing; thus, to the given Terms add the Sum, Seek in the first Column of the Table, and from the Number over against it subtract an Unit, the Remainder is the Number of Elections sought.

EXAMPLE.

How many Elections are there of the Letters of the Alphabet?

Look in the first Column of the Table for 25, and over against it is 16777216 = the 24th Power of 2.

Subtract 1

Rest 16777215, the Number of Elections of 24 Letters.

EXAMPLE II.

How many Locks whose Wards differ, may be unlock'd with a Key of 8 several Wards?

Answ

Answer 255 Locks, 8 whereof may have one single Ward, 28 Double Wards, 56 Treble Wards, 70 four Wards, 56 five Wards, 28 six Wards, 8 seven Wards, and one Lock eight Wards.

Of Variation of Quantities.

By *Variation of Quantities* is meant how many several Ways any given Number of Quantities may be changed, as in respect to their Places.

As *a. b.* may be changed into *b a,* and *a. b. c.* may be changed 6 Ways (*viz.*) *abc. acb. bac. bca. cab. cba.*

EXAMPLE.

How many Changes may be Rung on five Bells?

RULE.

Multiply 1, 2, 3, 4, 5, one into another, the last Product is the Answer.

	1	Admits of no Variation.
So on 5 Bells may be Rung 120 Changes.	2	
	2	2 Admits of 2 Variations.
	3	
And on 6 Bells may be Rung 720 Changes.	6	3 Admits of 6 Variations.
	4	
	24	4 Admits of 24 Variations.
And thus of any other Number of Bells.	5	
	120	5 Admits of 120 Variations.

EXAMPLE II.

A Young Scholar, but an Arithmetician, coming into a Town for the Convenience of a good Library, demands of a Gentleman with whom he lodged, what his Diet would cost for a Year, the Gentleman asks him 10 *l.* the Scholar answered he was not certain what time he might stay, and would know what he must give him for his Diet so long as he could place his Family (consisting of 6 Persons besides himself) every Day at Dinner in a contrary Position? The Gentleman considering of it, and thinking it could not be

long, tells him he would allow him his Diet so long for 5 Pounds, to which the Scholar assents.

The Question is, what he gave for his Table *per Annum* ?

The Changes on 7 Quantities will be found to be 5040, which divided by 365, the Days in a Year, gives 13 Years .808 or 10 Months and 15 Days, and so long must he Board the Scholar, according to the former Conditions, which will hardly amount to 7 Shillings and 3 Pence *per Annum*.

EXAMPLE II.

How many several Combinations with their Variations are there of Three Letters of the *English* Alphabet ?

RULE.

Multiply all its several Combinations 2024 by its Changes 6, and the Product 12144 is the Number of placing three Letters of the Alphabet with all its Variations.

From this Mutability of Variations and Combinations, it is no Marvel that by 24 Letters, there ariseth and is made such Variety of Languages in the World, and such infinite Number of Words in each Language, seeing the Diversity of Syllables produceth that Effect; and also by Interchanging and placing of Letters amongst the Vowels and among themselves, make those Syllables; for the Alphabet of 24 Letters, may be varied thus many times (*viz.*) 620448401733239439360000.

Now if you take in the Combinations with the several Variations of 2, 3, 4, 5, 6, *&c.* Letters; there may be made and composed such a vast Number of Words, that if a Man could read 50 Thousand Words in an Hour, which is more than the *Psalms of David* contain (a Task too great for any Man to perform) and if there were a Hundred Thousand Millions of Men, they would not speak these Words according to the hourly Proportion, before mentioned, in a Hundred thousand Years; a thing seeming most impossible and incredible, yet most certain and infallible in Computation.

Hence likewise, it may appear how many ways the Letters of a Name or Word may be varied, and differently disposed by way of Anagram; out of which those of Use may be gathered, neglecting the rest, as for Example, the Word *Roma*; consisting of 4 different Letters, may admit of 24 Changes, as hereafter.

Rom

Roma	*Orma*	*Mroa*	*Arom*
Roam	*Oram*	*Mrao*	*Armo*
Rmoa	*Omra*	*Mora*	*Aorm*
Rmao	*Omar*	*Moar*	*Aomr*
Raom	*Oarm*	*Maro*	*Amro*
Ramo	*Oamr*	*Maor*	*Amor*

Of which these, to wit, *Roma, Ramo, Oram, Mora, Maro, Armo, Amor*; Are only useful, and all the rest useless.

But if there be two or more Letters of a Sort, divide the whole Number of Changes, by the Changes of the Number of those Letters, and the Quotient is the Number of Changes desired.

So if the Word *Philippa* were given, which consisteth of 8 Letters, of which (without considering those which are of the same sort) the Changes will be 40320; but because I is twice repeated, I divide 40320 by 2, the Changes on two Letters, the Quote is 20160, and this divided again by 6 the Changes on 3 Quantities, because *P* is thrice repeated, gives in the Quotient 3360, which are the Changes in the Word *Philippa.*

Or if I had divided 40320 by 12 (because 2 times 6 is 12) the Quotient would give in the *Answer*, at one Operation, the same as before.

After this manner may be found the Variations or Changes which may be made of some particular *Latin* Verses, so as to keep the Rules of a true Verse, and the Sense Grammatically the same.

So if the Two following Verses were chosen (*viz.*)

Lex, Rex, Grex, Res, Spes, Jus, Thus, Sal, Sol, (bona) Lux, Laus.

Mars, Mors, Sors, Fraus, Fex, Styx, Nox, Crux, Pus, (mala) Vis, Lis.

It is very remarkable how many sundry Ways the same may be varied, and yet the Sense remain Good, and the Verse Grammatically true; for if we suppose the Words *Bona* and *Mala* continually to keep the same (to wit, the 10th) place, the rest being 11 in Number, indifferently changing place with any other in the same Verse; the Number of Variations of 11 Places will be 39916800, which doubled for the Number of Changes in both Verses, makes 79833600, which would compose above 249 *Folio* Volumes, each Volume to contain 2000 Pages, every Page

Page divided into two Parts, and every Part to contain 80 Verses, which at a Penny the Sheet, would amount to 518 Pounds, 15 Shillings; and supposing them bound for 5 Shillings a Volume, which is not dear, the Binding would cost 62 Pound 5 Shillings; and the worth of the whole would be 581 Pound.

Of Composition of Quantities.

By *Composition of Quantities* is meant, when in any number of given Quantities, as in Letters or Figures, one row is joined with another row of the same, or with 2, 3 or more other rows, as in placing, and the Chances of the Dice.

This differs from *Combination* and *Election*, in that there one Quantity is taken but once, here as oft as there are Quantities to be taken.

Though this be the most Composed way, that it is not difficult to be performed; for if the Compositions of two Quantities in 10 (or any other Number) were sought, it is but squaring the given Number; if of three Quantities in 10, it is Cubing the given Number; if of 4 Quantities, its Biquadratick will fit, and so increase the Powers, according as your number of Quantities increase,

EXAMPLE.

What number of Compositions of 3 Letters in 20? *Facit* 8000, the Cube of 20.

EXAMPLE II.

What number of Combinations of 6 Letters in 24, or in the whole Alphabet? *Answer* 191102976 the Cubo-cubick of 24.

EXAMPLE III.

What number of Compositions of 24 Letters of the Alphabet, accounting them by 1 and 1, by 2 and 2, by 3 and 3, and so on to 24? If we account each time 24, the Answer would be 1333735776850284124449081472843776; But since we are to find all the Numbers preceding in Geometrical Progression under it; to perform which, observe the following *Rule*.

As

As the Ratio *minus* Unity : is to the first Term : So the whole Power of the Ratio *minus* Unity : to the Sum of all the Terms.

Thus Stated.

As 23 : to 24 :: So 1333735776850284124449081472,843775 : to 1391724288887252999425128493402200, which is the Number of Compositions sought.

How many several Chances are there on 2, 3, 4, 5, and 6 Dice?

Answer, on 2 Dice are 36, on 3 Dice 216, on 4 Dice 1296, on 5 Dice 7776, and on 6 Dice 46656 Chances.

Casts.		*Points.*		*Chances.*	*Sum.*
2	12	1	1	1	1
3	11	1	2	2	2
4	10	1	3	2	3
		2	2	1	
5	9	1	4	2	4
		2	3	2	
6	8	1	5	2	5
		2	4	2	
		3	3	1	
7		1	6	2	6
		2	5	2	
		3	4	2	

The foregoing Table shews the several particular Chances on two Dice. The Sum of the Chances of 2 . 3 . 4 . 5 . 6 Casts are 15, and of 12 . 11 . 10 . 9 . 8, are 15. Then add the Chances on 7, (*viz.*) 6, gives 36, the Chances on 2 Dice.

The particular Chances on 2 Dice, otherwise formed, whereby the following Observations may be made.

1.1	1.2	1.3	1.4	1.5	1.6
2.1	2.2	2.3	2.4	2.5	2.6
3.1	3.2	3.3	3.4	3.5	3.6
4.1	4.2	4.3	4.4	4.5	4.6
5.1	5.2	5.3	5.4	5.5	5.6
6.1	6.2	6.3	6.4	6.5	6.6

OBSER-

OBSERVATIONS.

Here are 36 Chances the Square of 6, as before; all the Chances of 6 are placed in the lowermost and furthermost row, whence observe, That in the Square of 5, (*viz.*) 25, there is no 6; So there are 25 Chances without 6, and 11 where there is a 6: So that it is above 2 to 1 whoever throws 2 Dies, throws not a 6. In the Square of 4, (*viz.*) 16, there is neither 5 nor 6. In the Square of 3, (*viz.*) 9, there is neither 4, 5 nor 6. In the Square of 2, (*viz.*) 4, there is neither 3, 4, 5 nor 6. This may be applied to other Chances.

A TABLE *of the Powers of* 6, *and the Numbers under it.*

1	2	3	4	5	6
Lat.	*q*	*c*	*qq*	*qc*	*cb*
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	4096
5	25	125	625	3125	15625
6	36	216	1296	7776	46656

By this Table may be accounted, how many several Chances there are on 2, 3, 4, 5 or 6 Dice, and how many there are where there is no 6, or neither 5 nor 6, or neither 4, 5 nor 6, or neither 3, 4, 5 nor 6.

EXAMPLE.

On 4 Dice there are 1296 Chances, 625 where there is no 6, 256 where there is neither 5 nor 6, 81 where there is neither 4, 5, nor 6, and 16 where there is neither 3, 4, 5, nor 6; but if it were demanded, on how many Chances on 4 Dice, there is a 6, I answer 671; for 1296 *minus* 625, is equal to 671.

And 1040 Chances have either a 5 or 6; for 1296 *minus* 256, is equal to 1040.

Likewise 1215 Chances have either a 4, 5 or 6; for 1296 *minus* 81, is equal to 1215.

And

And lastly, 1280 Chances have either a 3, 4, 5, or 6; for 1296, Minus 16, is equal to 1280. And so of any other in the Table.

To conclude, I shall here mention a small Treatise written not long since, entitled *Artificial Versifying*, shewing any one, tho' of ordinary Capacity, that can Write and Read, tho' he understand not a Word of *Latin*, how to make Thousands of *Hexameter* and *Pentameter* Verses, which shall be good *Latin*, true Verse and perfect Sense; and that in two Hours time.

Which is performed by a Select Number of *Latin* Words, artfully digested into Tables, the more to amuse the Reader. The Words are these which follow.

In the Table of *Hexameters* the Words are,

1. *Turbida, Ignea, Pessima, Horrida, Aspera, Martia, Barbara, Lurida, Effera.*

2. *Fata, Signa, Damna, Bella, Vincla, Sistra, Castra, Scuta, Tela.*

3. *Sequi, Fori, Pati, Tuis, Domi, Patet, Pato, Palam, Ferunt.*

4. *Præmonstrant, Proritunt, Promittunt, Protendunt, Producunt, Monstrabunt, Causabunt, Prænarrant, Promulgant.*

5. *Tempora, Pocula, Prœlia, Verbera, Lumina, Fœdera, Agmina, Crimina, Sidera.*

6. *Dura, Sæpe, Quædam, Acerba, Prava, Multa, Dira, Nigra, Sæva.*

Here if you take one Word out of each Line, you will have a true *Hexameter* Verse.

In the Tables of *Pentameters* the Words are,

1. *Tetrica, Ardua, Perfida, Improba, Sordida, Impia, Tristia, Turpia, Noxia.*

2. *Præstabunt, Præscribunt, Concludunt, Prædicunt, Perficiunt, Consummant, Conglomerant, Significant, Procurant.*

3. *Dura, Acta, Vina, Verba, Dicta, Facta, Labra, Arma, Astra.*

4. *Dolosa, Pudenda, Proterva, Nefanda, Cruenta, Superba, Molesta, Sinistra, Maligna.*

5. *Nova, Aliis, Tibi, Viris, Scio, Mera, Malis, Vides, Mihi.*

Here likewise if you take a Word out of every Line, you will have a true *Pentameter* Verse.

Now if it were required to find how many Verses may be composed out of the aforegoing Words, seeing every

Line hath 9 Words; I find the Cubo-cube of 9, becauſe there are 6 Lines, which will be 531441, and ſo many Verſes may be made out of the Tables of *Hexameters*, without taking Notice of the *Permutation* of Places; for you may change moſt of the 1ſt and the 6th Line, into the 5th and 2d Line, which Verſes will Compoſe 30 Volumes as big, or bigger than *Virgil*.

Likewiſe in the Tables of *Pentameters* may be compoſed 59049 Verſes, being the Quadricubick Power of 9; becauſe there are 5 Lines, and 9 Words in every Line.

But if the *Combinations* of 6 in 54 for the *Hexameters*, or 5 in 45 in the *Pentameters* were required, the Anſwer will be in the firſt 25827165, in the latter 1221756; but becauſe we are not to take two Words in a Line, theſe Numbers cannot be admitted.

And in the laſt Place, becauſe the Contrivance may pleaſe and divert the Reader, we will here annex the Tables themſelves, with the Manner of their Uſe, and ſo conclude this Rule.

The Tables for Hexameters.

TABLE I.

T	i	p	h	a	m	b	l	e	u
g	e	o	s	a	a	u	f	r	n
ſ	r	p	r	r	r	f	b	e	s
r	e	t	b	i	e	i	a	i	i
r	i	a	d	r	d		m	d	a
a	r	a	a	a		a	a		
a									

TABLE II.

F	s	d	b	v	s	c	s	t	a
i	a	e	i	i	a	c	e	t	g
m	l	n	s	s	o	l	a	n	n
l	c	t	t	r	a		a	a	a
l	r	r	t						a
a	a	a							

TABLE III.

s	f	p	t	d	p	p	p	f	e
o	a	u	o	a	u	a	e	q	r
t	i	m	t	t	l	r	u	i	i
s	i	e	o	a	u	i			
	t		m	n					
			r						

TABLE IV.

p	p	p	p	p	m	c	p	p	r
r	r	o	r	o	a	r	r	æ	o
o	r	o	n	u	æ	o	m	r	m
t	d	s	s	n	m	o	i	i	e
u	t	a	a	u	n	t	t	n	c
r	b	r	l	s	a	t	d	u	a
u	r	g	t	n	u	u	n	b	n
a	a	r	t	n	n	t	u	t	n
n	a		t	t		n		t	t
n					t				t

TABLE V.

T	p	p	v	l	f	a	c	s	e
o	r	e	u	œ	g	r	i	m	c
æ	r	m	d	m	i	d	p	u	l
b	i	e	i	m	e	o	l	i	e
n	r	n	i	r	r	a	a	r	a
a	a	n	a	a			a		
	a								

TABLE VI.

D	s	q	a	p	m	d	n	s	u
æ	u	c	r	u	i	i	æ	r	p
æ	e	a	l	r	g	v	a	e	d
r	v	t	a	r	a			a	b
a	a		a				m	a	

The Tables for Pentameters.

TABLE I.

T	a	p	i	l	i	t	t	n	e
r	e	m	o	m	r	u	o	t	d
r	p	r	p	i	r	x	r	u	f
r	d	i	ſ	p	i	i	a	i	o
i	a	t	i	a	c		d	b	d
	i	a		a		a	a	a	
a									

TABLE II.

p	p	c	p	p	c	c	ſ	p	r
r	o	r	e	o	o	i	r	æ	æ
n	æ	r	n	n	g	o	ſ	ſ	c
d	f	ſ	g	n	ç	t	c	l	i
i	u	l	i	u	a	r	u	c	ç
m	o	f	r	b	i	d	u	i	m
m	i	a	u	b	u	n	u	a	e
c	n	n	u	n	t	n	n	r	a
t	t	n	t		t	t	a	n	
	t					n	t		
					t				

TABLE III.

D	a	v	v	d	f	ſ	a	a	u
c	i	e	i	a	a	r	ſ	r	t
n	r	c	c	b	m	t	a	a	a
b	t	t	r	a	r				a
a	a	a		a					

TABLE

TABLE IV.

D	p	p	n	c	ſ	m	ſ	m	o
u	r	e	r	u	o	i	a	l	d
o	ſ	u	p	l	n	l	o	e	t
a	e	e	e	i	i	ſ	n	e	n
p	r	ſ	ſ	g	a	d	r	d	t
b	t	t	n		a	v	a	a	a
a	r	a			a				
a									

TABLE V.

n	a	t	v	ſ	m	m	v	m	o
l	i	i	c	e	a	i	i	v	i
b	r	i	r	l	d	h	a	i	i
i	o	a	i	e	i		s		s
		s	s						

The Uſe of theſe Tables are *very eaſy*;

For ſuppoſe we would compoſe an Hexameter and Pentameter Verſe.

For the Hexameter, chuſe any ſix of the nine Digits, as ſuppoſe 345648, and for the Pentameter, any five of the ſaid nine Digits; as ſuppoſe 23479.

Firſt, For the Hexameter take three, the firſt Figure thereof towards the Left-hand, and looking in the firſt Table, count till you come to the third Square, where you will find the Letter *P* for the firſt Letter of the firſt Word, then counting forward till you come to the ninth place, which falls in the ſecond Column, and 2d Square, where you will find the Letter *e* for the ſecond Letter of the firſt Word; and counting forward nine places more, which falls in the 3d Column and firſt Square, where you may find the Letter *ſ*, and ſo counting every time, nine Places from the Letter laſt found, you will have the Word *Peſſima*, which is the firſt Word. Then taking the ſecond Figure 4 in the ſaid Number, and proceeding with it in the ſecond Table, ac-

cording

cording to the former Directions, you will find the second Word to be *Bella*, and thus running through the first six Tables with the said six Figures, according to the former Directions, you will find the first Verse to be this that follows,

Pessima Bella Domi monstrabunt verbera nigra.

And, if after this manner, you proceed with the other five Figures in the Table of Pentameters, you will find the other Verse to be,

Ardua concludunt verba molesta mihi,

Note, If the ninth Place chance to be blank, you may know your Word is finished.

Note also, Your Verses may more readily be found thus; find the first Letter as before, and run the Squares diagonally downward towards the Left-hand for the remaining Letters, if the Diagonals be too few, the Remainder may be found by counting nine forward, and running down Diagonally as before; so in the first Word before found, I find *Pes*, in the three first Diagonals, and counting nine forward, and looking diagonally, *sima*, which put together makes *Pessima*. And thus of any other.

Decimal Arithmetick.

Numeration.

WHAT a Decimal Fraction is, was shewed in the Introduction, but for the Learner's Benefit we shall again repeat it,

A Decimal Fraction is such whose Denominator is not expressed but understood, and is an Unit with as many Cyphers annexed, as there are places in the Numerator. $\frac{5}{10}$ will be expressed thus, .5; and $\frac{25}{100}$ thus, .25; and $\frac{125}{1000}$ thus, .125; &c.

And

And they have commonly a Point or Comma prefixed to diſtinguiſh them from an Integer.

Note, A Cypher placed to the Left-hand of an Integer, or to the Right-hand of a Decimal, neither increaſeth nor decreaſeth the Value; but placed to the Right-hand of an Integer increaſeth the Value, and to the Left-hand of a Decimal decreaſeth it. *Obſerve the following Table.*

The Table of Numeration.

Hund. of Mill.	Tens of Mill.	Millions.	Hund. of Thouſ.	Tens of Thouſ.	Thouſands.	Hundreds.	Tens.	Units.		Tenth Parts.	Hund. Parts.	Thouſand Parts.	Ten Thouſ. Parts.	Hund. Thouſ. Parts.	Millions of Parts.	Ten Mill. of Parts.	Hund. Mill. of Parts.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
Integers.										Decimals.							

In this Table you may obſerve, that as Integers increaſe in a tenfold Proportion to the Left-hand, ſo Decimal Fractions decreaſe in a tenfold Proportion to the Right-hand.

So 5, 50, 500, 5000 is Five, Fifty, Five Hundred, Five Thouſand.

And .5, .05, .005, .0005 is 5 Tenth, 5 Hundred, 5 Thouſand, 5 Ten Thouſ. Parts.

Reduction in Decimals.

By Reduction we find the Decimal of any Fractional Parts of Coin, Weight, Meaſure, &c. And on the contrary, reduce any Decimal Fraction given, into its Equivalent Fractional Parts of Coin, Weight, Meaſure, &c.

PROP.

PROP. I.

Any Vulgar Fraction given, to reduce the ſame into a Decimal Fraction of equal Value. To perform which, the Proportion is,

As the Denominator of the Vulgar Fraction given : is to the Numerator thereof : :

So is an Unit, with Cyphers annexed at Pleaſure, to the Decimal Fraction required.

Or thus,

Add a competent Number of Cyphers to the Numerator, and divide by the Denominator, the Quotient is the Decimal Fraction required.

EXAMPLE I.

Let it be required to find the Decimal Fraction of $\frac{3}{4}$.

See the Work.

```
4) 3.000 (.75   Facit .75
     ..
    28
   ----
    20
    20
   ----
     0
```

Note, The Cyphers added, are to be diſtinguiſhed by a Point or Comma, and you may annex what Number you pleaſe; but you muſt take notice what Cyphers you make uſe of; for ſo many muſt be cut off in the Quotient ; and if at any time it happens, as ſometimes it will, there be not a ſufficient Number in the Quotient, they muſt be ſupplied, by adding Cyphers to the Left-hand.

EXAMPLE II.

Reduce $\frac{1}{8}$ into a Decimal.

8)

8) 1.0000 (.125 *Facit* .125

8

20
16

40
40

0

EXAMPLE III.

Reduce $\frac{35}{721}$ into a Decimal.

721.) 35.060000 (.048543

2884

6160
5768

3920
3605

3150
2884

2660
2163

497 Remainder.

In many Cases, as in this Example, though you should annex a thousand Cyphers, yet your Decimal Fraction will not come up, but there will still be a Remainder, but if we bring it 5 or 6 places after the Separatrix it will be exact enough in most Cases, and the Remainder may be thrown away as of no Value, for if you suppose this was the Fraction of a Pound Sterling, this Decimal will be so near the Truth, as if a Pound were divided into a Million of Parts, it would not Err from the Truth, so much as one of those Parts.

And seeing I made use of 6 Cyphers in the Operation, and but 5 Figures in the Quotient, I added a Cypher to the Left-hand before I made my Separatrix.

EXAMPLE IV.

Reduce 9 Pence into a Decimal Fraction.

Seeing 240 Pence make a Pound Sterling, 9 Pence is equal to $\frac{9}{240}$ which reduce as before.

```
240) 9.0000 (.0375  the Dec. of 9 d.
     720
     ----
     1800
     1680
     ----
      1200
      1200
     -----
         0
```

EXAMPLE V.

Reduce 11 Shillings into a Decimal.

```
11 Shill. is 11/20    20) 11.000 (.55 = to 11 Shill.
                          100
                          ---
                           100
                           100
                          ---
                            0
```

But the Decimal answering any Number of Shillings may more quickly be found by halving the Number of Shillings given.

So for 11 Shill. ½ of 11 is 5, and 1 remains, to which suppose a Cypher annexed makes 10, whose half is 5 as you see.

```
11 Shill
--------
55 Dec
```

So the Decimal of 12 Shillings is .6; of 15 Shillings is .75; of one Shilling is .05; and so of any other.

EXAM-

EXAMPLE VI.

What is the Decimal of 7 Shillings and 6 Pence?

The Decimal of 7 Shillings by the last, is .35.

Then 6 is $\frac{6}{240}$, which reduced, is .025, to which adding .35, gives .375.

.35 = 7 Shillings.
.025 = 6 Pence.

.375 = 7 Shillings and 6 Pence.

But if you consider this, the Decimal of any number of Pence may be found by taking Parts of the Shilling, of which they consist.

So .05 being the Decimal of 1 Shilling.
$\frac{1}{2}$ = .025 is the Decimal of 6 Pence.

And $\frac{1}{2}$ of the Decimal of 6 Pence, is the Decimal of 3 *d.*
.025 is the Decimal of 6 Pence.
$\frac{1}{2}$ = .0125 is the Decimal of 3 Pence.

The 3d Part of the Decimal of 3 Pence, is the Decimal of a Penny, to wit .004166—and the 4th part of that, is the Decimal of a Farthing, to wit, .0010416.

.35 = 7 Shillings
.025 = 6 Pence
So the Dec. of 7 *s.* 7 *d* $\frac{1}{2}$ is .38125 .00625 = 1 *d* $\frac{1}{2}$

38125 = 7 *s.* 7 *d.* $\frac{1}{2}$

In the former Examples of Money, we have supposed the Integer to be a Pound Sterling; but if the Decimal of 6 Pence were required, and the Integer to be a Shilling, then the Decimal would be .5; which if the Integer had been a Pound, would have been the Decimal of 10 Shillings; whereby you may see the Decimal alters according as we take our Integer; so if we account a Penny to be the Integer, the Decimal of 1 Farthing is .25; of 2 Farthings is .5; of 3 Farthings is .75.

And you may see and take Notice, that these three last Decimal Fractions, are general Fractions in any Case; for .25 is equal to $\frac{1}{4}$ of any thing, .5 is equal to $\frac{1}{2}$ of any thing, .75 is equal to $\frac{3}{4}$ of any thing, as of a Pound, Shilling, Penny, Yard, Hundred, &c.

For if a Pound Sterling be the Integer, .5 is 10 Shillings; if a Shilling be the Integer .5 is 6 Pence; if a Penny be the Integer, .5 is 2 Farthings; if a Yard be the Integer, .5 is $\frac{1}{2}$ Yard; and so of any of the rest, which being considered, may be of good Use to the Learner.

EXAMPLE VII.

Let it be required to find the Decimal answering 16 Penny Weight, one Pound *Troy* being the Integer. Seeing there is 240 Penny Weight in a Pound, 16 Penny Weight is $\frac{16}{240}$, which reduced by the Examples aforegoing, will be .0666—And so infinitely.

```
240) 16.00000 (.0666
     1440
     ----
      1600
      1440
      ----
       160
```

EXAMPLE VIII.

What is the Decimal of 11 Pen. Wt. 16 Grains? Bring 11 Pen. Wt. 16 Grains into Grains, which are 280 Grains; then because there is 5760 Grains in a Pound, 280 Grains will be $\frac{280}{5760}$, or $\frac{28}{576}$, by cutting off a Cypher in each, which reduced as before, will be .048611.

```
5760) 280.000000 (.048611
      23040
      -----
       49600
       46080
       -----
        35200
        34560
        -----
          6400
          5760
          ----
           6400
           5760
```

EXAM-

EXAMPLE IX.

What is the Decimal of 3 Quarters and 14 Pound, one Hundred, or 112 Pound being the Integer? *Answer* .875.

The Decimal of $\frac{3}{4}$ is, as was said before, .75, and 14 Pound is $\frac{14}{112}$, which reduced is .125.

112) 14.000 (.125		
112		
280	Unto	.75
224	Add	.125
560	*Facit*	.875
560		
0		

So the Decimal of 2 Pints, one Gallon the Integer, will be .25, 45 Minutes of an Hour is .75.

These *Examples* being understood and considered, are sufficient to reduce any other Weights and Measures into Decimals; so we will conclude this *Proposition*.

PROP. II.

To find the Value of any Decimal Fraction in the known Parts of the Integer, as of *Coin*, *Weight*, *Measure*; to perform which, observe the following *Rule*.

RULE.

Multiply the Decimal given, by the number of Parts of the next inferior Denomination, cutting off as many Figures from the Product, as the Decimal given consists of; the Remainder, if any, multiplied by the Parts of the next inferior Denomination, cutting off as before. Thus must you do till the Decimal given be brought into its least Parts, the Parts signified by the Decimal, will be thrown over the Separatrix.

EXAM-

EXAMPLE I.

What is the Value of .725 of a Pound Sterling?
20 Shillings in a Pound.

Shill. 14.500
12 Pence in a Shilling.

1000
500

Pence 6.000 *Facit* 14 *s.* 6 *d.*

EXAMPLE II.

What is the Value of .696875 of a Pound Sterling?
20 Shillings in a Pound.

Shill. 13.937500
12 Pence in a Shilling.

s. *d.* *q.*
Facit 13 11 1

1875000
937500

Pence 11.250000
4 Farthings in a Penny.

Farth. 1.000000

EXAMPLE III.

What is .72065 of a Pound Sterling?
20 Shillings in a Pound.

Shill. 14.41300
12 Pence in a Shilling.

82600
41300

Pence 4.95600
4 Farthings in a Penny.

Farthings 3.82400 remain less than a Farthing, and so not to be accounted of.

Note,

Note, But the Value of any Decimal Fraction of a Pound Sterling may be more easily found; thus, For the first Figure after the Separatrix, is a double number of Shillings; if the second Figure be 5, or above, for the 5 account one Shilling more; then the 2d Figure, if under 5, or the Excess, if above 5, added to the 3d, is so many Farthings, remembring to abate 1 from the Farthings, if the Sum be above 23, and 2 from the Farthings if the Sum be above 40.

EXAMPLE.

In the Decimal .76565, the first Figure 7 doubled, is 14, which are Shillings; and because the 2d Figure is above 5, subtract 5 from it, and account one Shilling more; and the 15 Farthings are 3 Pence and 3 Farthings: So the Value of it is 15 Shillings and 3 Pence 3 Farthings; as for the rest of the Figures, they being but the Fraction of a Farthing, are inconsiderable in Practice.

So the Value of .6666 of a Pound will be 13 Shillings and 4 Pence. Of .2065 will be 4 Shillings and 3 half Pence; and so of any other.

EXAMPLE IV.

What is the Value of .625 of a Pound *Troy* Weight?

	.625	
	12	Ounces in a Pound.
	1250	
Facit 7 *Oz.* 10 *pen. wt.*	625	
Ounces	7.500	
	20	Penny Wt. in an Ounce.
Penny Wt.	10,000	

EXAM-

EXAMPLE V.

What is the Value of .6725 of a Hundred Weight?

	4	Quarters in a Hundred.
Quarters	2.6900	
	28	Pounds in a Quarter.
	55200	
	13800	
Pounds	19.3200	
	16	Ounces in a Pound.
	19200	
	3200	
Ounces	5.1200	*Facit* 2 *q.* 19 *l.* 5 *oz.*

And the Value of .6125 of a Yard, is 1 Foot and 10 Inches. Of .725 of a Gallon, is almost 6 Pints.

But lest these ways of finding the Decimals of any Parts, as likewise the Value of any Decimal, should seem tedious, we have annexed Tables of the most eminent known Parts of *Money*, *Weight*, *Measure*, &c.

The Tables follow.

Decimal

Decimal Tables of Coin, Weight and Measure.

TABLE I.

English *Coin, one Pound the Integ.*

Shillings	Decimals	Shillings	Decimals
19	.95	9	.45
18	.9	8	.4
17	.85	7	.35
16	.8	6	.3
15	.75	5	.25
14	.7	4	.2
13	.65	3	.15
12	.6	2	.1
11	.55	1	.05
10	.5		

Pence	*Decimals.*
11	.045833
10	.041666
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016666
3	.0125
2	.008333
1	.004166

Farth.	*Decimals.*
3	.003125
2	.0020833
1	.0010416
½	.0005208
¼	.0002604

TABLE II.

English *Coin, a Noble the Integer.*

Shillings	*Decim.*
6	.9
5	.75
4	.6
3	.45
2	.3
1	.15

Pence	*Decimals.*
11	.1375
10	.1250
9	.1125
8	.1
7	.0875
6	.075
5	.0625
4	.05
3	.0375
2	.025
1	.0125

Farth.	*Decimals.*
3	.009375
2	.00625
1	.003125
½	.0015625
¼	.00078175

TABLE III.

English *Coin, one Shill. long Measure, one Foot the Integer.*

Pence / *Inches*	*Decim.*
11	.916666
10	.833333
9	.75
8	.666666
7	.583333
6	.5
5	.416666
4	.333333
3	.25
2	.166666
1	.083333

Farthings / *Q. Inches*	*Dec.*
3	.0625
2	.041666
1	.020833
½	.010416
¼	.0052083

TABLE IV.

Troy Weight, one Pound the Integer

Ounces the same as Pence in the last Table.

Pen. wt.	*Decim.*
19	.079166
18	.075
17	.070833
16	.066666
15	.0625

The rest of the Table.

14	.058333
13	.054166
12	.05
11	.045833
10	.041696
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016666
3	.0125
2	.008333
1	.004166

Grains	Decimals
23	.003993
22	.003819
21	.003646
20	.003472
19	.003298
18	.003125
17	.002951
16	.002778
15	.002604
14	.002431
13	.002257
12	.002083
11	.001910
10	.001736
9	.001562
8	.001389
7	.001215
6	.001042
5	.000868
4	.000694
3	.000521
2	.000347
1	.000173
½	.000086

TABLE V.

Averdupoiz wt. 112 lb. the Integ.

Quarter Hundred	Decimals
3	.75
2	.5
1	.25

Pounds	Decimals
27	.241071
26	.232143
25	.223214
24	.214286
23	.205357
22	.196428
21	.1875
20	.178571
19	.169643
18	.160714
17	.151785
16	.142857
15	.133928
14	.125
13	.116071
12	.107143
11	.098214
10	.089286
9	.080357
8	.071428
7	.0625
6	.053571
5	.044643
4	.035714
3	.026785
2	.017857
1	.008928

The rest of the Table.

Ounces	Decim.
15	.008370
14	.007812
13	.007254
12	.006696
11	.006138
10	.005580
9	.005022
8	.004464
7	.003906
6	.003348
5	.002790
4	.002232
3	.001674
2	.001116
1	.000558

Quar. Oz.	Decim.
3	.000418
2	.000279
1	.000139

TABLE VI.

Troy Weight, 1 Oz the Integer.

Penny-weight the same as Shillings in the first Table.

Grains	Decim.
23	.047916
22	.045833
21	.04375
20	.041666
19	.039583
18	.0375

The rest of the Table.

17	.035416
16	.033333
15	.03125
14	.029166
13	.027083
12	.025
11	.022916
10	.020833
9	.01875
8	.016666
7	.014583
6	.0125
5	.010416
4	.008333
3	.00625
2	.004166
1	.002083

TABLE VII.

Averdupoiz *wt. 1 lb. the Integer.*

Ounces	*Decimals.*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

The rest of the Table.

Drams	*Decimals*
15	.058593
14	.054687
13	.050781
12	.046875
11	.042968
10	.039062
9	.035156
8	.03125
7	.027343
6	.023437
5	.019531
4	.015625
3	.011718
2	.007812
1	.003906

TABLE VIII.

Liquid Measure one Gallon; dry Measure one qr. the Integer.

Pints	*Dec.*	*Bush.*
7	.875	7
6	.75	6
5	.625	5
4	.5	4
3	.375	3
2	.25	2
1	.125	1

Qr.Pts.	*D.*	*Pecks*
3	.09375	3
2	.0625	2
1	.03125	1

The rest of the Table.

Decim.	*qr.*	*Peck*
.0234375	3	
.015625	2	
.0078125	1	

Decimals	*Pints.*
.005859	3
.003906	2
.001953	1

TABLE IX.

Time; One Year the Integer.

Months the same as Pence in the third Table.

Days	*Decimals.*
30	.082191
29	.079452
28	.076712
27	.073972
26	.071233
25	.068493
24	.065753
23	.063014
22	.060274
21	.057534
20	.054794
19	.052055
18	.049315
17	.046575
16	.043836
15	.041096
14	.038356

The rest of the Table.	
13	.035616
12	.032876
11	.030137
10	.027397
9	.024657
8	.021917
7	.019178
6	.016438
5	.013698
4	.010959
3	.008219
2	.005479
1	.002739

TABLE X.

Time, One Day the Integer.

Hours	Decimals.
23	.958333
22	.916666
21	.875
20	.833333
19	.791666
18	.75
17	.708333
16	.66666
15	.625
14	.583333
13	.541666
12	.5
11	.458333
10	.416666
9	.375
8	.333333
7	.291666
6	.25
5	.208333
4	.166666

The rest of the Table.	
3	.125
2	.083333
1	.041666

Minutes	Decim.
59	.040972
58	.040277
57	.039583
56	.038888
55	.038194
54	.0375
53	.036805
52	.036111
51	.035416
50	.034722
49	.034027
48	.033333
47	.032638
46	.031944
45	.03125
44	.030555
43	.029861
42	.029166
41	.028472
40	.027777
39	.027083
38	.026388
37	.025694
36	.025
35	.024305
34	.023611
33	.022916
32	.022222
31	.021527
30	.020833
29	.021138
28	.019444
27	.01875
26	.018055
25	.017361

The rest of the Table.	
24	.016666
23	.015972
22	.015277
21	.014583
20	.013888
19	.013194
18	.0125
17	.011805
16	.011111
15	.010416
14	.009722
13	.009027
12	.008333
11	.007638
10	.006942
9	.00625
8	.005555
7	.004861
6	.004166
5	.003472
4	.002777
3	.002083
2	.001388
1	.000694

TABLE XI.

Cloath Measure, One Yard the Int.

Quarters	Decim.
3	.75
2	.5
1	.25

Nails	Decimals.
3	.1375
2	.125
1	.0125

The Use of the Tables.

Concerning the Construction of these Tables, we need to say nothing, that being sufficiently shewn in the first nine Examples of this Rule, but proceed to their Use in a Proposition or two, and so conclude this Rule.

PROP. I.

To find the Decimal answering any Fractional Part of Coin, Weight, Measure, *&c.*

This is for the most part given by Inspection, or at most by a single Addition of two or three Numbers.

EXAMPLE I.

What is the Decimal answering 17 Shill. one *l.* being the Integer?

Seek in the first Table for 17 Shillings, in the Tablet of Shillings, and against it in the Column adjoyning is .85; the Decimal required. So the Decimal of 5 Shillings, one Noble being the Integer, by the second Table is found to be .75; and the Decimal of 8, one Shilling being the Integer, by the third Table is .666666; which is likewise the Decimal of 8 Inches, or 8 Months, a Foot being the Integer in the one, and a Year in the other; likewise the Decimal of 21 Pounds by the fifth Table, will be .1875; and so of any other.

EXAMPLE II.

What is the Decimal answering 7*s.* 9*d.* 1 *q.* a Pound Sterling being the Integer?

By the First Table	7 Shillings is	.35
	9 Pence is	.0375
	1 Farthing is	.001041
	The Decimal of 7 *s.* 9*d.* 1 *q.* is =	.388541

So the Decimal of 3 *qr.* 7 *lb.* 8 *oz.* by the fifth Table, will be found to be .816964.

For the Decimal of 3 Quarters is	.75
of 7 Pound is	.0625
of 8 Ounces is	.004464
of 3 *qr.* 7 *lb.* 8 *oz.* is	.816964

After

After the ſame manner the Decimal of 3 *s.* 9 *d.* a Noble the Integer, by the ſecond Table, will be found to be .5625; and ſo of any other.

The ſecond Table was added, becauſe ſome may diſlike the firſt Table, in reſpect ſome parts of Coin will not be exactly expreſſed in Decimals by it, and though infinitely near the Truth, yet will never come up.

But if you make a Noble the Integer, you may expreſs any Parts exactly by it, as may be ſeen by the Table it ſelf.

PROP. II.

Any Decimal Fraction of Coin, Weight, Meaſure, being given, to find the Value thereof.

This is but the Converſe of the laſt Propoſition, and he that underſtands that, cannot be ignorant of this; however, take an Example or Two.

EXAMPLE I.

Let .65 be the Decimal Fraction of a Pound Sterling, and let the Value thereof be required.

Seek in the firſt Table of *Engliſh* Coin for .65, and in the Column of Shillings over-againſt it I find 13 Shillings, the Value of the Decimal given: So .75 Parts of a Pound Sterling will be found to be 15 Shill. but if the Integer had been a Hundred Weight, the Value thereof by the fifth Table will be found to be ¾, or 3 Quarters.

Likewiſe .049315, being the Decimal of a Year, by Table the 9th, will be found to be 18 Days.

EXAMPLE II.

But ſometimes your Decimal given cannot be found at one time, then uſe the following Method.

So if the Value of .46725 of a Pound Troy were required. Seek in the fourth Table, and becauſe therein the Ounces are not expreſſed, becauſe they are the ſame as Pence; in the third Table find the Ounces there; and the Penny-weight and Grains in the fourth Table, and the Value will be fouud to be 5 Ounces, 12 Penny-weight and 3 Grains.

See the following Work.

The

The Remainder being less than a Grain, is inconsiderable, and so not taken notice of.

Decimal given	.46725	
Nearest	.41666	= 5 Ounces.
Remainder	.05059	
Nearest	.05	= 12 Pen. wt.
Remainder	.00059	
Nearest	.00051	= 3 Grains.
	8	

So if the Value of .77777 of a Hundred Weight were sought, it would be found to be 3 *qr.* 3 *lb.* 1 *oz.* ¾.

See the Work.

Decimal given	.77777	
Nearest less	.75	= 3 Quarters.
Remainder	.02777	
Nearest less	.02678	= 3 Pound.
Remainder	.00099	
Nearest less	.00055	= 1 Ounce.
Remainder	.00044	
Nearest less	.00041	= 3 Qr. Ounce.
	3 Neglect.	

And so of any other.

It remains only, we should say something of the third Table, and so conclude.

This Table is of excellent Use, being not only a Decimal Table of Pence, a Shilling being the Integer; but likewise of Inches, Months, Dozens, or any other Weight or Measure where the Integer is divided into 12 Parts: It will likewise give the Decimal of any Pence or Farthings, supposing a Pound Sterling were the Integer; for if to the Decimal of any Number of Pence in the Table, you postpone a Cypher, and take ½ that Number, it shall be the Decimal of the same Number of Pence, a Pound being the Integer.

So if the Decimal of 7 Pence were required, and a Pound the Integer, it would be found to be .029166.

The Decimal of 7 Pence in the Table is	.583333
The same with a Cypher postponed is	.0583333
One half of the last is	.0291666 = 7*d.*

And

And that this is the Decimal of 7 *d.* may be proved by the first Table.

The Use of this Table being so excellent, it ought by every Learner to be got by Heart, which is easy to do, by reading the Numbers as is expressed underneath.

For 11 *d.* Read, Nine, One and all Sixes.
10 —— Eight and all Threes.
9 —— Seven, Five.
8 —— All Sixes.
7 —— Five, Eight, and all Threes.
6 —— Five.
5 —— Four, One, and all Sixes.
4 —— All Threes.
3 —— Two, Five.
2 —— One, and all Sixes.
1 —— Nought, Eight, and all Threes.

Addition in DECIMALS.

ADDITION *of Decimals* is not much different from *Addition of Integers,* only you must take care to keep Units under Units in Integers, and Tenths under Tenths in Decimal Parts.

EXAMPLE I.

Let it be required to add .7125 of a Pound to .42 of a Pound: The Sum is 1.1325 or 1 *l.* 2 *s.* 7 *d.* 3 *q.* $\frac{2}{10}$

Place your Numbers thus .7125 Not thus .7125
.42 .42

And the Sum will be 1.1325 Not .7167

Note, When you have added your Decimals together, so many must be cut off with a Dash of your Pen, as that Decimal Number consists of, which in your Example contains the most Places, the rest if any, are Integers; as may be seen in the Examples.

So

	l.		*l.*
So the Sum of	.7426 846 7.612 5.5	is	14.7006

Sum 14.7006

	l.
And the Sum of	421.625 46.1625 5.21925 .416786 74.8 6618.1 24424.712
Will be	31591.035536

But if your Numbers given to be added are not all of the same Denomination, they must be brought into Fractions of like Denominations, as in the following Example is done.

Let it be required to add .725 of a Pound, and .625 of a Shilling, into one Sum.

First, Find what Decimal of a Pound .625 will Represent, which is easily done if you prepone a Cypher, and half the Number is the Decimal of a Pound.

The Number with a Cypher preponed is .0625, ½ is .03125.

Then I add $\left\{\begin{array}{r} .725 \\ .03125 \end{array}\right\}$ *Sic de cæteris.*

The Sum is .75625

Subtraction in Decimals.

Subtraction in Decimals differs but little from *Subtraction in Integers*, only in placing your Numbers you must, as in Addition, keep Units under Units in Integers, and Tenths under Tenths in Decimal Parts.

EXAMPLE.

Let it be required to subtract .617, from .84125, which are to be placed thus.

From .84125
Subt. .617

The Remainder is = .22425

So if from 25.75
You Subtract 6.9845

There will Remain 18.7655

If the Decimal Parts in either Number have fewer Places than the other, the Vacancy is to be supplied by annexing so many Cyphers as will make them equal, or supposing them to be annexed. As here,

Cyphers annexed.		Cyphers supposed annexed.
From 426.4500		From 426.45
Sub. 129.6925	The same Number	Sub. 129.6925
Rest 296.7575		The Remainder 296.7575

But if your Numbers given to be Subtracted are not of the same Denomination, you must as in Addition, bring them into one Denomination, as in the following Example.

Let it be required to Subtract .03125 of an Ounce Troy, from .0625 of a Pound Troy.

Seeing one is the Decimal of an Ounce, and the other the Decimal of a Pound, bring them both into the Decimal of a Pound, by dividing .03125 the Decimal of an Ounce, by 12 the Ounces in a Pound, and it will give .002604.

.03125 Then { From .0625
$\frac{1}{12}$.002604 { Sub .002604

pw. gr.
Rest .059896 = 14 . 8

Or you may bring them both into the Decimal of an Ounce by multiplying .0625 the Decimal of a Pound, by 12 the Ounces in a Pound, which is the Converse of the last,

last, and it will give .7500 or .75, both being the same.

.0625	Then	From .75	
12		Sub. .03125	
			pw. gr.
.7500		Rest .71875 =	14 . 8

And so of any other.

Multiplication in Decimals.

1. *Multiplication in Decimals,* both in placing your Figures, and in the Work it self, differs nothing at all from Multiplication of Integers, only when your Work is finished, you must take Care that with a Dash of your Pen you make as many Places of Decimals in your Product as there were places of Decimals, both in your Multiplier and Multiplicand, but in case of want in your Product, annex Cyphers to the Left-hand.

2. In Multiplication of Decimals, it will be convenient to make that Number the Multiplicand which contains most places, though sometimes it may be less in Quantity: And, *Note,* That if both Terms to be multiplied, be Decimals, the Product will be a Decimal, or if both be mixt, that is, if both Terms consist both of Integers and Decimals, the Product will be mixt, but if one be mixt and the other a Decimal, the Product will sometimes be mixt, sometimes a Decimal.

EXAMPLE I.

Let it be required to multiply .75 by .425, the Product will be found to be .31875.

Mul.	.425	Multiplicand.
By	.75	Multiplier.
	2125	
	2975	
Facit	.31875	

EXAMPLE II.

The length of a Board is seven Foot .615 Parts.
The breadth of the Board is one Foot .15 Parts.
What is the Superficial Content?

Facit 8.75725, or 8 Foot $\frac{3}{4}$.

Mul.	7.615	Multiplicand,
By	1.15	Multiplier.

38075
7615
7615

Facit 8.75725 The Product.

EXAMPLE III.

Let it be required to multiply 2 Shill. 6 Pence, by 2 Shill. 6 Pence, one Pound being supposed to be the Integer. The Decimal answering 2 *s*. 6 *d*. or $\frac{1}{8}$ of a Pound is .125

Then I Mul.	.125	Multiplicand,
By the same	.125	Multiplier.

625
250
125

	l.	s.	d.	q.
Facit .015625 =	00	00	3	3

The same Question performed by Vulgar Fractions.

Mul. $\frac{1}{8}$ of a Pound,
By $\frac{1}{8}$ of a Pound.

	d.	q.
Facit $\frac{1}{64}$ of a Pound =	3	3

The Value of $\frac{1}{64}$ of a Pound, by the 4th Note, in Reduction of Vulgar Fractions, is equal to 3 *d*. 3 *q*. as before or if you reduce $\frac{1}{64}$ into a Decimal by Proposition the first in *Reduction of Decimals*, the Decimal will be the same as in the former Work, which may be a Proof of the Question.

Think it not strange that 2 *s*. 6 *d*. multiplied by 2 *s*. 6 *d*. produceth but 3 *d*. 3 *q*. but you must understand that Fractions multiplied together become less, in the same Proportio[n]

portion as Integers by multiplying become greater.

Some of our Pretenders to Art deny this, but for the Benefit of the Young Learner, and to stop the others Mouths, we shall give you a Demonstration thereof by the first of the second of *Euclid*.

Any two Numbers being to be multiplied together, if you divide either or both into as many Parts as you please, if then you multiply those Parts one by another, the Sum of those Products will be equal to the Product of one Number multiplied by another.

Let us divide the former Numbers, one into two Parts, and the other into three Parts.

First, Let us divide one into 1 *s.* and 1 *s.* and 6 *d.* and the other into 2 *s.* and into 6 *d.* then multiply those Parts, one by another as followeth.

First, 6 *d.* by 6 *d.* or .025 by .025 is	=.000625
Secondly, 1 *s.* by 6 *d.* or .05 by .025 is	=.00125
Thirdly, 1 *s.* by 6 *d.* or .05 by .025 is	=.00125
Fourthly, 6 *d.* by 2 *s.* or .025 by .1 is	=.0025
Fifthly, 1 *s.* by 2 *s.* or .05 by .1 is	=.005
Sixthly, 1 *s.* by 2 *s.* or .05 by .1 is	=.005
Products.	
Sum of the Products =	.015625

Which Product is the same as was found by the Multiplication of the two Numbers before, which shews the Work to be right.

But suppose the former Question were propounded, and a Shilling to be the Integer, then the Work would have been as underneath, and the Product would be 6.25 or 6 *s.* 3 *d.*

See the Work.

$$\begin{array}{r} 2.5 \\ 2.5 \\ \hline 125 \\ 50 \\ \hline \textit{Facit}\ \ 6.25 \end{array}$$

Thus you may see your Product will alter in Value, according as you alter your Integer.

EXAMPLE IV.

Let it be required to multiply 5 *l.* 12 *s.* 9 *d.* by 3 *l.* 6 *s.* 3 *d.*

	s.	*d.*		
Decimal answering	12	9	is	.6375
And of	6	3	is	.3125

```
Then Multiply  5.6375
By             3.3125
               ------
               281875
              112750
             56375
           169125
          169125
          ----------
The Product 18.67421875
```

The Product 18.67421875 = 18 . 13 . 5 . 3 1/4 (*l. s. d. q.*)

But seeing we have for the most part, but occasion for three or four Figures after the Separatrix, and sometimes the Multiplications are tedious and long; we will therefore give you a Rule how you may contract your Work, and yet secure what places of Decimals you please. To do which observe the following Method.

Having set down your Multiplicand as usual, set the Units place of your Multiplier under that Figure in your Multiplicand which stands as far from Unity as the last Figure of your Product is desired to stand, and write the rest in the Inverse Order, then multiply by your Multiplier, as usual; only, *Note*, That you need only begin in your Multiplicand with that Figure that stands over the Figure you multiply by; having a respect to the Increase that would come from the following Figures of the Multiplicand, placing every single Product exactly even at the Right-hand, contrary to the common way, and adding them as they stand, you must cut off so many Figures in your Product as was designed, which you may better understand by the Work of the following Example.

See the last Question wrought and but 3 Figures cut off.

Multi-

Multiplicand	5.6375
Multiplier Transverse	5213.3
	16913
	1691
	56
	11
	3
	18.674

Here you may see the Work contracted much, and the Product to three Places as was before.

So if .125 were to be multiplied by .125, as in the third Example beforegoing, and to have four Figures of Decimals after the Separatrix.

See the Work.

Multiplicand	.125	
Multiplier Inverse	521.	
	125	
	25	
	6	
	d.	q.
	.0156=3	3

Here because there was but three Places, I prepone a Cypher, and the Product to four Places is the same as before.

When a Decimal Fraction or mixt Number is to be multiplied by an Unit with Cyphers (as 10, 100, 1000, &c.) you need only to remove the Separatrix so many Plaçes towards the Right-hand as there are Cyphers annexed to the Unit. So if .1278 were to be multiplied,

$$\text{By}\begin{cases}10\\100\\1000\\10000\end{cases}\text{The Product will be}\begin{cases}1.278\\12.78\\127.8\\1278.\end{cases}$$

Division in Decimals.

Division in Decimals, differs nothing from Division of Integers, either in placing the Numbers, or in the Work it self: All the Difficulty being in discovering the true Value of the Quotient, which to do, observe the following Rule, which is but the Converse of that in Multiplication, and is this that follows.

As

As many Figures as are cut off in the Dividend, so many must be cut off in the Divisor and Quotient; or thus, As many Figures must be cut off in the Quotient as will make those cut off in the Divisor, equal to those in the Dividend, taking notice if there be not so many in the Quotient, you must add Cyphers to the Left-hand. Note also, If your Dividend be an Integer or have less cut off than is in the Divisor, it is convenient you add Cyphers to the Dividend, till they be equal or more, then the Work will be easy.

The following Examples will make all plain.

EXAMPLE I.

Where the Dividend is a mixt Number, and the Divisor an Integer. Divide 742.651 by 41.

See the Work. *l.* *s.* *d.*

```
41) 742.651 (18.113=18 . 2 . 3 1/4
    41
    ---
    332
    328
    ---
      46
      41
      ---
       55
       41
       ---
       141
       123
       ---
        18 Remains.
```

The Quotient is 18.113.

EXAMPLE II.

Where both are mixt Numbers.

Divide 4672.565, by 25.635.

See the Work.

25,635) 4672.565 (182

In this Example because there is like cut off in both, the Quotient is an Integer. And with adding of Cyphers, you may bring it as far after the Separatrix as you please.

```
25635
------
210906
205080
------
  58265
  51270
------
   6995 Remainder.
```

EXAMPLE III.

Where both Numbers are Decimals; Divide .75 by .0125.

Seeing I cannot divide, I add Cyphers to the Dividend, to wit, two, and there will be alike cut off in both; then as in the last Example, the Quotient will be an Integer.

See the Work.

```
.0125) .7500 (60
        ..
        750
       -----
         00
```

Facit 60 in the Quotient.

By which you may observe, that as Multiplication of Fractions decreaseth their Value, so Division of Fractions increases the Value contrary in both to the Nature of Integers.

This last Example is the same as if it were demanded to divide 15 Shill. by 3 Pence, the Quotient will be found to be 60 Pound. The Proof is easy by Multiplication.

For if we multiply 3 *d.* or .0125 by 60 *l.* the Quotient will be .75 or 15 Shill. as you may see by the Work.

```
 .0125
    60
 ------
 .7500 = 15 Shill.
```

Supposing still a Pound Sterling to be the Integer.

EXAMPLE IV.

Where the Dividend is an Integer and the Divisor a Decimal, let it be required to divide 1425, by .6252.

Here before Division can be well made, it will be convenient to add a competent Number of Cyphers.

If you only require the Integral Part of the Quotient, add so many Cyphers to the Dividend as there are Decimal Parts in your Divisor, then your Quotient will be wholly Integral; But if you require Decimal Parts, so many Cyphers more must be added (besides the Number to make them equal) as you design to have Decimal Parts in your Quotient.

Let us in this Question have three places of Decimals after the Integral part of the Quotient.

See the Work.

```
.6252) 1425.0000000 (2279.270
        .......
       12504
       -----
        17460
        12504
        -----
         49560
         43764
         -----
          57960
          56268
          -----
           16920
           12504
           -----
            44160
            43764
            -----
             3960 Remainder.
```

The Quotient will be 2279.270

EXAMPLE V.

Where the Dividend is a Decimal, and the Divisor an Integer. Let us divide .13975, by 43.

See

When the Division was finished, there were but 3 Figures in my Quotient, and seeing there should be 5 cut off, I therefore annex 2 Cyphers to the Left-hand, as may be seen in the Example.

See the Work.

```
43) .13975 (.00325
      ...
    129
    ---
     107
      86
     ---
     215
     215
     ---
       0
```

And if 5.29125 were divided by 42.5; the Quotient would be .1245.

See the Work.

```
42.5) 5.29125 (.1245
        ...
      425
      ---
      1041
       850
      ----
      1912
      1700
      ----
       2125
       2125
       ----
          0
```

When any Decimal Fraction or mixt Number is to be divided by an Unit with any Number of Cyphers annexed, it is but removing the Separatrix so many Places towards the Left-hand, as there were Cyphers annexed to the Unit.

So if 17.28 were given to be divided.

By	10	The Quotient will be	1.728
	100		.1728
	1000		.01728
	10000		.001728

By what goeth before, it may be observed, that if the Dividend be greater than the Divisor, the Quotient will

 either

either be an Integer, or a mixt Number, but if the Divisor be greater, the Quotient will be a Decimal.

Multiplication and Division in Decimals (as in Integers) interchangeably prove each other.

To prove Multiplication divide the Product by the Multiplier, quotes the Multiplicand, or by the Multiplicand quotes the Multiplier.

To prove Division, multiply the Quotient by the Divisor, produceth the Dividend, or by the Dividend, produceth the Divisor.

Before we leave Division in Decimals, we will give the Learner the Resolution of two excellent Problems, which will be of good Use.

The first is, having a Multiplicator, to find a Divisor.

Divide an Unit with Cyphers by the Multiplicator, the Quotient will be the Divisor sought.

EXAMPLE.

What Divisor is that, by which dividing 7315, shall give a Quotient equal to the Product of the same Number, multiplied by 125? *Facit* .008.

See the Work.

```
125) 1.000 (.008
     1.000
     -----
         0
```

The PROOF.

```
  7315          .008) 7315.000 (914375
   125                72
 -----                --
 36575                 8
 14630                35
 7315                 32
 -----                --
914375                30
                      24
                      --
                      60
                      56
                      --
                      40
                      40
                      --
                       0
```

Here you may see the Product and Quotient are the same.

Then

The second is, having a Divisor, to find a Multiplicator.

This is but the Converse of the former, for if you divide Unity, with Cyphers annexed by the given Divisor, the Quotient will be the Multiplicator sought.

EXAMPLE.

What Multiplicator is that by which multiplying 7315, shall give a Product equal to the Quotient of the same Number, divided by .008? *Facit* .125.

See the Work.

```
.008) 1.000 (125
      8
      --
       20
       16
       --
        40
        40
        --
         0
```

The Proof is in the last.

Golden Rule in Decimals.

We shall not in this Place need to give you a Definition of the *Rule of Three*, that being sufficiently done in *Vulgar Arithmetick*.

For seeing the Rule of Three in Decimals is the same, both in the stating and working of the Question, as in the Rule of Three before taught, respect being had to the Rules in Decimals aforegoing, which if well understood, any Question of the Golden Rule, though consisting of never so cross Fractional Parts, will receive its Resolution as easily, as if the Question were composed of Integers only, which shall be made plain in the following Examples.

EXAMPLE I.

If 7 Yards and Three Quarters of Cloath cost 2 *l.* 12 *s.* 9 *d.* what will 140 Yards and half of the same Cloath cost?

The Fractional Parts reduced to Decimals by the Rules aforegoing, and stated as taught in the Rule of Three in Vulgar Arithmetick: The Work will stand as follows.

If

```
       Yd.    l.         Yd.
   If 7.75 : 2.6375 : : 140.5
             140.5
           ---------
            131875
           105500
           26375
        -------------
   7.75) 370.56875 (47.815
              .....
          3100
          ----
          6056
          5425
          ----
           6318
           6200
           -----
            1187
             775
            ----
            4125
            3875
            ----
             250
```

l. s. d. q.
Ans. 47 . 16 . 3 . 3

QUESTION II.

If a Chest of Sugar weighing 7 *C.* 2 *qr.* 14 *lb.* cost 36 *l.* 12 *s.* 9 *d.* What will 2 *C.* 1 *qr.* 21 *lb.* of the same Sugar cost?

The Fractional Parts reduced into Decimals, and stated as before taught. The Work will stand as underneath.

```
      C.        lb.         C.
   If 7.625 : 36.6375 : : 2.4375
               2.4375
           -----------
             1831875
            2564625
           1099125
          1465500
          732750
       ---------------
   7.625) 89.30390625 (11.71198
       ---------------
   Remainder 5875
```

Ans. 11 *l.* 14 *s.* 2 *d.* 3 *q.*

QUESTION

QUEST. III.

A Grocer buys 24 Tunns 12 Hundred 2 Quarters 14 Pound and 12 Ounces of Tobacco, for 3678 Pound 6 Shill. and 4 Pence; What will one Ounce of this Tobacco cost?

The Question reduced and stated, will stand thus as beneath.

C. *lb.* *C.*
If 492.8817 : 3678.3166 :: .000558
.000558

492.8817) 2.0525007 (.00416

The Work at large is neglected, only the Product and Quotient. } *Answer* 1 *d.* the Ounce, *ferè.*

QUEST. IV.

If 16 Pioneers do make a Trench in one Month and 14 Days, How many Pioneers will make the same Trench in 12 Days? *Ans.* 56 Pioneers.

See the Work.

M. *P.* *M.*
If 1.5 : 16 :: .42857
1.5

80
16

.42857) 24.00000 (56

214285

257150
257142

8

In my Product, because I had but one Decimal place, I annexed 4 Cyphers, to equal the Number of Decimal Places in my Division, that so my Quotient might be an Integer.

QUEST. V.

If when Wheat is sold for 12 Shill. the Quarter, the half Penny white-Loaf ought to weigh one Pound one Ounce, and 12 Penny-Weight; What must the half Penny white-Loaf

Loaf weigh, when Wheat is sold for 1 Pound 16 Shill. and 3 Pence the Quarter? *Answer*, The half Penny white-Loaf ought to weigh 4 Ounces and 11 Penny-weight, *&c.*

```
            l.     lb.      l.
     If .6 : 1.1333 :: 1.8125
                 .6
             --------
   1.8125 ) .68000 (.3751
             54375
             ------
             136250
             126875
             ------
              93750
              90625
              -----
              31250
```

QUEST. VI.

What length of a Board 9 Inches broad will make a square Foot, when 12 times 12, or 144 Inches make one Foot? Say, If 12 in breadth require 12 in length, what will 9 in breadth require? *Answer*, 16 Inches in length.

See the Work.

```
         B.   L.   B.
     If 12 : 12 :: 9
               12
              ----
          9) 144 (16
              9
             ---
              54
              54
             ---
               0
```

Double Golden Rule in Decimals.

We shall here give you an Example or two in the *Double Rule of Three in Decimals*, or *Rule of Plural Proportion*, and so conclude this Rule.

QUEST.

QUESTION I.

If three Labourers in two Months and twelve Days Thrash 221 Quarters three Bushels and two Pecks of Corn, How much will nine Labourers thrash in one Month, two Weeks and five Days?

Lab. — *Quar.* *Lab.*
First say, If 3 thrash 221.4375, what will 9 thrash?
Quar.
Facit 664.3158

Say again, If 2.42857 Months thrash 664.3158 Quar. what will 1.67857 Months thrash? *Facit* 459.155 equal to 459 Quarters one Bushel, and one Peck.

QUEST. II.

Mr. *Bridges* in his *Lex Mercatoria*, Page 222. hath the following Question.

If 100 *l.* in 12 Months gain 8 *l.* what will 7390 *l.* 13 *s.* 11 *d.* 3 *q.* gain in 9 Months? And further saith, the most methodical way of working this Question will require about 300 Figures more than the Practical Way, he shews in the Work of the said Question, *p.* 188 of the said Book, which we will examine.

First, I say, If 100 : 8 : : 7390.6991
8

Facit 591.25592 = one Year's Gain.

M. *l.* *M.*
Say again, If 12 : 591.25592 : : 9
9

5321.30328

$\frac{1}{12}$ 443.4419 = 443 *l.* 8 *s.* 10 *d.* the *Ans.*

Here are but about 40 Figures besides the *Answer*, taking in the stating of the Question, and all which we might have contracted to much fewer; and his Practical way before mentioned, hath above 60 Figures besides the *Answer*, and how he will find a more methodical Way I know not.

Mr. *Bridges* had no Reason to undervalue Decimal Arithmetick so much, if he had notice thereof; It and the Logarithms being two of the most famous Inventions the pre-

 ceding

ceding Ages have been Masters of, and if we use the Practical Way, its convenient the Fractional Parts be reduced into Decimals, as in the annexed Operation is manifest.

First, I multiply by 8, cutting off 5 Fig. *viz.* 3 for the Decimal, and 2 instead of dividing by 100, that done, I took half for 6 Months, and half of 6 Months for 3 Months, which two Numbers added make 443 *l.* 8 *s.* 10 *d.* as before.	7390.699 8 ——— 591.25592 ——— 295.6279 147.8139 ——— 443.4418=443 *l.* 8. *s.* 10 *d.*

QUEST. III.

If 2 Angels be equal to 20 Shillings, and 15 Shill. equal to 3 Crowns, and 60 Crowns equal to 15 Pounds, and 13 Pounds equal to 12 Guineas: How many Angels will countervail 650 Guineas?

s. *Ang.* *s.*
First, I say, If 20 : 2 :: 15 : *Facit* 1.5 Angels.
Crow. *Ang.* *Crow.*
Secondly, If 3 : 15 :: 60 : *Facit* 30 Angels.
l. *Ang.* *l.*
Thirdly, If 15 : 30 :: 13 : *Facit* 26 Angels.
Guin. *Ang.* *Guin.*
Lastly, If 12 : 26 :: 650 : *Facit* 1408.333 Angels.
Answ. 1408 333 *Ang.* or, 1408 *Ang.* 3 *Shill.* and 4 Pence.

The last Question may be wrought by Division only, by placing your Numbers as underneath.

Here if you multip. the 1st, 3d, 5th, 7th, and 9th for a Dividend; and the 2d, 4th, 6th, and 8th for a Divisor, the Quotient is the *Answer* which you may try at your Leisure.

THUS,

If 2 *Angels* equal 20 *Shillings.*
And 15 *Shillings* equal 3 *Crowns.*
And 60 *Crowns* equal 15 *Pounds.*
And 13 *Pounds* equal 12 *Guineas.*
What will 650 *Guineas* equal?
Answer, 1408 *Angels* $\frac{1}{3}$.

In this Question it was required to know, how many of the first would equal such a Number of the last, and by some is called the Compound Rule Descending: But if it had been required to know, how many of the last would countervail such a Number of the first; then, in this Question you must have mulplied the 2d, 4th, 6th, 8th, and 9th, for a Dividend, and the Product of the 1st, 3d, 5th, and 7th, would have been Divisor; and this is commonly called the Compound Rule Ascending.

QUEST. IV.

There is a Cistern hath three Cocks, the first will empty the Cistern in a Quarter of an Hour, the second in half an Hour, the third in three quarters of an Hour, in what time will the three Cocks empty this Cistern?

	H.	C.	H.	
First, If	.25 :	1 ::	1 :	*Facit* 4 Cisterns.
2dly, If	.5 :	1 ::	1 :	*Facit* 2 Cisterns.
3dly, If	.75 :	1 ::	1 :	*Facit* 1.333 Cisterns.
				7.333

	C.	H.	C.
Then say, If	7.333 :	1 ::	1
		1	

7.333) 1.0000 (.13635 equal to 8 Minutes, and 11 Seconds, *fere.*

```
 7333
 ----
 26670
 21999
 -----
  46600
  43998
  -----
   26020
   21990
   -----
    40210
```

QUEST. V.

A Conduit hath a Cock, which running into a Cistern, will fill it in 10 Minutes, or $\frac{1}{6}$ of an Hour. This Cistern hath 3 Cocks, one before, and one on each side; that before will empty the Cistern in an Hour, that on the right side in $\frac{1}{2}$ an Hour, that on the left in $\frac{3}{4}$ of an Hour; in what time will this Cistern be filled, all running?

H. C. H. C.

First say, If .1666 : 1 :: 1 : *Facit* 6 the filling Cock.
Secondly, If 1 : 1 :: 1 : *Facit* 1 the emptying Cock.
Thirdly, If .75 : 1 :: 1 : *Facit* 1.333 emptying Cock.
Fourthly, If .5 : 1 :: 1 : *Facit* 2 the emptying Cock.

Filling Cistern = 6
Empt. Cistern = 4.333

Difference 1.666

C. H. C.

Then say, If 1.666 : 1 :: 1 : *Facit* .6 Hours, or 36 Minutes the Answer.

QUEST. VI.

A Cock of a Conduit runneth into a Cistern, and filleth it in 5 Hours; this Cistern hath a Cock that will empty it in 12 Hours. In what time will the Cistern be filled, if both run at once?

First, I say, If 5 Hours fill one Cistern, what will 1 Hour fill? *Facit* .2 of a Cistern.

Say again, If 12 Hours empty one Cistern, what will one Hour empty? *Facit* .08333 of a Cistern.

From .2 - - the filling Cock.
Subt. .0833 the emptying Cock.

Diff. .1166

Then say, If .1166 of a Cistern require one Hour, what will one Cistern require? *Facit* 8.5645, or 8 Hours, 33 Minutes and 50 Seconds, the Answer.

QUEST.

QUEST. XVII.

If Wine worth 15 *l.* 12 *s.* 9 *d.* be sufficient for the Ordinary of 100 Men, when it is worth 25 *l.* 15 *s.* *per* Tun; how many Men will 3 Pounds worth satisfie, when Wine is worth 50 *l.* *per* Tun?

l. *Men.* *l.* *Men.*

First say if 15.6375 : 100 :: 3 *Facit* 19.184654.

Say again, If 25.75 *l.* *per* Tun suffice 19.184654, what number of Men will 50 *l.* *per* Tun require? *Facit* 10 *Men, fere.*

Rule of Practice *in Decimals.*

ALtho' we have treated before of the *Rule of Practice*, yet because many times your Question may consist of Integers, and Fractional Parts, and seeing many times Questions may be more easily wrought, by reducing the Parts into Decimals, whereby the Operation will be the same as in Integers; we thought fit to give the Learner a touch thereof, no one having done it before.

First, If at any time your Question consists of the Aliquot Parts of two Shillings, your Question may be wrought at one Operation, without reducing the Answer into Pounds, for 2 Shillings being the Decimal Part of a Pound, the Answer at 2 Shillings is found by Inspection, by separating the last Figure towards the Right Hand from the rest, for a Decimal Fraction, the rest is Pounds.

EXAMPLES.

(1.)

At 6 *s.* the Yard, what will 64.3 Yards cost?

The Answer at 2 *s.* is 6.43 *l.* and 6 *s.* $\frac{1}{4}$, of which is 16.075 *l.* or 16 *l.* 1 *s.* 6 *d.*

(2.)

At 8 *d.* the Yard, what will 144 Yards cost?

Answ. at 2 *s.* is 14.4

$\frac{1}{3}$ for 8 *d.* is 4.8 or 4 *l.* 16 *s.*

Answer 4 *l.* 16 *s.*

More

More EXAMPLES.

(3.)

At 4 *d.* the Yard, what will 144 Yards cost?

	14.4
$\frac{1}{6}$ part is	2.4

Answer 2 *l.* 8 *s.*

(4.)

At 3 *d.* the Yard, what will 145 Yards cost?

	14.5
$\frac{1}{8}$ part is	1.8125

Answer 1 *l.* 16 *s.* 3 *d.*

And if your Question consist not of Aliquot Parts of 2 Shillings, you may easily divide it into Aliquot Parts, as in the *Examples* following.

(1.)

At 9 *d.* the Yard, what will 724 Yards cost?

$\frac{1}{10}$ Yards is	72.4
$\frac{1}{4}$ for 6 *d.* is	18.1
$\frac{1}{2}$ of 6 *d.* for 3 *d.* is	9.05
Sum	27.15

Answer 27 *l.* 3 *s.*

(2.)

At 16 *d.* the Yard, what will 721 Yards cost?

$\frac{1}{10}$ is	72.1
$\frac{1}{3}$ for 8 *d.* is	24.0333
the same	24.0333
Sum	48.0666

Answer 48 *l.* 1 *s.* 4 *d.*

And so of any other.

Now seeing half the number of Shillings, is the Decimal thereof, Any Question of Practice, consisting of any number of Shillings, may be answered by an easy Multiplication, and the Answer given in Pounds, and Parts of a Pound, the Value of which Fraction, by the compendious method of Valuing a Fraction of a Pound Sterling, taught in *Reduction* aforegoing, may be known by Inspection.

EXAMPLES.

(1.)

At 12 *s.* the Yard, what will 72 Yards cost?

Mult.	72
By	.6
Prod.	43.2

Answer 43 *l.* 4 *s.*

(2.)

At 15 *s.* the Yard, what will 65 Yards cost?

Mult.	65
By	.75
	325
	455
Prod.	48.75

Answer 48 *l.* 15 *s.*

(3.)

At 1 *l.* 13 *s.* 9 *d.* the Yard, what will 48 Yards cost?

For one Pound is	48 *l.*
Prod. by .65 for 13 *s.* is	31.2
$\frac{1}{4}$ of 4.8 for 6 *d.* is	1.2
$\frac{1}{2}$ of that for 2 *d.* is	.6
Answer	81 *l.* 0 *s.*

(4.)

At 14 *s.* 8 *d.* the Yard, what will 144 Yards cost?

Mult. 144
By .7

100.8
4.8 $\frac{1}{3}$ of 14.4 for 8 *d.*

Pr. 105.6 *Answ.* 105 *l.* 12 *s.*

But if at any time, as often it will happen, that your given Number hath a Fraction annexed, such Fraction is to be reduced into a Decimal; So will the Operation be as easy as if it consisted of Integers only.

EXAMPLE.

I. At 8 *d.* the Yard, what will 672$\frac{3}{5}$ Yards cost?

Your Number after Reduction stands thus,	672.6
$\frac{1}{10}$ for 2 Shillings is	67.26
$\frac{1}{3}$ of 2 Shillings for 8 *d.* is	22.42

Answer is 22 *l.* 8 *s.* 4 *d.* 3 *q.* $\frac{2}{10}$ or $\frac{1}{5}$

II. At 3 *s.* 6 *d.* the Gross, what will 25 $\frac{3}{4}$ Gross cost?
Your Number after Reduction stands thus 25.75 Gross
Mult. by the Decimal of 3 *s.* 6 *d.* to wit, .175

$$
\begin{array}{r}
12875 \\
18025 \\
2575 \\
\hline
4.50625
\end{array}
$$

Answer 4 *l.* 10 *s.* 1 *d.* $\frac{1}{2}$

Take a Question or two in *Averdupoise Weight.*

At 5 *l.* 15 *s.* 7 *d.* $\frac{3}{4}$ the Hundred, what will 218 *C.* 3 *qr.* and 24 *lb.* cost?

See the Work at large.

The Weight reduced stands thus,	218.96428
Multiply by	5.78219
	197067852
	43792856
	43792856
See Kersey *upon* Wingate, Page 332.	175171424
	153274996
	109482140
Answ. 1266 *l.* 2 *s.* 3 *d.* 3 *q. fere.*	1266.114966612

The Work by Contraction in Multiplication of Decimals.

Multiplicand	218.964
Multiplicator Inverse	92287.5
	1094821
	153275
	17517
	438
	44
	19
Answer 1266 *l.* 2 *s.* 3 *d.* 3 *q. fere,* as before.	1266.114

QUEST. II.

At 1 *l.* 17 *s.* 5 *d.* $\frac{1}{4}$ the Hundred, what will 57 C. 2 q. 17 *l.* 11 *oz.* cost?

The Multiplicand	57.6579
Multiplicator Inverse	28178.1
	576579
	461263
	40360
	577
	461
	40
Answer 107 *l.* 18 *s.* 6 *d.* $\frac{3}{4}$	107.9280

Take

Take a Question or two in *Lead Weight*.

QUEST. I.

At 9 *l.* 17 *s.* 5 *d.* the Fodder of Lead, what will 17 Fodder, 14 Hundred, 1 Quarter, and 17 Pound cost? 19 Hundred and half, one Fodder.

The Multiplicand 17.73855
Multiplicator Inverse 38078.9

See Spiedell's *Arithmetical Extraction*, Pag. 95 and 96, *where you may find the Work of this Question, whereby you may see the facility of this Operation from that.*	1596471
	141908
	12415
	142
	1
	175.0937

Answer 175 *l.* 1 *s.* 10 *d.* ½ +

But lest the Learner should stumble in reducing these sorts of Weights into Decimals, and seeing they are useful Operations, I thought it convenient, for the Learner's Advantage, to annex a *Decimal Table* for that purpose; and it is as followeth.

A Table of Lead Weight.

Lead Weight in Decimals, one Fodder the Integer.

Hund	Decimals
19	.974350
18	.923076
17	.871794
16	.820512
15	.769230
14	.717948
13	.666666
12	.615384
11	.564102
10	.512820
9	.461538
8	.410256
7	.358974
6	.307692
5	.256410
4	.205128
3	.153846
2	.102564
1	.051282

Quarter Hundred	Decimals.
3	.038461
2	.025641
1	.012820

Pounds	Decimal
27	.0123626
26	.0119047
25	.0114469
24	.0109890
23	.0105311
22	.0100732
21	.0096253
20	.0091575
19	.0086996
18	.0082417
17	.0077838
16	.0073260
15	.0068681
14	.0064102
13	.0059523
12	.0054945
11	.0050366
10	.0045787
9	.0041208
8	.0036630
7	.0032051
6	.0027472
5	.0022893
4	.0018315
3	.0013736
2	.0009157
1	.0004578

Ounces	Decim.
15	.0004292
14	.0004006
13	.0003720
12	.0003434
11	.0003148
10	.0002861
9	.0002575
8	.0002289
7	.0002003
6	.0001717
5	.0001430
4	.0001144
3	.0000858
2	.0000572
1	.0000286

Quar. Oz.	Decim
3	.0000214
2	.0000143
1	.0000071

QUEST.

QUEST. II.

At 9 *l.* 11 *s.* 1 *d.* $\frac{1}{2}$ the Fodder, what will 18 Fodder, 13 Hundred, 3 Quarters, 17 Pound, 13 Ounces cost?

The Weight reduced by the foregoing Table, is 18.7133
Multiplicator Inverse 52655.9

	1684197
	93567
	9357
	1122
	37
Answer 178 *l.* 16 *s.* 7 *d.* fere.	9
	Product 178.8289

Note, Tho' some of the foregoing Questions be wrought by *Multiplication in Decimals*, and not by the Practical way of Aliquot Parts before taught, the Reason whereof is, That in those, and the like particular Questions, the Practical way, is more intricate, tedious, and requires more Figures than the Method here used, as may be observed, if the Reader peruse Mr. *Spiedell's Arithmetical Extraction*, Pag. 95, 96, and 97, in the Practical way of the said Questions.

Here follows a Question or two in Exchange, and so we will conclude this Rule.

QUEST. I.

A Merchant hath received a Bill of Exchange for 593 $\frac{1}{2}$ Pieces, at 7 Shill. 9 $\frac{1}{4}$ Pence *per* Piece, what Sterling Money will they amount to? *Answer* 231 *l.* 16 *s.* 8 *d.* 2 *q.* $\frac{1}{4}$.

The Work.

Number given	593.5
$\frac{1}{4}$ for 5 Shillings is	148.375
$\frac{1}{10}$ for 2 Shillings is	59.35
$\frac{1}{10}$ of 5 Shill. for 6 *d.* is	14.8375
$\frac{1}{2}$ of 6 *d.* for 3 *d.* is	7.41875
$\frac{1}{4}$ of 3 *d.* for 3 *q.* is	1.85468
Sum	231.83593

QUEST. II.

One hath changed 759 double Pistolets at 11 *s.* 8 *d.* $\frac{1}{2}$ *per* Piece, what will they come to? *Answer* 444 *l.* 6 *s.* 7 *d.* $\frac{1}{2}$.

Given Number	759
$\frac{1}{2}$ for 10 Shillings	379.5
$\frac{1}{10}$ of 10 *s.* for 1 *s.*	37.95
$\frac{1}{2}$ of 1 *s.* for 6 *d.*	18.975
$\frac{1}{3}$ of 6 *d.* for 2 *d.*	6.325
$\frac{1}{4}$ of 2 *d.* for 2 *q.*	1.58125
The Sum	444.33125

Hereunto let us annex a compendious Method of Buying or Selling, by the Hundred Neat, or Hundred *Averdupoise*; as oft as your Question is but of a small Price.

For the little, or true Hundred, for as many Farthings as the Pound costs, account twice so many Shillings, and once so many Pence.

For the great Hundred, or 112 Pound, as many Farthings as the Pound costs, twice so many Shillings, and once so many Groats the Hundred Gross will cost.

EXAMPLE *in both.*

I. At 3 *d.* $\frac{1}{2}$ the Pound, what will 100 Pound cost?

	l.	*s.*	*d.*
3 *d.* $\frac{1}{2}$ is 14 Farth. twice so many Shill. are 28, or	1	8	0
Once so many Pence are 14, or	0	1	2
Answer 1 *l.* 9 *s.* 2 *d.* Sum	1	9	2

II. At 2 *d.* 1 *q.* the Pound, what will the Hundred *Averdupoise*, or 112 Pound cost?

	l.	*s.*	*d.*
2 *d.* $\frac{1}{4}$ is 9 Farth. twice so many Shillings are	0	18	0
Once so many Groats is	0	03	0
Answer 1 *l.* 1 *s.* Sum	1	01	0

I

I shall in this Place annex one Question, to make the Learner a good Husband, if possible.

The Question is, That if one square Yard of Land cost a Penny, what will buy an Acre, 160 Perches being an Acre, and 7 Yards a Perch?

Yards in a Perch	7	Perches in an Acre	160
Multip. by it self	7	Square Yards in a Perch	49
Product	49		1440
Sqr. Yds in a Perch			640
		Yards in an Acre	7840

At 1 *d.* the Yard, what will 7840 Yards cost?

65|3 = 4 *d.*

Facit 32 *l.* 13 *s.* 4 *d.*

And the yearly Rent which 32 *l.* 13 *s.* 4 *d.* will purchase at 6 *per Cent.* Compound Interest, or the Annual Rent of an Acre, will, by the Rules in Compound Interest following, be found to be 1.96 *l.* or 1 *l.* 19 *s.* 2 *d.* 1 *q.* $\frac{6}{10}$, very near 40 Shillings.

Whereby it is evident, That he that spends one Penny, spends, or makes away a square Yard of as good Land as most in *England*, from him and his Heirs for ever.

And it is a Question whether *England* be worth 20 Shillings an Acre Annually, taking one Acre with another. How much good Land we make away, it is easy to judge.

And he that spends a Penny a Day, spends one Pound, one half Pound, one Groat, and one Penny; and so by Consequence, two Pence a Day, will be two Pound, two half Pound, two Groats, and two Pence; and three Pence three Pound, three half Pound, three Groats and three Pence, &c. *per* Year.

Extraction

Extraction of the *Square Root.*

A Square Number is that which is contained under two equal Numbers, or which is equally equal.

So 4 is a ſquare Number, contained under 2 equal Numbers; to wit, 2 and 2; for two times 2 is 4: And the ſquare Number 9 is contained under 3 and 3; for 3 times 3 makes 9; and of the reſt as in the following Table.

A Table of Squares, with their Genitive equal Numbers, as far as the firſt 9 Digits.

Equal Numbers					Squares
1	into	1	is		1
2	into	2	is		4
3	into	3	is		9
4	into	4	is		16
5	into	5	is		25
6	into	6	is		36
7	into	7	is		49
8	into	8	is		64
9	into	9	is		81

And when it is required to extract the ſquare Root of any given Number, we have nothing to do but to find that equal Number, of which it is compoſed: So if the Root of 16 were required, it would be found to be 4, as in the ſaid Table.

Here 4 is the Root, called by ſome the firſt Power, and 16 is the Square called the ſecond Power.

Of Numbers to be extracted, are three Sorts.

Firſt, Single.

Secondly, Compound.

Thirdly, Irrational.

Single are ſuch Squares as are compoſed, or made up of any of the 9 Digits; of which ſort are thoſe in the aforegoing Table.

Compound

Compound are all such Squares that are composed of more Figures than one, as 100, whose Root is 10; 121, whose Root is 11; or 144, whose Root is 12, &c.

Irrational are all such Squares, whose Roots cannot be discovered by Art exactly, neither in whole Numbers or Fractions, but something will still remain, there being no Proportion yet found betwixt an irrational Number and its Root; such Numbers are 3 . 7 . 19 . 74 . 156 . 751, &c.

The Extraction of the square Root is not much unlike Division; only there our Divisor is fix'd, here we are to seek a new one for each Operation.

The Root of any single square Number is found by Inspection, as in the foregoing Table may be seen.

But if it be a Compound square Number, it must be prepared by pointing thus: make a Point under your Unit's place, and omitting one, point every other Figure: And as many Points as your Number contains, so many Figures will your Root consist of.

Then proceed by the following Directions.

A Rule to be got by Heart.

The Root of your first Period you
Must place in Quote, if you work true,
Whose Square from your said Period then
You must subduct; and to th' Remain
Another Period being brought,
You must divide, as here is taught,
By th' double of your Quote, but see
Your Unit's place you do leave free;
Which place will be supplied by th' Square
Of your next quoted Figure there:
Next multiply, subduct, and then
Repeat your Work unto the End,
And if your Number be Irrational,
Add pairs of Cyphers for a Decimal.

EXAM-

EXAMPLE.

Let it be required to find the Square Root of 451584.

Having pointed it as in the Work, shews the Root will have 3 Places.

The Work.

```
451584 ( 6
36
——
9
```

1. Seek the greatest Root of your first Period 45, which by your Table you will find to be 6, which place in your Quotient, and the Square thereof under 45, your first Period sub. 36 from 45, rest 9. This is your first Work, and is no more to be repeated.

```
451584 ( 6
36
——
)915
```

2. To the Remainder bring down your next Period 15, makes 915 for a Dividend, or as some call it a Resolvend, as you may see in the Work.

```
    451584 (67
    36
    ——
127)915
    889
    ——
```

3. Double your Quote 6, makes 12 for a Divisor, then seek how oft 12 in 91; or how oft 1 in 9 (reserving the Unit's place for the Square of my sought Figure) which I find to be 7, which I place in my Quotient; and to save trouble of Addition to the right Hand of my Divisor as a part thereof, making it 127, then multiplying 127 by 7, the Prod. I place under my Dividend or Resolvend as you see.

This Work is every time to be repeated.

```
    451584 (67
    36
    ——
127)915
    889
    ——
   )2684
```

4. Subtract 889 from 915, rest 26, to which I bring down my third and last Period 84, then shall I have 2684 for a new Dividend, or Resolvend, as you may see in the Work itself.

5. Double

5. Double your Quotient 67, *facit* 134 for a new Divisor, then I ask how oft 134 in 268, (still reserving my Unit's place in the Dividend) or which is the same, how oft 1 in 2.

```
  451584 (672
   · · ·
   36
   ——
127)915
    889
    ———
1342)2684
     2684
     ————
        0
```

Answ. 2 times, which I place in my Quotient, and likewise on the right Hand of my Divisor, making it 1342, then multiplying 1342 by 7, the Product, to wit, 2684 I place under my Dividend, and seeing they are equal, and that nothing remains, I find my Number was a Square rational Number, and the Root is 672.

To prove your Work, multiply 672 = Root
By 672

```
  1344
 4704
4032
——————
451584 = given Number.
```

After the like manner the Square Root of 2985984 would be found to be 1728.

But if your Number to be extracted, have a remainder, then you may know it is Irrational, and the Root cannot be got exact: Although by adding Cyphers, you may come as near the Truth as you please.

EXAMPLE.

Let it be required to extract the Square Root of 160; or which is the same, to find the Length of one Side of a Square Acre.

Having pointed my Number, and wrought as before, I find 12 for my nearest Root, and 16 to remain, to which adding two Cyphers, I find my next Figure to be 6, which I cut off from the rest, as part of a Decimal Fraction, which by continually adding pairs of Cyphers to each Remainder, I increase to 5 places, which is exact enough, not wanting 2 Parts, if Unity were divided into a hundred thousand Parts; for if I square 12.64911, it will produce 159.9999837921.

Thus the Square Root of any mixt Number may be found, the fractional Part first reduced into even Places of Decimals, or supply'd, if need be; so if the Square Root of $17\frac{1}{2}$ were required to 3 places of Decimals, the Work would stand as here, and the Square Root would be 4.183.

See the Work.

```
        160(12.64911
        1
       22)060
           44
      246)1600
          1476
     2524)12400
          10096
    25289)230400
          227601
   252981)279900
          252981
  2529821(2691900
          2529821
           162079
```

See the Work.

```
   17.500000(4.183
   16
  81)150
      81
  828(6900
      6624
 8363)27600
      25089
       2511
```

The

The Square Root of a vulgar Fraction, that is commensurable to its Root, may easily be found, by extracting the Square Root of the Numerator for the Numerator of the Root, and likewise the Square Root of the Denominator, for the Denominator of the said Root, which Fraction is the Root sought. So if the Square Root of $\frac{9}{49}$, were required, it would be found to be $\frac{3}{7}$, for the Square Root of 9 is 3, of 49 is 7, equal to $\frac{3}{7}$: and so of any other.

After this manner may the Square Root of a mixt Number, which is commensurable to its Root, be easily found.

But if your Fraction be incommensurable to its Root, then the best way will be to reduce it into a Decimal. And extract the Root as before taught.

So if the Square Root of $\frac{3}{80}$ were required unto 4 places of Decimals, it would be .1936, as you see in the Work.

$\frac{3}{80}$ is equal to .0375

```
then .0375 (.1936
       1
      ---
  29) 275
      261
      ----
 383) 1400
      1149
      -----
3866) 25160
      23196
      -----
       1904
```

And so farther if you please.

But if you have it to fall in some Operation, you may prefix its radical Sign before it thus, $\sqrt{\frac{3}{80}}$; and so of any other.

In the last place I will show how to find the Square Root of an irrational Number nearly, without the help of Decimals, being a useful Notion for such as understand not those Fractions : And it is thus.

After you have found the integral Part of your Root to its Quadruple, add Unity for the Denominator of the fractional part, and the Remainder doubled is Numerator ;

So the Root of 160 in this Method will be 12[illegible]; and thus of any other.

Extraction of the *Cube Root*.

1. A Cube Number is that which is contained under 3 equal Numbers, or which is equally equal.

So 8 is a Cube Number, contained under 3 equal Numbers to wit, 2.2 and 2, for two times 2 is 4, and 2 times 4 is 8; and the Cube Number 27, is contained under 3.3, and 3, for 3 times 3 is 9, and 3 times 9 is 27; and of the rest as in the following Table.

A Table of Cubes, with their Genitive equal Numbers as far as the 9 Digits.

Equal Numbers.							Cubes
1	into	1	into	1	is		1
2	into	2	into	2	is		8
3	into	3	into	3	is		27
4	into	4	into	4	is		64
5	into	5	into	5	is		125
6	into	6	into	6	is		216
7	into	7	into	7	is		343
8	into	8	into	8	is		512
9	into	9	into	9	is		729

And when it is required to extract the Cube Root of any given Number, we have nothing to do but to find that equal Number of which it is composed; so if the Root of 64 were required, it would be found to be 4, as in the Table.

Here 4 is the Root, or first Power, and 4 times 4 is 16, the second Power, and 4 times 16 is 64, or the third Power, which is the Cube.

Of Cube Numbers to be extracted, are three Sorts.

First,

First, Single.
Secondly, Compound.
Thirdly, Irrational.

Single are all such Squares as are composed, or made up of any of the 9 Digits, of which sort are those in the aforegoing Table.

Compound are all such Cubes that are composed of more Figures than one, as 1000, whose Root is 10, 1331, whose Root is 11, or 1728, whose Root is 12, *&c.*

Irrational are all such Cubes, whose Root cannot be discovered by Art exactly, neither in whole Numbers, nor Fractions, but something will still remain, there being no Proportion yet found betwixt an Irrational or surd Number, and its Root; such Numbers are 5. 7. 36. 160. 1526, *&c.*

The Extraction of the Cube Root participates something of the Nature of Division, yet a deal more difficult. The Root of any single Cube Number is found by Inspection, as in the foregoing Table may be seen.

But if it be a Compound Cube Number, it must be prepared by pointing thus: Make a Point under your Units place, and omitting Two, point every third Figure, and as many Points as your Number contains, so many Figures will your Root consist of. Then proceed by the following Directions.

A Rule to get by Heart.

The Cube of your first Period take,
And of its Root a Quotient make;
Which Root into a Cube must grow,
And from your Period taken fro:
To the Remainder then you must
Bring down another Period just,
Which being done, then you must see
Your Number straight divided be
By just 300 *times the Square*
Of what your Quotient Figures bear;
Which do, so that you in may take
The Fact your Quotient Figures make,
Last, squar'd and multiplied by th' Rest
And Product Thirty times exprest,

The

The Cube of your last found Figure too,
You must put in, if Right you do;
Repeat your Work and so descend,
From Point to Point unto the End;
That done, if ought remain there shall,
Add trebled Cyphers for a Decimal.

EXAMPLE.

Let it be required to Extract the Cube Root of 46656.

1. First, Point your Number as directed, whereby you may see the Root will have but two Places.

2. Seek the greatest Root of your first Period 46, which by the aforegoing Table you will find to be 3, which place in your Quotient, and the Cube thereof 27 place under 46, Subtract 27 from 46, and there will rest 19, as you see, if you observe the Work: This is your first Work, and no more to be repeated.

```
46656 (3
 .  .
27
——
19
```

3. To your Remainder 19, bring down your next and last Period 656, and it will make 19656 for a Dividend, then Square your Quotient, 3 makes 9, which multiply by 300, produceth 2700 for a Divisor. Seek, how oft 2 in 19; *Ans.* but 6 times, because of the Increase that will come from my Quotient, then I multiply my Divisor by 6, and the Product 16200 I place orderly under my Dividend, having separated them with a small Line, then proceed to find the Encrease coming from my Quotient, thus square your last Figure 6, *facit* 36, which multiply by the rest of your Quotient here by 3, *facit* 108, and this by 30, *facit* 3240, which place orderly under my last Number 16200, then Cube the Figure last placed in your Quotient, here 6 *facit* 216, which place orderly under your last Number 3240, and add your 3 Subducends (for so many you have in every Operation after the first) into one Sum, *facit* 19656; and seeing it is equal to my Dividend, and no more Periods to bring down, I see my Work is finished, and my Number a right Cube Number, and the Root is 36.

Note, As many Operations or Periods as you have, except the first, so oft this last Work is to be repeated.

See

See the Work.

```
      46656 (36 Quote equal the Root.
      27
      ----
2700) 19656 Dividend.
      -----
      16200 ⎫
       3240 ⎬ Subducends.
        216 ⎭
      -----
Sum   19656 From Dividend Sub.
      -----
Rest     00
```

PROOF.

```
Root    36          Square 1296
        36          Root     36
        ---               -----
        216                7776
       108                3888
       ----               -----
Square 1296               46656 Cube.
```

EXAMPLE II.

Let it be required to find the Cube Root of this Number 673373097125.

1. First, I point my Number, by which I see my Root will have four Places.

2. Next, Seek the greatest Root of your first Period 673, which by the Table is 8, which place in your Quote, and the Cube thereof 512, place under 672, and Subtract, Rest 161; this is the first Work, and no more to be repeated.

```
673373097125 (8
512
---
161
```

3. To the Remainder 161, bring down your next Period 373, makes 161373 for a Dividend, to which 19200 (being 300 times the Square of 3 your Quotient) is Divisor; and

and confidering how oft my Divifor is contained in my Dividend (fo as to allow place for my Subducends) I find it 7 times, place 7 in the Quotient, by which multiplying my Divifor, the Product I place under my Dividend for my firft Subducend ; next I Square my laft Figure 7, which multiplied by 8, and then by 30, gives 11760 for my fecond Subducend, which I place under my laft, and the Cube of 7 my laft quoted Figure is my third Subducend, which I place under the other two, and adding them, the Sum is 146503, which I Subtract from my Dividend, and the Remainder is 14870, then will the Work appear thus,

673373097125 (87

512

1. Divifor 19200) 161373 Dividend 1.

134400
11760 } Subducends.
343

Sum 146503 From Dividend Subtract.

Reft 14870

3. To this Remainder bring down your next Period 097, then will your fecond Dividend be 14870097, to which 2270700 (being 300 times the Square of your Quotient 87) is Divifor, and dividing by the Caution before given, I find the next Figure of my Root to be 6, and my firft Subducend is 13624200, fquare 6 *facit* 36, which multiplied by 87, makes 3132, and this by 30, gives 93960 for my fecond Subducend, and the Cube of 6, which is 216, is my third Subducend, which placed as before taught, and as you fee, in the Work, and then added, the Sum is 137183761, which I fubtract from my laft Dividend, and the Remainder is 1151721.

Then will the Work appear as in the following Operation.

673373

673373097125 (876

512

1\. Divisor 19200) 161373 Dividend (1.)

134400 }
11760 } Subducends.
343 }

Sum 146503 From Dividend Sub.

2\. Divisor 2270700) 14870097 Dividend (2.)

13624200 }
93960 } Subducends.
216 }

Sum 13718376 From Dividend Sub.

Rest 1151721

4\. To this Remainder bring down the last Period 125, and your third and last Dividend will be 1151721125, to which 230212800 is Divisor, which is 300 times the Square of 876 your Quotient, and Dividing as before, I find my fourth Figure to be 5, and my first Subducend is 1151064000, and mul. 876 by the Square of 5, and that by 30, gives 657000 for my second Subducend, and the Cube of 5, to wit, 125 is my third Subducend, which added into one Sum, makes 1151721125, and seeing it is equal to my last Dividend, and no more to bring down; I see my Work is finished, and the Number given, a right Cube Number, and my Root sought is 8765. And the whole Work appears as here.

67337309 7125 (8765

512

(1.) Divisor 19200) 161373 Dividend (1.)

134400
11760 } Subducends.
343

Sum = 146503 From Dividend Sub.

(2.) Divisor 2270700) 14870097 Dividend (2.)

13624200
93960 } Subducends.
216

Sum 13718376 From Dividend Sub.

(3.) Divisor 230212800) 1151721125 Dividend (3.)

1151064000
657000 } Subducends.
125

Sum 1151721125 From Divid. Sub.

Rest 00

PROOF.

Root 8765 } Mul.
8765

43825
52590
61355
70120

76825225 Square.

Multiply 76825225 Square
By 8765 Root

384126125
460951350
537776575
614601800

67337309 7125 Cube.

But if your Number to be extracted have a Remainder, it is then an irrational Number, and the exact Root cannot by

by Art be discovered, though you may find it near enough for Practice; if to the Remainder in every Operation you add 3 Cyphers, and so work as far as you will.

EXAMPLE.

Let it be required to extract the Cube Root of 282, or which is the same, to find the side of that Cubical Vessel which shall just contain a Gallon of Ale, being 282 Solid Inches.

Seeing there will be but one Point in the given Number, the Integral Part of the Root is found by inspection by your Table, and is six Inches; then adding three Cyphers to every Remainder throughout the whole Operation, I find the Fractional Part to three Places of Decimals to be .557; so that the side of the Cubical Vessel is six Inches and 557 Parts of a Thousand. And thus of any other.

The Work.

```
           282 (6.557
           216
10800)   66000
         54000
          4500
           125
         58625
1267500)  7375000
          6337500
            48750
              125
          6386375
128707500) 988625000
           900952500
             962850
                343
           901915693
Remainder   86709307
```

And after this manner may the Cube of any Fraction, or mixt Number be found, by reducing the fractional Part into Decimals, either of 3, 6, 9 or 12 places, as you desire your Root to be less or more exact.

So if the Cube Root of $\frac{3}{4}$ were required, the Work to 3 places of Decimals would stand thus, and the Root will be .908.

See the Work.

$$
\begin{array}{rl}
 & .750000000(.908 \\
 & 729 \\
\hline
2430000) & 21000000 \\
\hline
 & 19840000 \\
 & 172800 \\
 & 512 \\
\hline
 & 20013312 \\
\hline
\text{Rest} & 986688
\end{array}
$$

So the Cube Root of 25 $\frac{5}{8}$ will be 2.9481, and so of any other.

But if your mixt Number or Fraction be commensurable to its Root, then you may extract the Cube Root of the Numerator for the Numerator of the Root, and the Cube Root of the Denominator for the Denominator of the said Root; so, the Cube Root of $\frac{27}{64}$ will be $\frac{3}{4}$, for the Cube Root of 27 is 3, and of 64 is 4, which is $\frac{3}{4}$; and so of any other.

But if your Fraction or mixt Number be Incommensurable to its Root, you must work as before, or if you have no present Occasion for it, you may prefix its radical Sign, So the Cube Root of $\frac{11}{15}$ would be expressed thus, $\sqrt{c}\,\frac{11}{15}$ or $\sqrt{}\frac{11}{15}$; and so of any other.

As in the Square Root, so here I will shew you how to find the Cube Root of an Irrational Number near, without the Use of Decimal Fractions, and it is thus.

After you have found the Integral Part of your Root, to the Treble thereof add Unity, and that Sum added to the Square of the said Root tripled is the Denominator, to which the Remainder is Numerator; so the Cube Root of 282 will be found to be $\frac{66}{127}$, which is near enough for ordinary Practice, or which is the same, if you find the difference betwixt the Cube of the Root, and the Cube of the Root *Plus Unity*, you have the Denominator as before.

For

For the Cube of the Root 6 is 216, and the Cube of 7, *viz.* the Root *Plus Unity* is 343, their Difference is 127, which is the Denominator as before.

The Use of the Square and Cube Roots.

HERE follow some Uses of the *Square and Cube Roots*, both in Arithmetick and Geometry.

PROBLEM I.

To find a Mean proportional between any two Numbers given.

RULE.

The Square Root of the Product of the given Numbers is the Mean Proportional sought.

So a Mean Proportional betwixt 16 and 64 will be 32.

This Problem is of excellent Use in finding the side of a Square equal to any Parallellogram, *Rhombus*, *Rhomboides*, Triangule, or Regular Polygon.

For if in a Parallellogram you suppose the two sides, or in a *Rhombus* or *Rhomboides* the side and Perpendicular falling thereon: In a Triangle, the Base and $\frac{1}{2}$ the Perpend. or Perpendicular and $\frac{1}{2}$ the Base; and in a Regular Polygon the $\frac{1}{2}$ Perimeter and Perpendicular, or $\frac{1}{2}$ Perpendicular and Perimeter; I say, if you suppose them as two Numbers given, and by the foregoing Problem find a Mean Proportional given, is the side of a Square equal sought.

From this Problem by consequence follows *Prob.* 2.

PROB. II.

To find the side of a Square equal in *Area*, to any given Superficies whatsoever.

RULE.

The Square Root of the Content of any given Superficies is the side of the Square equal sought.

So

So if the Content of a given Circle be 160, the side of the Square equal will be $12\frac{32}{49}$ *fere*, or more exact in Decimals 12.64911.

Here if you suppose the Content to be the Product of two Numbers, as in many Cases it is, it will be the same as to find a Mean Proportional betwixt those two Numbers.

PROB. III.

The Area of a Circle given, to find the Diameter.

RULE.

As 355 to 452, or as 1 to 1.273239 : : so the Area : to the Square of the Diameter.

What length of a Cord will fit to tye to a Cow-Tail, the other End fixed in the Ground to let her have liberty of eating an Acre of Grass and no more, supposing the Cow and Tail to be 5 Yards and half.

Say, as 355 : to 452 : : so 160 being the Area of a Circle whose Content is an Acre : to 203.7183, whose Square Root is the Diameter, (*viz.*) 14.273 Perches, the Semidiameter is 7.136, from which subtract one Perch for the Cow and Tail, rest 6.136 Perch for the length of the Cord.

PROB. IV.

The Area of a Circle given to find the Periphery.

RULE.

Say as 113 to 1420, or as 1 to 12.56637, so the Area to the Square of the Periphery.

So if the Area of a Circle be 160, the Periphery will be found to be 44.84 *fere*.

PROB. V.

The Sum of the Squares of two Numbers together, with the Square of the $\frac{1}{2}$ Sum being given, to find the Numbers.

RULE.

From the Sum of the Squares subtract the doubled Square of the $\frac{1}{2}$ Sum, half the Remainder is the Square of $\frac{1}{2}$ their Difference ; and if to the $\frac{1}{2}$ Sum you add their half Difference, you have the greater Number, and by Subtraction, the less.

Let the Sum of the Squares of 2 Numb. be 3161, and the Square of their $\frac{1}{2}$ Sum 1560.25 and let the Two Numbers be sought.

The

The doubled Square of the $\frac{1}{2}$ Sum is 3120.5, which sub. from the Sum of the Squares 3161, there will rest 40.5 half of which is 20.25, whose Square Root is 4.5, and is the $\frac{1}{2}$ diff. which add to the Square Root of 1560.25 (*viz.*) 39.5, and it will give 44, the greater Number; and if you Sub. 4.5 from 39.5 you have the less, to wit, 35.

PROB. VI.

The Sum of the Squares of two Numbers, together with the Square of their $\frac{1}{2}$ diff. being given, to find the Numbers themselves.

RULE.

From the $\frac{1}{2}$ Sum subtract the Square of $\frac{1}{2}$ their Diff. the Remainder is the Square of the $\frac{1}{2}$ Sum of those two Numbers, then work by the last.

Let the Sum of the Squares of 2 Numbers be 3161, and the Square of their half diff. is 20.25; I demand the two Numbers.

Half the Sum of the Squares is 1580.5 from which Sub. 20.25 the Square of their half diff. rest 1560.25, whose Square Root is 39.5, which is the $\frac{1}{2}$ Sum; and the Square Root of 20. 25, is 4.5, then the Sum of 4.5 and 39.5 is 44, the greater Number; and their diff. is 35, the less Number.

PROB. VII.

The Sum of the Squares of the half Sum, and half the difference of two Numbers, with one of them being given, to find the other. The Rule follows.

From the doubled Sum of the said Squares subtract the Square of the given Number, the Remainder is the Square of the Number required.

Let the Sum of the Squares of $\frac{1}{2}$ the Sum, and $\frac{1}{2}$ the diff. of 2 Numbers be 1580.5; and let the less Number be 35, from 3161, the double Sum of the Squares subtract 1225, the Square of 35, the Remainder is 1936, whose Square Root is 44, which is the other Number.

PROB. VIII.

Any two Sides of a right angled Triangle being given to find the third Side.

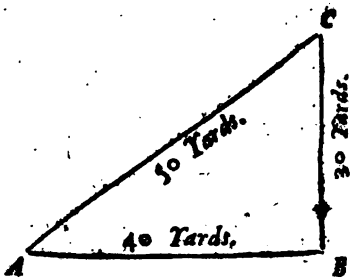
In this useful Problem lies hid a great part of the Mathematicks, the Invention whereof is fathered upon *Pythagoras*, the Demonstration thereof, *Euclid* hath in the 47th Proposi-

tion

on of the first Book of his *Elements of Geometry*; where it is proved that the Square of the Hypothenuse, or longest side of a right angled Triangle, is equal to the Sum of the Squares of the Base and Perpendicular, or the other two Sides.

In the annexed Triangle *A B C*, let the Base or Ground *A B*, represent the Breadth of a Moat or Ditch, and let the Perpendicular *B C*, represent the height of a Castle, Tower, or City Wall, and let the Hypothenuse or longest side represent the length of a scaling Ladder.

Let the Base *A B*, or the Breadth of the Ditch be 40 Yards, and the Perpendicular *B C*, or the Height of the Wall be 30 Yards; What length will the Hypothenuse *A C*, or the Scaling Ladder be?



RULE.

The Square Root of the Sum of the Squares of the Base and Perpendicular, is the length of the Hypothenuse.

Ans. 50 Yards the length of the Ladder.

For the Square of the Base 40 is 1600
And the Square of the Perpend. 30 is 900

The Sum is 2500 (50 the Root.

25

000

But if the Breadth of the Ditch were required, and the Perpend. and Hypothenuse were given, then this is

The RULE,

The Square Root of the Difference of the Squares of the Hypothenuse and Perpendicular, is the Length of the Base, or Breadth of the Ditch.

For

For the Square of the Hypot. *AC* is	2500
And the Square of the Perpend. *BC* is	900
Diff.	1600(40 Root
Here you see the Base is 40.	16
	000

And if *BC* were required from the given sides *AB*, and *AC*, then the square Root of the Difference of the Distance of the Squares of the Hypothenuse and Base, is the Height of the Perpendicular, or *BC*.

PROB. IX.

To divide a Number given by Extream and Mean Proportion.

RULE.

Multiply the Square of your given Number by 5, and divide the Prod. by 4, and from the square Root of the Quotient Subtract $\frac{1}{2}$ your given Number, the Remainder is the greater Portion, which subtract from the whole, gives the less.

Let the given Number be 12, whose Square is 144, which multiplied by 5, produceth 720, which Product divided by 4, gives 180, whose square Root is $13\frac{2}{5}\frac{2}{3}$, from which subtract $6\frac{1}{2}$, your given Number, rest $7\frac{2}{5}\frac{2}{3}$ for your greater part, which subtracted from the whole Number 12 gives $4\frac{3}{5}\frac{1}{3}$ for the less.

PROB. X.

Any Number of Men being given, to form them into a Square Battle, or to find the Number of Ranks and File.

RULE.

The square Root of the Number of Men given, is the Number of Men either in Rank or File.

Let there be an Army of Men of 32400, and let us form them into a Square Battle; extract the Square Root of 32400, and it will be found to be 180, which shews there will be 180 Men in Rank, and as many in File.

See the Work.

$$\begin{array}{rl} & 32400\ (180 \text{ Root.} \\ & 1 \\ \hline 28) & 224 \\ & 224 \\ \hline & 000 \end{array}$$

Thus much shall suffice for the Use of the square Root: We will now proceed to some Uses of the Cube Root.

The chief Use of the Cube Root, is to find out a Proportion between like Solids, as Globes, Cylinders, Cubes, &c.

PROB. I.

If a Bullet of Brass of 8 Inches Diameter weigh 72 Pound, What shall a Bullet of Brass weigh, whose Diameter is 4 Inches?

RULE.

Since like Solids are in Triple Proportion to their Homologous Sides, Diameters, Lines, &c. it holds,

As the Cube of the Diameter given:
To the Weight thereof ::
So the Cube of the Diameter sought:
To the Weight thereof:

See the Work.

$$\begin{array}{rl} & \text{C.D.}\quad lb.\quad \text{C.D.} \\ \text{If} & 512 : 72 :: 64 \\ & \quad\quad 64 \\ \hline & \quad\quad 288 \\ & \quad 432 \\ \hline 512) & 4608\ (9 \\ & 4608 \\ \hline & \quad 0 \end{array}$$

Facit 9 Pound.

PROB. II.

If a Ship of 100 Tun be 44 Foot long at the Keel, of what length shall the Keel be of a Ship of 220 Tun?

Say,

Say, as 100 : to the Cube of 44, (viz.) 85184 :: So 220 : to 187404.8, whose Cube Root is 57.225, the Length of the Keel sought.

PROB. III.

The Side of a Cube being given, to find the Side of that Cube that shall be double, treble, &c, in Quantity to the given Cube.

RULE.

Cube your Side given, and multiply it by 2, 3, &c. The Cube Root of the Product is the Side sought.

There is a Cubical Vessel whose Side is 12 Inches, and it is required to find the side of that Vessel which shall contain 3 times as much.

The Cube of 12 is 1728
Multiply by 3 — 3

5184 Product.

The Cube Root of which Product is 17.306, the Side sought.

After the same Method may you find a Side that shall contain $\frac{1}{2}$ as much, $\frac{1}{4}$ as much, or any other given Quantity.

PROB. IV.

To find the Side of a Cube that shall be equal in Solidity to any given Solid, as Globe, Cylinder, Prism, Cone, or such like.

RULE.

The Cube Root of the Solid Content of any Solid Body given, is the Side of the Cube of equal Solidity.

So, if the Content of a Globe were found to be 15625 solid Inches, seek the Cube Root of 15625, which is 25, which is the Side of a Cube of equal Capacity.

PROB. V.

Between 2 Numbers given to find, 2 mean Proportionals.

RULE.

Multiply the less Extream by the Cube Root of the Quotient, of the greater Extream divided by the less, the Product is the lesser of the two mean Proportionals, which

multiplied by the said Cube Root, gives the greater Mean sought: So if two mean Proportionals betwixt 6 and 162, were sought, they would be found to be 18 and 54, for 162 divided by 6, quotes 27, whose Cube Root is 3, by which multiplying 6, the less Extream, gives 18 for the less Mean; and 18 multiplied by the same Root 3, gives 54 for the greater; or if you divide the greater Mean by the same Root, it quotes the greater Mean as before.

PROB. VI.

The Concave Diameter of 2 Guns being known, together with the Quantity of Gun-Powder sufficient to charge one, to find what will be sufficient to charge the other. The Capacities are one to another, as the Cubes of their Diameters.

EXAMPLE.

If .43 Pound of Gun-powder be sufficient to charge a Gun, whose Concave Diameter is one Inch $\frac{1}{2}$ or 1.5, how much Gun-powder will suffice to charge a Gun whose Concave Diameter is 7 Inches? *Ans.* 43 Pound and $\frac{7}{10}$.

Say, as 3.375 is to .43 :: so 343 is 43.7 Pound.

But if the Gun-powder given and required, be of different Strengths, the Question requires two Operations, the first of which finds the Quantity either of a stronger or weaker sort, and the Proportion is Inverse, the 2d is as in this Example. Many more Uses might be named, but let this suffice in this place.

Simple INTEREST.

WE shall proceed now to Interest of Money, wherein the greatest and most useful Practice of *Decimal Arithmetick* consists.

When a Sum of Money is lent by one to another for any time agreed on, and an Allowance granted for the Loan of the same: Here the Money so Lent is called the Principal or Stock, and the Allowance or Gain is called the Interest.

Interest

Interest formerly was very high, *viz.* 12, 10, or 8, *per Cent. per Annum*, but by an Act of Parliament made in *August*, 1651, it was brought down to 6 *per Cent. per Annum*, and so still continues, above which no Person dare pretend to take, nor is any obliged to give.

Notwithstanding a sufficient Person may have Money in most Places at 5 *per Cent. per Annum*, and in some Places for less.

Interest is either Simple or Compound.

When a Sum of Money is lent, and the Interest thereof when due is not paid, but kept in the Borrower's Hands, and yet becomes not a part of the Principal, then it is called *Simple Interest*.

The Business of *Simple Interest* is performed by a Rank of Numbers Arithmetically proportional, from which naturally will arise this Theorem.

If a pair of Ranks of Numbers shall be so posited, as to have the same common Ratio betwixt every pair of Correspondents; then it follows; That the Numbers themselves, their Correspondent Sums, and Correspondent Differences, have the same common Ratio.

ILLUSTRATION.

2 . 8		.6 . 3.0	
3 . 12		.7 . 3.5	
4 . 16		.8 . 4.0	
5 . 20		.9 . 4.5	
14 . 56		3.0 . 15.0	

In the first pair of Ranks the Ratio is 4, then you may take any Number in the first Rank of the first Pair, suppose 4, then it holds;

As 4 to 16 : : so 14 : to 56, & *Contra* understand the same in the second pair of Ranks.

In the Solution of Questions of Simple Interest, four Things are to be considered.

First, The Principal or Money lent: Secondly, The Time for which it is lent: Thirdly, The Rate or Gain, suppose of one Pound in a Year: And, Fourthly, The amount.

Any

Any three of these being given, to find the fourth, as in these four *Propositions* following.

P R O P. I.

Principal, Rate, and Time given, to find the Amount.

R U L E.

To the Product of the Rate, multiplied by the Time, add Unity; that Sum multiplied by the Principal, gives the Amount.

E X A M P L E.

What will 20 *l.* amount to, forborn 7 Years, at 6 *per Cent.* Simple Interest?

Principal 20 *l.* Rate .06 *l.* Time 7 *Years.*

The Work.

```
 .06
   7
 ----
1.42
  20
 ----
28.40
```

Answer 28 *l.* 8 *s.*

E X A M P L E II.

What will 36 Pound amount to, lent from *May* the 9th, 1712. until *November* the 17th next following, simple Interest being computed at 5 *per Cent.* Principal 36, Rate .05, Time .526? *Answer* 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$.

13 Months = 1 Year
4 Weeks = 1 Month
7 Days = 1 Week

```
    .526
     .05
 -------
 1.02630
      36
 -------
  615780
 307890
 -------
36.94680
```

And

And seeing the Time is both given and required in Years and Parts, we have annexed a *Decimal Table* thereof; supposing a Year the Integer, and divided, as noted before; the Use is the same as in other Decimal Tables, and needs no Explication.

Betwixt *May* the 9th, and *November* the 17th, are 192 Days, gathered as in the annexed Work; and by the annexed Decimal Table is found to be 6 Months, 2 Weeks, and 3 Days; the Decimal of which is .526027; and because the 4th Figure is a Cypher, we have only used three Places, they being sufficient to find the Days in every Month, observe this old Rule.

May	22
June	30
July	31
Aug.	31
Sept.	30
Octob.	31
Nov.	17
Sum	192

A Decimal T A B L E *of Time, one Year the Integer.*

Mon.	*Dec.*	*Days*
12	.920548	336
11	.843835	308
10	.767123	280
9	.690411	252
8	.613698	224
7	.536986	196
6	.460274	168
5	.383561	140
4	.306849	112
3	.230137	84
2	.153424	56
1	.076712	28
Weeks	***Dec.***	***Days***
3	.057534	21
2	.038356	14
1	.019178	7
Days	***Decimals***	
6	.016438	6
5	.013698	5
4	.010959	4
3	.008219	3
2	.005479	2
1	.002739	1

Thirty Days hath September,
April, June *and* November;
All the rest have Thirty One,
Excepting February *alone,*
Which hath but 28 Days clear,
And 29 every Leap Year.

And

And that nothing may be wanting, we have added a small Table, which gives the number of Days betwixt any two given Times with much ease, as for *Example.*

The TABLE.				
31	334	*January*	00	31
59	306	*February*	31	28
90	275	*March*	59	31
120	245	*April*	90	30
151	214	*May*	120	31
181	184	*June*	151	30
212	153	*July*	181	31
243	122	*August*	212	31
273	92	*September*	243	30
304	61	*October*	273	31
334	31	*November*	304	30
365	00	*December*	334	31

1. From the beginning of the Year, to the 11th of *July*, what number of Days?

Over-against *July*, on the Right-hand, I find 181, and 11 Days more make 192, the Answer.

2. Betwixt *May* the 9th, and *September* the 17th next following how many Days? Subtract *May* 120 *plus* 9, equal 129; from *September* 243 *plus* 17, equal 260, rest 131, the number of Days sought.

3. From the 5th of *November*, 1712, to the 16th of *May* 1713, how many Days?

Add 25, the complement of 5, to 30 (the Days in *November*) to 31 found on the Left-hand *November*, and to that Sum add 120, for *May*, *plus* 16, the Sum is 192; and those are the Number of Days sought.

But to proceed,

PROP. II.

The Amount, Rate, and Time being given, to find the Principal.

RULE.

To the Rate multiplied by the Time, add Unity, by which dividing the Amount, quotes the Principal.

EXAMPLE.

What present Money will pay a Debt of 28 *l.* 8 *s.* due seven Years hence, at 6 *per Cent.* simple Interest, Amount 28.4 *l.* Rate .06, Time 7 Years? *Answer* 20 *l.*

The

The Work.

```
   .06
     7
  ----
  .42) 28.4 (20 l.
1.42) 28.4 (20 l.
       284
       ---
         0
```

EXAMPLE II.

What present Money will pay a Debt of 36.9468, or 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$ due 192 Days, or 6 Months, 3 Weeks and 3 Days hence, Rebate being allowed, at 5 *per Cent.* Simple Intereft. Amount 36.9468, Rate .05. Time .526. *Anfwer* 36 *l.*

The Work.

```
      .526
       .05
   -------
   1.02630) 36.9468 (36 l.
            307890
            ------
             615780
             615780
             ------
                  0
```

PROP. III.

Principal, Amount, and Rate given, to find the Time.

RULE.

From the Amount Subtract the Principal, the Difference divided by the Product of the Principal and Rate, gives the Time.

EXAMPLE.

In what time will 20 *l.* raife a Stock of 28 *l.* 8 *s.* or 28.4 at 6 *per Cent.* Simple Intereft? Principal 20 *l.* Amount 28.4 Rate .06. *Anfwer* in 7 Years.

The Work.

```
                28.4
  20            20
 .06           ----
----     1.20) 8.40 (7
1.20           840
               ----
                  0
```

EXAMPLE II.

In what time will 36 Pound amount to, 36.9468, or 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$ at 5 *per Cent.* Simple Interest? Principal 36, Amount 36.9468, Rate .05. *Answer* in 6 Months, 3 Weeks and 3 Days.

The Work.

```
              36.9468
              36.
              -------
       1.80)  .9468      (.526 = 192 Days, or
 36                        6 Months, 3 Weeks,
 05            900         and 3 Days.
1.80          -----
----           468
               360
              -----
               1080
               1080
              -----
                 0
```

PROP. IV.

The Principal, Time, and Amount given, to find the Rate.

RULE.

From the Amount subtract the Principal, the Remainder divided by the Product of the Principal and Time, gives the Rate.

EXAM-

EXAMPLE.

At what rate of Simple Interest will 20 *l.* amount to, 28.4 in 7 Years? *Answer* 6 *per Cent.*

The Work.

```
 20   28.4
  7   20
---  -----
140)  8.40 (06
      840
     -----
        0
```

EXAMPLE II.

If 36 *l.* amount to 36.9468, or 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$ in 6 Months, 3 Weeks and 3 Days, what rate of Simple Interest is implied in this Bargain? Principal 36, Time .526, Amount 36.9468. *Answer* 5 *per Cent.*

The Work.

```
  .526          36.9468
   36           36.0
 ------        -------
  3156   18.936) .94680 (.05
 1578            .94680
 ------          ------
 18.936             0
```

The two first Propositions being of most common Use, I have annexed two Tables, which give the Amount and Rebate, or the present Worth of any Sum of Money for any time under 32 Years; the Construction of them lies in the *Proposition's* themselves, by making one Pound the Principal in the first, and one Pound the Amount in the second; or if you divide Unity by the Number in the first Table, the Quotient is the Numbers in the second.

TABLE I. Shewing the Amount of one Pound for 31 Years, at 5 and 6 per Cent. *Simple Interest.*			TABLE II. Shewing the Rebate of One Pound for 31 Years, at 5 and 6 per Cent. *Simple Interest.*	
Years	5	6	5	6
1	1.05	1.06	.952380	.943396
2	1.10	1.12	.909091	.892857
3	1.15	1.18	.865561	.847457
4	1.20	1.24	.833333	.806451
5	1.25	1.30	.800000	.769230
6	1.30	1.36	.769230	.735294
7	1.35	1.42	.740740	.704225
8	1.40	1.48	.714286	.675675
9	1.45	1.54	.689655	.649359
10	1.50	1.60	.666666	.625000
11	1.55	1.66	.645161	.602409
12	1.60	1.72	.625000	.581395
13	1.65	1.78	.606060	.561797
14	1.70	1.84	.588235	.543478
15	1.75	1.90	.571428	.526315
16	1.80	1.96	.555555	.510204
17	1.85	2.02	.540540	.495049
18	1.90	2.08	.526315	.480769
19	1.95	2.14	.512820	.467289
20	2.00	2.20	.500000	.454545
21	2.05	2.26	.487804	.442477
22	2.10	2.32	.476190	.431034
23	2.15	2.38	.465116	.420168
24	2.20	2.44	.454545	.409836
25	2.25	2.50	.444444	.400000
26	2.30	2.56	.434781	.390625
27	2.35	2.62	.425532	.381679
28	2.40	2.68	.416666	.373134
29	2.45	2.74	.408163	.364963
30	2.50	2.80	.400000	.351143
31	2.55	2.86	.392157	.349650

In

In the Use of these Tables, the Proportion runs

As 1 : to the Tabular Number :: So the Sum of Money given : to the Answer sought ; and seeing the first Number is an Unit, the whole Work requires only a single Multiplication, as in the Examples following may be seen.

EXAMPLE I.

In the Use of the first Table.

What will 20 *l.* amount to, forborn 7 Years, at 6 *per Cent.* Simple Interest? *Answer* 28 *l.* 8 *s.*

Tabular Number answering 7 Years, and under 6 *per Cent.* in the first Table is 1.42

Multiply by 20

28.40

EXAMPLE II.

What will 3 *l.* 17 *s.* 6 *d.* amount to, forborn 21 Years, at 5 *per Cent.* Simple Interest? *Answ.* 7 *l.* 18 *s.* 10 *d.* $\frac{1}{2}$.

Tabular Number under 5 *per Cent.* and answering 21 Years, is 2.05

Multiply by the Principal 3.875

```
   1025
  1435
 1640
615
7.94375
```

EXAMPLE III.

In the Use of the second Table.

What ready Money will pay a Debt of 28 *l.* 8 *s.* due 7 Years hence, at 6 *per Cent.* Simple Interest? *Answer* 20 *l.*

Tabular Number against 7 Years, and under 6 *per Cent.* is .704225

Multiply by the Debt 28.4

```
  2816900
 5633800
1408450
19.9999900
```

PROP.

EXAMPLE IV.

What ready Money will pay a Debt of 12 *l.* 12 *s.* 6 *d.* due 20 Years hence, Rebate being allow'd, at 5 *per Cent.* Simple Interest? *Answer* 6 *l.* 6 *s.* 3 *d.*

Tabular Number against 20 Years, and under 5 *per Cent.* is .500000
Multiply by the Debt 12.625

Product = 6.3125

And thus may any Question of this Nature be resolved, if the Time given be even Years, but if not, use the Propositions themselves, as was shewn before.

SECT. II.

But if your Question be concerning Annual Rents, Payments or Annuities to be Bought or Sold for some Time, then you are to consider it under these four Particulars,

First, The Annuity or Pension.

Secondly, The Time of Continuance.

Thirdly, the Rate of Interest.

Fourthly, The present Worth.

Any three of these being given, the fourth thence may be found, as in the four following Propositions may be seen.

PROPOSITION I.

Annuity, Rate, and Time given, to find the present Worth.

RULE.

To the Square of the Time, multiplied by the Rate, add the double of the Time, from which subtract the Rate multiplied by the Time, the Remainder multiplied by the Annuity, and that Product divided by the double of the Rate, multiplied by the Time, *plus* 2, the Quotient is the present Worth.

EXAMPLE.

What is an Annuity of 20 *l. per Annum* clear Value, to be sold for 7 Years, worth in ready Money, Simple Interest, being computed at 6 *per. Cent*? *Ans.* 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$, *fere.*

Annuity

Annuity 20 *l.* Time 7 Years, Rate .06.

The Work.

```
                                                   .12
                              16.52                  7
                                 20               ----
49 = square Time           -------                2.84
.06 = the Rate     2.84)330.40 (116.338
----                        ..
2.94                     284
14.     .06              ---
----      7              464
16.94   ---              284
   42   .42              ----
-----                    1800
16.52                    1704
                         ----
                          960
                          852
                         ----
                         1080
                          852
                         ----
```

EXAMPLE II.

There is an Annuity of 12 *l.* 10 *s.* *per Annum* to continue 21 Years, what is it worth at 5 *per Cent.* Simple Interest? *Answer* 192 *l.* 1 *s.* 5 *d.* $\frac{1}{2}$.

Annuity 12.5 *l.* Time 21 Years, Rate .05.

The Work.

```
                           12.5 Annuity
                             63
    21                      ----
    21         21           375
   ----         2           750
    21        ----     4.1) 787.5 (192.0731
   42          42            ..
   ----       ----          41
   441         21          ----
   .05        .05           377
  -----       ----          369
  22.05       1.05          ----
  42.                        85
  -----                      82
  64.05                     ----
   1.05                      300
  -----                      287
  63.                       ----
                              130
                              123
                             ----
                               70
                               41
                             ----
                               29
```

PROP. II.

Present Worth, Time, and Rate given, to find the Annuity.

RULE.

Multiply the double of the Rate, multiplied by the Time *plus* 2, by the present Worth; that Product divided by the Square of the Time, multiplied by the Rate *plus* the double of the Time, *minus* the Rate multiplied by the Time, the Quotient is the Annuity.

EXAMPLE.

What Annuity to continue 7 Years at 6 *per Cent.* Simple Interest, will 20 *l.* Purchase? *Anf.* 3 *l.* 8 *s.* 9 *d.* $\frac{1}{4}$ *fere.*

Present Worth 20 *l.* Time 7 Years, Rate .06.

The Work.

.12	
7	49
	.06
2.84	
20	2.94
	14
16.52) 56.80 (3.4382	
4956	16.94
	.42
7240	16.52
6608	
6320	
4956	
13640	
13216	
4240	

EXAMPLE II.

What Annuity to continue 21 Years, will 192.0731 *l.* or 192 *l.* 1 *s.* 5 *d.* $\frac{1}{2}$ purchase, at 5 *per Cent.* Simple Interest?

Present Worth 192.0731 *l.* Time 21 Years, Rate .05.

RULE

RULE.

Present worth	192.0735 —
Product of the double of the Rate, multiplied by the Time *plus* 2, is	4.10

63) 7875 (12.5
63
157
126
315
315
0

Answer 12 *l.* 10 *s.*

This is the Converse of *Question* the 2d, and *Proposition* the first, aforegoing.

PROP. III.

The Annuity, present Worth, and Time being given, to find the Rate of Interest.

RULE.

The Product of the Annuity and Time, *minus* the present Worth, being multiplied by 2, and divided by the Sum of double the present Worth, multiplied by the Time *plus* the Annuity, multiplied by the Time *minus* the Annuity, multiplied by the square of the Time, the Quotient is the Rate.

EXAMPLE I.

At what rate of simple Interest will 116.338, or 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$ Purchase an Annuity of 20 *l.* to continue for 7 Years? *Answer* 6 *per Cent.* the Rate.

Annuity 20 *l.* present Worth 116.338; Time 7 Years.

The Work.

116.338	20	20	20
2	7	7	49
232.676	140	140	180
7		116.338	80
1628.732		23.662	980
140		2	
1768.732	788 732) 47.3240 (.06		
980			
788.732			

EXAMPLE II.

At what Rate of Simple Interest will 250 *l.* Purchase an Annuity of 30 *l. per Annum,* to continue 10 Years? *Answer* 4 *l.* 6 *s.* 11 *d.* ½ *fere* the Interest *per Centum* sought.

Annuity 30 *l.* present Worth 250, Time 10 Years.

The Work.

2300) 100.0000 (.043478

PROP. IV.

Annuity, Rate, and present Worth being given, to find the Time of Continuance.

RULE.

To the doubled Product of the present worth, multiplied by the Rate, add the Product of the Annuity multiplied by the Rate, the square of the Difference betwixt this last Sum (which we may call *A*) and double the Annuity (which may be called *B*) added to the octuple Product of present Worth; Annuity and Rate multiplied one into another, the square Root of this last Sum added or subtracted to, or from, the former Difference according as *A* was either greater or less than *B*; this last Sum or Difference, divided by double the Annuity, multiplied by the Rate, quotes the time sought.

EXAMPLE.

In what time will 7 Pound *per Annum* pay a Debt of *l.*
120.4 or 120 *l.* 8 *s.* at 6 *per Cent.* simple Interest?

Annuity

Annuity 7 *l.* Rate .06 *l.* Present Worth 120 *l.* 4 *s.*

Ans. In 25 Years.

The Work.

```
Present worth 120 4    Annuity 7
    Rate          .06      Rate .06
            -------        ----
            7.224           .42
    Mul. by     2                     Doub. of Ann.  14
            -------                   Rate          .06
           14.448                                   ----
    Add       .42                                    .84
            -------
           14.868 = A
           14.000 = B Double the Annuity.
            -------
             .868   Difference.   Present Worth 120.4
             .868                 Rate            .06
            -------                          ---------
             6944                                7 224
            5208                  Annuity            7
           6944                              ---------
            -------                             50.568
          .753424                 Mul. by            8
        404.544                              ---------
       ----------                 Prod.        404 544
        405.297424   ( 20.132
         . . . . .      .868 = former Difference.
       4              ------
 -----------          21. Sum A was Greater than B.
 401) 00529
        401
 -----------
 4023) 12874                .84) 21.00 ( 25 Years.
       12069
 -------------                   168
 40262) 80524                   -----
        80524                    420
 -------------                   420
            0
```

PROP. V.

To the aforegoing four Propositions we may add a fifth which is by halving the Annuity, Time and Rate given, to find the Amount.

RULE.

From the given Number of Years subtract an Unit, half of the Remainder, multiplied by the Product of the Rate and Time, and to this Product adding the Number of Years given, the Summ multiplied by the Annuity gives the Amount.

EXAMPLE.

An Annuity of 20 *l. per Annum* is forborn 7 Years, what will then be due at 5 *per Cent.* Simple Interest? Annuity 20 *l.* Time 7 Years, Rate .05. *Answer* 161 *l.*

The Work.

½ 6 = time — 1	.35	.05
3	3	7
	1.05	.35
	7	
	8.05	
	20	
	161.00	

EXAMPLE II.

A Tradesman binds his Son an Apprentice for 7 Years, and at the same time lets an Annuity of 36 *l.* 15 *s.* run to the Expiration of the said Term, that it may be a Stock for his Son. The Question is, What this Stock will be, accounting Simple Interest at 6 *per Cent*? *Ans.* 303 *l.* 11 *s.* 1 *d.* ¼ *fere.*

The

The Work.

7	7	36.75
.06	1	8.26
.42	6	22050
3		7350
1.26	3	29400
7		303.5550
8.26		

The First, Second and Fifth Propositions being of most common Use, we have annexed Tables fitted thereto, at 5 and 6 *per Cent.* as in the first Section, whereby the Answer may be found at one single Operation, as in the Use of them may be seen.

We need not say any thing of their Composition, the Propositions themselves, being the Fountains whence they were drawn.

The

Years.	The present Worth of one Pound Annuities for 31 Years at 5 and 6 per Cent. Simple Interest. TABLE I.		The Annuity that one Pound will Purchase for 31 Years at 5 and 6 per Cent. Simple Interest. TABLE II.		Amount of Pound An[nuities] for 31 Y. [at] 5 and 6 p[er] C. simp. In[terest]. TAB. III	
	5	6	5	6	5	6
1	0.952386	0.943396	1.05	1.06	1.00	1.0
2	1.863636	1.839285	.536585	.543689	2.05	2.0
3	2.739130	2.694915	.365079	.371069	3.15	3.1
4	3.583333	3.516129	.279069	.284403	4.30	4.3
5	4.400000	4.307692	.227272	.232142	5.50	5.6
6	5.192307	5.073529	.192607	.197101	6.75	6.9
7	5.962962	5.816901	.167702	.171913	8.05	8.2
8	6.714285	6.540546	.148936	.152892	9.40	9.6
9	7.448275	7.246753	.134259	.137992	10.80	11.1
10	8.166666	7.937500	.122041	.125984	12.25	12.7
11	8.870967	8.614457	.114545	.116084	13.75	14.3
12	9.562500	9.279069	.104575	.107738	15.30	15.96
13	10.242424	9.932584	.097633	.100678	16.90	17.68
14	10.911764	10.853260	.091644	.092184	18.55	19.46
15	11.571428	11.210526	.086420	.089207	20.25	21.30
16	12.222222	11.836734	.081318	.084482	22.00	23.20
17	12.864864	12.455445	.077731	.080286	23.80	25.16
18	13.500000	13.057067	.074074	.076527	25.65	27.18
19	14.128205	13.672897	.070780	.073137	27.55	29.26
20	14.750000	14.272727	.068644	.070063	29.50	31.40
21	15.365853	14.865044	.065079	.067262	31.50	33.60
22	15.976190	15.456896	.062593	.064697	33.55	35.86
23	16.581393	16.042016	.060308	.052336	35.65	38.18
24	17.181818	16.602459	.058201	.060158	37.80	40.56
25	17.777777	17.200000	.056250	.058139	40.00	43.00
26	18.369565	17.773437	.054438	.056263	42.25	45.50
27	18.957447	18.343511	.052749	.054515	44.55	48.06
28	19.541666	18.910447	.051172	.052880	46.90	50.68
29	20.122449	19.472627	.049695	.051349	49.30	53.36
30	20.700000	20.035714	.048309	.049911	51.75	56.10
31	21.274510	20.594404	.047005	.048556	54.25	58.90

The

The Use of these Tables are the same with those aforegoing as in the following Examples may be seen.

EXAMPLE I.

In the Use of the first Table.

There is an Annuity of 20 *l. per Annum* to continue for 7 Years, to be sold for ready Money, what is the Value thereof, allowing Simple Interest at 6 *per Cent*?

Tabular Number, answering 7 Years and under 6 *per Cent*.

In the first Table is	5.816901
Multiply by	20
Answer 116 *l.* 6 *s.* 9 *d.* ¼ *fere.*	116.338020

This agrees with Example the first, in the first Proposition of this Section.

Note, But if this were wrought by the common Tables of Simple Interest, shewing the present Worth of Annuities, printed in several Books of Arithmetick, the Answer would be found to be 113 *l.* 19 *s.* 6 *d.* 3 *q. fere.* So if their Tables be true, he that gives 116 *l.* 6 *s.* 9 *d.* ¼ is cheated of 2 *l.* 7 *s.* 2 *d.* ½, but this we will try.

A lends *B* 113 *l.* 19 *s.* 6 *d.* ¾ for 7 Years, which at the End thereof amounts to 161 *l.* 16 *s.* 11 *d.* ½, this is not deny'd. At the same time *B* delivers up to *A* an Annual Rent of 20 *l. per Annum*, to continue the same Term, and accordingly receiv'd it Annually. Now the Question is, whether the reception of these 7 Annual Payments, quit the Scores betwixt *A* and *B*, both being obliged under the same Rate of Interest, for by their Tables it must; but it may appear to any considerate Man it doth not, which I prove thus.

	Years.				
If 100 *l.* paid before it be due	6	be worth	136	what 20 *l. facit*	27.20
	5		130		26.00
	4		124		24.80
	3		118		23.60
	2		112		22.40
	1		106		21.20
	0		100		20.00
				Summ is =	165.20

Thus

Thus you may see the 7 Annual Payments are worth, at the End of 7 Years 165 *l.* 4 *s.* and the whole Amount that *A* could claim of *B*, was but 161 *l.* 16 *s.* 11 *d.* $\frac{1}{2}$; therefore *A* is in *B*'s Debt 3 *l.* 7 *s.* 0 *d.* $\frac{1}{2}$.

But by this Table of ours, if *A* had lent to *B* 116.338 or 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$, it would upon the Condition aforesaid, have amounted to 165.2 or 165 *l.* 4 *s.* whereby it is evident our Table is grounded on a firm Foundation. See *Dary in his Interest Epitomiz'd.*

EXAMPLE II.

An Annuity of 51 *l.* 15 *s.* to continue 21 Years is to be sold for ready Money, what is it worth at 5 *per Cent.* Simple Interest? *Anf.* 795 *l.* 3 *s.* 8 *d. fere.*

Tabular Number answering 21 Years and under 5 *per Cent.* in the first Table, is 15.365853

Multiply by 51.75

```
     15.365853
         51.75
   -----------
      76829265
    107560971
    15365853
   76829265
   -----------
   795.18289275
```

EXAMPLE III.

In the Use of the second Table.

What Annuity to continue 7 Years, will 20 *l.* purchase at 5 *per Cent.* Simple Interest? *Anf.* 3 *l.* 7 *s.* 1 *d. fere.*

Tabular Number answering 7 Years, and under 5 *per Cent.* in Table the Second, is .167702

Multiply by 20

```
   .167702
        20
  --------
  3.354040
```

QUEST. II.

What Annuity, to continue 21 Years will 65 *l.* 10 *s.* purchase at 6 *per Cent.* Simple Interest?

Tabular

Tabular Number answering 21, and under 6 *per Cent.* in the second Table, is .067262

Multiply by 65.5

336310
336310
403572

Answ. 4 *l.* 8 *s.* 1 *d.* $\frac{1}{2}$. 4.4056610

EXAMPLE I.

In the Use of the Third Table.

An Annuity of 20 *l. per Annum* is forborn for 7 Years, what will then be due at 5 *per Cent.* Simple Interest? *Answer* 161 *l.*

Tabular Number answering 7 Years and under 5 *per Cent.* in the third Table, is 8.05

Multiply by 20

161.00

EXAMPLE II.

What will an Annuity of 36 *l.* 11 *s.* 3 *d.* amount to, forborn 21 Years at 6 *per Cent.* Simple Interest? *Answer* 1228 *l.* 10 *s.*

Tabular Number answering 21 Years, and under 6 *per Cent.* in Table the Third, is 33.60

Multiply 36.5625
By 33.6

2193750
1096875
1096875

1228.50000

Thus may any other Question be resolved if the time given be even Years, not exceeding 31 Years; and the Rate either 5 or 6 *per Cent.* If otherwise, you must use the Propositions themselves.

Here follow some more Questions for the Use of the said Tables, wherein appears some more Variety.

QUEST. I.

A hath an Annuity of 20 *l. per Annum* to continue 7 Years. *B* hath an Annuity of 5 *l.* 10 *s.* to continue 21 Years. These two Persons would change Annuities, and allow each other Simple Interest at 6 *per Cent.* The Question is, who must pay Money, and how much? *Answer*, *A* must receive from *B* 34 *l.* 11 *s.* 7 *d.* $\frac{2}{4}$ on the Conditions aforesaid.

Seek by Table the First, in the second or last Head of Tables, the present Worth of 20 *l. per Annum* to continue 7 Years, which you may see by the Work will be 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$.

$$\begin{array}{r} 5.816901 \\ 20 \\ \hline A = 116.338020 \end{array}$$

Seek likewise by the same Table, the present worth of 5 *l.* 10 *s. per Annum* to continue the same time, which by the Work you may see will be 81 *l.* 15 *s.* 1 *d.* $\frac{3}{4}$.

$$\begin{array}{r} 14.865044 \\ 5.5 \\ \hline 74325220 \\ 74325220 \\ \hline B = 81.7577420 \end{array}$$

	l.	*s.*	*d.*	*q.*
From	116 :	06 :	9 :	1
Sub.	81 :	15 :	1 :	3
Rest	34 :	11 :	7 :	2

QUEST. II.

A lends to *B* 360 *l.* upon a Mortgage of Land, whose Rent is 75 *l. per Ann.* *B* keeps his Money 5 Years, during which time *A* receives the said Rent; at the end of which time,

time they come to Account. The Question is, whether this Rent hath cleared the Mortgage, if not, what must *B* pay to *A*, Simple Interest at 6 *per Cent.* being accounted on both sides?

First, Seek by the first Table in the former Head of Tables, the Amount of 360 *l.* to continue 5 Years at 6 *per Cent.* Simple Interest, which you will find to be 468 *l.*

$$\begin{array}{r} 1.30 \\ 360 \\ \hline 78 \\ 39 \\ \hline 468.00 \end{array}$$

Secondly, Seek by Table the Third in the second Head of Tables, what an Annuity of 75 *l.* *per Annum* will amount to, forborn 5 Years at 6 *per Cent.* Simple Interest, which you will find to be 420 *l.* which *B* must pay unto *A* before he hath discharged his Mortgage.

$$\begin{array}{r} 5.60 \\ 75 \\ \hline 2800 \\ 3920 \\ \hline 42.000 \end{array}$$

QUEST. III.

A Merchant is indebted 360 *l.* the Creditor is contented to receive the same at 10 equal Payments, the Debtor allowing for the Forbearance of the same Money after the Rate of 6 *per Cent.* Simple Interest. The Question is, what those Payments ought to be? *Answer*, 45 *l.* 7 *s.* 1 *d.*

Seek in the second Table of the last Head of Tables, for the Numbers over-against 10 Years, and under 6 *per Cent.* which Number multiplied by 360 gives the Answer.

See the Work.

.125984
360

755904
377952

45.354240

QUEST. IV.

A Gentleman bequeathed 1500 *l.* to his Daughter to be paid her at 14 Years end. The Executors desire to pay ready Money, so they may be abated after the Rate of 5 *per Cent.* Simple Interest. The Question is, what ready Money will pay this Legacy? *Ans.* 882 *l.* 7 *s.* 0 *d.* $\frac{1}{2}$.

Seek in the second Table in the first Head of Tables, for the Number over-against the 14*th* Year, and under 5 *per Cent.* which multiplied by 1500 gives the Answer.

See the Work.

.588235
1500

2941175
588235

882.352500

QUEST. V.

A Gentleman hath 160 *l.* which he would lay out to Purchase 20 *l. per Annum.* How many Years must the said Annuity continue, Simple Interest being computed at 6 *per Cent?*

First, Divide 160 by 20 Quotes 8, which shews he gives 8 Years Purchase for the said Annuity.

Secondly, Seek in the first Table of the last Head of Tables, and under 6 *per Cent.* for the Number 8; but not finding it, I look the next less, which is 7.937500, and over-against it is 10 Years, which shews it must continue somewhat above 10 Years; and to find which, say, As the Difference betwixt this and the next bigger Number

.676957

.676957 to 1 : : so the Difference of the next less and 1 : (*viz.*) .0625 to .092 Parts of a Year, equal to 1 Month and 5 Days, accounting 13 to the Year. So his Annuity must continue 10 Years, 1 Month, and 5 Days.

Let these suffice in this Place.

We should now proceed to *Compound Interest* and *Annuities*, but these things being best performed by Logarithms, we will therefore treat of them when we come into *Logarithmical Arithmetick*.

We shall now proceed to other Parts of Arithmetick, as *Trett* and *Tare*, *Barter*, *Fellowship*, &c.

Rules in Trett *and* Tare, *&c.*

THese are Allowances commonly used among Merchants, in such Commodities as are sold by Weight.

Tare is the Weight of the Bag, Chest, Hogshead, *&c.* wherein the Goods are carried or put.

Trett is an Allowance of 4 *lb.* in 100 or 104 *lb.* for Goods, wherein is loss, as *Treacle*, *Sugar*, &c.

Cloff is an Allowance of 2 Pound upon every Draught which exceedeth 336 Pound, or 300 Gross Weight.

Subtile is the Weight of the Goods when the Tare is Subtracted, but not the Trett.

Neat Weight is the Remainder when both (if both be allow'd) are taken away.

QUESTION I.

In 6 Bags of Raisins, marked with the Gross Weight, as followeth, Tare 20 Pound *per* Bag, what Neat Weight?

First,

First, Multiply 20, the Weight of each Bag, by 6, the Number of Bags, produceth 120, which divided by 112 gives 1 C. 0 *q*. 8 *lb.* which subtracted from the Gross, gives the Neat Wt.

	C.	*q.*	*lb.*	
A	7	2	11	
B	8	1	17	
C	6	3	14	
D	9	0	10	
E	8	2	17	
F	6	1	20	
	47	0	05	Gross Wt.
Sub.	1	0	8	Tare.
Ans.	45	3	25	Neat Wt.

```
        20
         6
        ---C.   q.   lb.
112)120(1   0    8
    112
    ---
      8
```

QUEST. II.

In 346 *C.* 3 *q.* 11 *lb.* Gross, Tare 12 *lb. per C.* how many *C.* Neat?

First find the Pounds Gross

```
        C.    q.   lb.
Thus   346    3    12
         4
       ----
       1387 Quarters.
         28
       -----
      11098
      2775
      -----
      38848 lb. Gross.
       4162 Tare Sub.
      ------ C.    q.   lb.
112) 34686 (309    2    22 Neat Wt.
     336
     ----
      1086
      1008
      ----
Rest    78 lb. = to 2 q. 22 lb.
```

To find the Tare.

```
        lb.  lb.     lb.
Say if 112 : 12 :: 38848
                       12
                    -----
                    77696
                    38848
                    ------
              112)466176(4162 = (the Tare
                  448
                  ---
                   181
                   112
                   ---
                    697
                    672
                    ---
                     256
                     224
                     ---
               Rem.   32 Inconsid.
```

QUEST.

QUEST. III.

In 674 *C.* 2 *q.* 16 *lb.* Groſs Weight, Tare 14 *lb. per C.* what Neat Weight?

	C.	q.	lb.	
$\frac{1}{8}$	674	2	16	Groſs Wt.
	84	1	9	Tare.
	590	1	7	Neat Wt.

When Tare, as here, is an Aliquot Part of 112, take ſuch Part of the Groſs Weight, which Subtracted from the Groſs leaves the Neat Weight, as in the Example I take $\frac{1}{8}$ Part for 14 *lb.*

Any Aliquot Part is ſhewn in the annexed Table.

Aliquot Parts of 112 *lb.*	
7 is	$= \frac{1}{16}$
14 is	$= \frac{1}{8}$
16 is	$= \frac{1}{7}$
28 is	$= \frac{1}{4}$
56 is	$= \frac{1}{2}$
84 is	$= \frac{3}{4}$

The Tare in the ſecond Queſtion might have been found more quickly, as in the following Queſtion, where the Tare is no Aliquot Part of 112.

QUEST. IV.

In 246 *C.* 2 *q.* 16 *lb.* Groſs, Tare 21 *lb. per C.* How many Hundred Neat? *Anſ.* 200 *C.* 1 *q.* 16 *lb.*

Reduce the Groſs into Pounds thus,

C.	q.	lb.
246	2	16
4		
986		
28		
7894		
1973		
27624 *lb.* Groſs		
5179 $\frac{1}{2}$ Tare.		

To find the Tare.

C.	q.	lb.
246	2	16
21		
246		
492		
5166		
10 $\frac{1}{2}$		
3		
5179 $\frac{1}{2}$ Tare.		

Here to find the Tare I multiply the Groſs *C.* by 21, and for the 2 Quarters I took $\frac{1}{2}$ the 21 Pound, and becauſe 16 is $\frac{1}{7}$ of 112, I took $\frac{1}{7}$ of 21, which is 3, which added is the Tare, as in the Work.

	C.	q.	lb.	
112)22444 $\frac{1}{2}$ (	200	1	16 $\frac{1}{2}$	Neat Wt.
224				
		q.	lb.	
044		1	16 $\frac{1}{2}$.	

The

The last Question may be answered as in the annexed Work; for though 21 *lb.* is not an Aliquot Part of 112 *l.* it may be parted into Aliquot Parts, (*viz.*) 14 and 7. For 14 *lb.* take $\frac{1}{8}$ Part, and for 7 *lb.* take $\frac{1}{2}$ of 14, those two added together give the Tare, which subtracted from the Gross, leave the Neat Weight as before. Thus you may do in many Cases, which the Learner should observe.

	C.	q.	lb.	
$\frac{1}{8}$	246	2	16	Gross Wt.
$\frac{1}{2}$	30	3	9	Tare.
	15	1	18 $\frac{1}{2}$	
Sub.	46	0	27 $\frac{1}{2}$	
	200	1	16 $\frac{1}{2}$	Neat Wt.

QUEST. V.

In 72 *C.* 3 *q.* 12 *lb.* Gross, Tare 16 *lb. per C.* Trett 4 *lb. per* 104. How many *C.* Neat?

The Tare is found as before, which Subtracted from the Gross, leaves the Subtile, which I divide by 26, which is $\frac{1}{4}$ of 104 quotes the Trett, which taken from the Subtile leaves the Neat Weight.

	C.	lb.	q.	
$\frac{1}{7}$	72	3	12	Gross Wt.
	10	1	18	Tare.
$\frac{1}{26}$	62	1	22	Subtile.
	2	1	17	Trett.
	60	0	05	Neat Wt.

QUEST. VI.

In 6 Hogsheads of Tobacco, containing 56 *C.* 2 *q.* 20 *lb.* Gross, Tare 30 *lb. per* Hogshead, 4 *lb. per* 104 Trett, and 2 *lb.* upon every 300 Gross Weight for Cloff; What will the Neat Weight cost at 6 *d.* $\frac{1}{2}$ *per* Pound; 51 *l.* 11 *s.* 8 *d.* being deducted for Custom and other Charges? *Answer*, 108 *l.* 11 *s.* 9 *d.*

See the following Work.

See the Work.

lb.	Mul. 30	336)5931(17 for the Cloff.
112	By 6	
56		336
	180 Tare.	
672		2571
560	26)6168(237 $\frac{1}{13}$ Trett.	2352
56	52	
20		219
	96	
6348 Grofs.	78	At 6 *d.* $\frac{1}{2}$, Wt. 5914?
180 Tare.		
	188	2957
6168 Subtile.	182	246 = 5
237 Trett.		
	6	320|3 = 5
5931		160 = 3 = 5
17 Cloff.		51 = 11 = 8
5914 Neat.		*Anf.* 108 = 11 = 9

The Rule of BARTER.

BArter is a Rule by which Merchants or others exchange Goods of feveral Prices and Quantities, fo as to receive no Lofs by fuch Truck or Change.

Obferve the Nature and Work of the following Queftions.

QUEST. I.

How many Pound of Sugar at 4 *d.* $\frac{1}{2}$ *per* Pound, muft be given in Barter for 60 Groce of Incle, at 8 *s.* 8 *d. per* Groce?

First, By Practice find the Value of the 60 Groce of Incle at 8 *s.* 8 *d. per* Groce, which will be 26 *l.* which divided by the Decimal of 4 *d.* $\frac{1}{2}$, which is .01875, Quotes 1386 $\frac{2}{3}$, the Number of Pounds fought.

s. d.
At 8 8 Groce, Wt. 60 Groce?

24
2

facit 26 *l.*

 .01875

·01875) 26.00000 (1386.666

1875	At 4 *d.* $\frac{1}{2}$ *lb.* Wt.	1386 $\frac{2}{3}$?
7250		
5625		
16250		
15000		
12500		
11250		
1250		

Proof	17	6	6
	8	13	3
			3
	26	00	0

Or the last Question may be wrought thus, bring the Price of a Groce, (*viz.*) 8 *s.* 8 *d.* into Pence, which will be 104. then say,

If 4 *d.* $\frac{1}{2}$, or 4.5 become 104 *d.* what will 60 become? *Facit* 1386 $\frac{2}{3}$, as before.

But you may observe the Terms are not methodically Stated; but because the 3d multiplied by the 2d, and divided by the 1st, gives the true Answer, we have so placed the Numbers: The Reader therefore is to consider such Cases, that he be at no loss; for the Reason is evident.

QUEST. II.

A hath 2 Packs of Cotton-Wool at 240 *lb. per* Pack, and at 15 *d. per* Pound ready Money; in Barter he will have 16 *d.* $\frac{1}{2}$. *B* hath Broad Beds at 5 *s. per* Yard ready Money. The Question is, to know how *B* must raise his Beds in Barter *per* Yard, that he be no loser, and how many Yards will be Equivalent to the 2 Packs of Cotton?

Find first how much he must raise his Beds in Barter, thus, if 15 *d.* become 16 *d.* $\frac{1}{2}$, what will 5 *s*? *Facit* 5 *s.* 6 *d.* this done, at 16 *d.* $\frac{1}{2}$ *per* Pound, what will 2 Packs or 480 *l.* cost? And *Note*, that as many Pence as the Pound costs, so many Pounds will the Pack cost. So the two Packs will cost 33 Pound: Then as in the last Question, divide 33 by the Decimal of 5 *s* 6 *d.* (*viz.*) .275, Quotes 120, the Number of Yards sought.

See

See the Work.

```
          d.    d.      s.
First, if 15 : 16.5 : : 5
                         5
                 ----
            15) 82.5 (5.5     Facit 5 s. 6 d.
                75
                --
                 75
                 75
                 --
                  0

Secondly, .275)33.000(120 Yards.
              275
              ---      At 5 s. 6 d. Yd. Wt. 120 ?
               550                          ---
               550                           30
               ---                            3
                00   Proof                  ---
                                  Facit      33
```

QUEST. III.

A hath 100 Yards of Kersey at 3 *s. per* Yard ready Money, which he Barters with *B* at 3 *s.* 6 *d.* taking small Hair-Buttons at 15 *d. per* Groce, which are but worth 12 *d.* How many Groce of Buttons will pay for the Kersey; and whether doth *A* or *B* get the better Bargain, and by how much?

```
First, at 3 s. 6 d. per Yd. what 100 Yds?
                             ----
                             12.5
                              5.0
                             ----
                     Facit   17.5 or 17 l. 10 s.
```

Secondly, .0625) 17.5000 (280 Numb. of Gr. sought.

```
1250
----
5000
5000
----
   0
```

Thirdly, At 3 *s.* Yd. Wt. 100 ?

```
 10
  5
---
Facit 15 l. A
```

Fourthly, At 12 *d.* Gr. what 28.0 ?

Facit 14 *B*

A's Goods are worth 15 *l.* and *B*'s Goods worth 14 *l.* by which it is evident *B* gets the better bargain by 1 Pound, or 20 Shillings.

But if it were required to know by how much *per Cent.* *B* hath the better Bargain; then say, if 14 *l.* become 15 *l.* what will 100 *l.* become ? *Ans.* 7 *l.* $\frac{1}{7}$, or 7.142 the Gain *per Cent.*

```
14) 1500 (107.142
    14
    ---
     100
      98
     ---
       20
       14
      ---
        60
        56
       ---
        40
```

QUEST.

QUEST. IV.

A hath Linnen Cloth at 10 *d.* the Ell ready Money, in Barter 12 *d.* *B* hath 3610 Pound of Sugar at 7 *d.* the Pound ready Money, and would have of *A* 35 *l.* in ready Money, the rest in Linnen Cloth. The Question is, what Rate the Sugar bears in Barter, and how much Linnen Cloth *A* must give unto *B*?

It is evident that as much Sugar as *B* receives ready Money for, which is 35 Pounds worth, he ought but to have 7 *d.* *per* Pound for it.

However first find the Barter thus, if 10 *d* : 12 *d* :: 7 *d* : *facit* 8 $\frac{4}{10}$, then find the Quantity of Sugar *A* must have for his 35 *l.* thus if $\frac{7}{240}$: $\frac{1}{1}$ :: $\frac{35}{1}$ *l.* *facit* 1200 *lb.* of Sugar, which subtract from 3610, resteth 2410, which at 8 *d.* $\frac{4}{10}$ the Pound, will amount to 84 *l.* 7 *s.* or 84.35, which divided by the Decimal of 1 Shilling, *viz.* .05 gives 1687, the Quantity of Linnen Cloth in Ells, which was required.

l.

The Rate the Sugar bears in Barter, is 8.4 *B* must receive 1687 Ells of Linnen Cloth, and 35 *l.* in ready Money.

QUEST. V.

Two Merchants Barter, *A* hath 20 Hundred of Cheese, at 21 *s.* 6 *d.* the Hundred. *B* hath 8 Pieces of *Irish* Cloth at 3 *l.* 14 *s.* *per* Piece. The Question is whether must receive Money, and how much? *Ans.* *A* must pay to B, 8 *l.* 2 *s.*

First say at 21 *s.* 6 *d.* the C, what 20 C?

	1	
Ans. 21 *l.* 10 *s.* *A*'s Goods	0	10
Facit	21	10

Secondly say, at 3 *l.* 14 *s.* Pie. what 8 Pieces?

				24	
Ans.	29	12 *B*'s Goods.		4	
Sub.	21	10		1	12
Rest	8	2	*Facit*	29	12

QUEST. VI.

A Barters with *B* Silk Stockings at 30 *s. per* Pair, which are vendible but for 26 *s.* and would have ⅓ ready Money, and gain 10 *l. per Cent.* for Stuff at 4 *s. per* Yard ready Money, how must the Yard of Stuff be valued to equal the Barter?

	l. *l.* *s.*		*s.*
First I say, if	100 : 110 : what 30?		*Facit* 33
	30		
	33.00		

Then because *A* would have ⅓ in ready Money, the Barter only resteth in the ⅔, which is the Remainder; wherefore subtract ⅓ of 33, which is 11, from the true Price 26, resteth 15, and from it self, to wit 33, leaves 22, then say,

If 15 : 22 : : 4		*Ans.* at 5 *s.* $\frac{13}{15}$
4		
15) 88 (5 $\frac{13}{15}$		
75		
13		

QUEST. VII.

A Barters with *B* Cloves at 6 *s.* for 7 *s.* 6 *d. per* Pound, and is willing to lose 10 *l. per Cent.* to have ⅓ ready Money, What's the just Price of a Yard of Velvet deliver'd at 22 *s.* to equal the Barter?

	l. *l.* *s.*	
First say, if	100 : 90 : : 7.5	
	75	
	450	
	630	
	6.750	*Facit* 6 *s.* 9 *d.*

Next for $\frac{1}{3}$ ready Money, which he desires, abate from 7 *s.* 6 *d.* the Bartering Price $\frac{1}{3}$ thereof, which is 2 *s.* 6 *d.* rest 5 *s.* then take 2 *s.* 6 *d.* from 6 *s.* 9 *d.* rest 4 *s.* 3 *d.*

```
               s.    s.    s.
Then say, If 5 : 4.25 : : 22
                  22
               -------
                 850
                850
               -------   s.
            5) 93.50 (18.7 equal to the just price of
                           a Yard of Velvet.
               5
               --
               43
               40
               --
                35
                35
                --
                 0
```

The Rule of Fellowship.

THE *Rule of Fellowship* is for Merchants or other Traders, where they have joint Stocks in Company, to distribute unto every one his proportional Share of the Gain or Loss, according to his Stock laid out.

It is divided into two Parts, commonly called the *single*, and *double Rule of Fellowship*, of which in their Order.

In the *single Rule*, Having the Particular Stocks, and the whole Gain or Loss, to find each particular Gain or Loss, observe this general Rule.

As the total Sum of the Stocks :
  To the total Gain or Loss : :
So each Man's particular Stock :
  To each Man's particular Gain or Loss.

QUEST.

QUESTION I.

Three Merchants put in Money together, *A*, *B*, and *C*; *A* put in 20 *l.* *B* put in 30 *l.* *C* put in 40 *l.* they gained 180 *l.* what is each Man's part of the Gain?

A	20	
B	30	The total Sum 90 *l.* Gain 180 *l.*
C	40	
Sum	90	

Then say,

First, For *A*'s Share; If 90 *l.* : 180 *l.* :: 20 *l.*

```
          20
     ---------
     90) 3600 (40 for A.
         360
     ---------
          00
```

Secondly, For *B*'s Share; If 90 *l.* : 180 *l.* :: 30 *l.*

```
          30
     ---------
     90) 5400 (60 for B.
         540
     ---------
          00
```

Thirdly, For *C*'s Share; If 90 *l.* : 180 *l.* :: 40 *l.*

```
Part.                  40
A's 40 l.         ---------
B's 60            90) 7200 (80 for C.
C's 80                720
-------           ---------
Sum 180 for Proof.     00
```

But if you consider the Observation belonging to the Question, in the *Rule of Three*, you may contract your Work in this and the like Questions, as was there intimated; for if you divide your second Number 180, by your first Number 90, and by the Quotient 2, multiplying *A*, *B*, and *C*'s

Stock

Stock, (*viz.*) 20, 30, 40, produceth 40, 60, 80, *A*, *B* and *C*'s Gains.

See the Work.

90) 180 (2	Mult. 20 *l.*	Mult. 30
180	By 2	By 2
0	40 = *A*'s Gain	60 = *B*'s Gain

Mult. 40	*A* 40 Gain
By 2	*B* 60 Gain
	C 80 Gain
80 = *C*'s Gain	180 Proof.

But if you consider this Question, the Answer may more quickly be found yet; for seeing the Gain was double to the whole Stock; each Man's Gain will be double to his Stock, and such Considerations as these may be of good Use in many Cases.

QUEST. II.

A Chapman breaking, ows unto four Men these following Sums of Money, *Viz.*

		l.	*s.*	*d.*
To	*A*	21	9	6
	B	72	19	3
	C	144	13	9
	D	264	17	6
Sum=		474	00	0

His whole Estate is found to be but 148 *l.* 2 *s.* 6 *d.* What must each have of the same, and what will it be *per* Pound?

If you Work this Question the contracted way, the Quotient will be the Answer *per* Pound, to wit 6 *s.* 3 *d.*

See the whole Work.

474) 148.125 (.3125 = 6 *s.* 3 *d.* what each must have per Pound.

```
 1422
 ----
  592
  474
 ----
  1185
  948
 ----
  2370
  2370
 ----
    0
```

Reduce the broken Parts of each Man's Money into Decimals, which multiply by .3125, gives in the several Products what each must have.

```
Mult. 114.6875    Mult.    72.9625    Mult. 21.475
By      .3125     By         .3125    By     .3125
   ----------        -----------         ---------
      5734375          3648125             107375
     2293750          1459250              42950
    1146875           729625               21475
   3440625           2188875              64425
 --------------   --------------      ------------
 C=35.83984375    B=22.80078125       A=6.7109375
```

```
 Mult. 264.875               l.             l.    s.  d.  q.
 By      .3125     A = 6.7109375      =     6 : 14 : 2 : 2
   -----------     B = 22.80078125    =    22 : 16 : 0 : 0
      1324375      C = 35.83984375    =    35 : 16 : 9 : 3
      529750       D = 82.7734375     =    82 : 15 : 5 : 3
    1146875        -----------------------------------------
    794625         Proof 148.12500000 = 148 : 02 : 6 : 0
  -----------
 D=82.7734375
```

QUEST. III.

A, *B* and *C* put in Money together, *A* put in 20 *l.* *B* and *C* put in 85 *l.* they gained 63 *l.* of which *B* took up 21 *l.* What did *A* and *C* gain, and *B* and *C* put in?

Firs

First, Find *A*'s Gain, thus, If 105 the Sum of all the Stocks, Gain 63 *l.* what will 20 *l.* *A*'s Stock gain? *Facit* 12 *l.* for *A*'s Gain.

Secondly, Find *B*'s Stock, thus, If 12 *l.* which is *A*'s Gain, come from 20 *l.* *A*'s Stock; what will 21 *l.* come from, which is *B*'s Gain? *Facit* 35 *l.* which is *B*'s Stock; then *C*'s Stock must be the Remainder to 85, (*viz.*) 50 *l.* And if you subtract *A* and *B*'s Gain 33 from the whole Gain 63, rests 30 for *C*'s Gain.

QUEST. IV.

A, *B* and *C* put in Money together, *A* put in 20 *l.* *B* 30 *l.* *C* a Sum unknown; they gained 36 *l.* *C* took up 16 *l.* what did *A* and *B* gain, and *C* put in?

Subtract *C*'s Gain, 16 *l.* from the whole Gain 36 *l.* resteth 20 *l.* Then say, If 50 *l.* *A* and *B*'s Stock gain 20, what will *A*'s Stock, 20 *l.* gain? *Facit* 8 for *A*, then *B* must have 12: To find *C*'s Stock, say, if 8 Pound, which is *A*'s Gain, come from 20: What will 16 *l.* *C*'s Gain come from? *Facit* 40 *l.* which *C* put in.

QUEST. V.

A, *B* and *C* put in 360 *l.* and gained 270 *l.* of which as oft as *A* took up 3 *l.* *B* took up 5 *l.* and as oft as *B* took up 5 *l.* *C* took up 7 *l.* what did each gain, and put in.

Suppose some Number for *A*, that the rest of the Parts may be taken without Fractions: As suppose *A* had 9 *l.* then *B* must have 15 *l.* and *C* 21 *l.* whose Sum is 45.

$$\text{Then the Proportion is, as } 45 \text{ to } 270, \text{ So} :: \begin{matrix} 9 \\ 15 \\ 21 \end{matrix} \text{ to } \begin{matrix} 54 \\ 90 \\ 126 \end{matrix}$$

But you had better find a common Multiplicator, as in the contracted Way.

For the Stocks.	For the Gains.
45) 360 (8 = Com. Mult.	45) 270 (6 = Com. Mult.
360	270
0	0

Then 8 times 9 is 72 *A*
And 8 times 15 is 120 *B*
And 8 times 21 is 168 *C*

Proof = 360

Then 6 times 9 is 54 *A*
And 6 times 15 is 90 *B*
And 6 times 21 is 126 *C*

Proof = 270

QUEST. VI.

Two Merchants Company, *A* put in 36 Pound, and taketh $\frac{3}{5}$ of the Gain, what did *B* put in?

If *A* take up $\frac{3}{5}$, *B* must needs have $\frac{2}{5}$.

Then say, If $\frac{3}{5}$: 36 :: $\frac{2}{5}$: *Facit* 24 for *B*.

```
        2
      ----
   3) 72 (24
      6
      --
      12
      12
      --
       0
```

Seeing the Denominators of the Fractions are equal, I neglect them and Work with the Numerators.

QUEST. VII.

Two Merchants Company, *A* put in 20 *l.* and *B* put in 144 *Ducats*; they gain'd 97 *l.* 10 *s.* of which *A* took up 30 *l.* what is the Value of a *Ducat*? *Answer* 6 *s.* 3 *d.*

First find a Stock for *B*, equivalent to *A*'s Stock, thus.

```
                         l.  l.   l.
                      If 30 : 20 :: 67.5
144) 45.000 (.3125 =                20
       to 6 s. 3 d.                ----
     432               30) 1350.0 (45 l. = to 144 Duc.
     ---                   120
      180                  ---
      144                   150
      ---                   150
       360                  ---
       288                    0
       ---
        720
        720
        ---
          0
```

QUEST.

QUEST. VIII.

Two Merchants put in Money together, and gained 120 *l.* their Agreement was, that *A* should have 10 *l. per Cent.* gain, and *B* 8 *l. per Cent.* what must each have?

Suppose each Man's Gain *per Cent.* to be his Stock, so *A*'s Stock will be 10 *l.* and *B*'s Stock 8 *l.* whose Sum is 18 *l.* then say, If 18 *l.* : 120 *l.* :: 10 *l.* *Facit* $66\frac{2}{3}$ for *A*'s Gain, then *B* must have $53\frac{1}{3}$.

18 : 120 :: 10
        10

18) 1200 ($66\frac{12}{18} = \frac{2}{3}$ *A*'s Part
108

120   120
108      $66.\frac{2}{3}$

12      $53\frac{1}{3}$ *B*'s Part

QUEST. IX.

A, *B*, *C*, and *D* put in Money together, and gained a Sum of Money, of which *A*, *B* and *C* took 60 *l.* *B*, *C* and *D* took 90 *l.* *C*, *D* and *A* took 80 *l.* and *D*, *A* and *B* took up 70 *l.* what distinct Gain did each take up?

Add these 4 Numbers into one Sum, which makes 300 *l.* in which each Man's Money is named 3 times, therefore take $\frac{1}{3}$ of it makes 100 *l.* for the whole Gain; from which subtract *A*, *B* and *C*'s Gain 60 *l.* leaves *D*'s Gain 40 *l.* and from the same Sum 100 *l.* subtract what *B*, *C* and *D* took up, leaves 10 *l.* for *A*; and from the same Sum 100 *l.* subtract what *C*, *D* and *A* took up, to wit 80 *l.* leaves 20 *l.* for *B*'s Gain. And lastly, If from 100 *l.* you subtract what *D*, *A* and *B* took up, 70 *l.* leaves 30 *l.* for *C*'s Gain. So *A* had 10 *l.* *B* 20 *l.* *C* 30 *l.* *D* 40 *l.*

QUEST.

QUEST. X.

Four Men bought a Hive of Bees for 20 Shillings, of which *A* must pay $\frac{1}{3}$, *B* $\frac{1}{4}$, *C* $\frac{1}{5}$, *D* $\frac{1}{6}$, what must each pay of the 20 Shillings?

Here if you take the natural Parts of 20 Shillings as they are expressed in the Question;

$$\text{Then}\left\{\begin{array}{lcr} & & s.\ \ d. \\ \frac{1}{3}\text{ of }20 & = & 6 : 8 \\ \frac{1}{4}\text{ of }20 & = & 5 : 0 \\ \frac{1}{5}\text{ of }20 & = & 4 : 0 \\ \frac{1}{6}\text{ of }20 & = & 3 : 4 \\ \hline \text{Sum} & = & 19 : 0 \end{array}\right.$$

But the Sum to be paid is 20 *s.* wherefore 20 *s.* must be divided into such Proportion as the Parts bear one to another.

Wherefore reduce the Fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, into a common Denominator, which will be $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$, and neglecting the Denominators, add the Numerators into one Sum, which will be 57; then the Proportion is

$$\text{As } 57 : \text{to } \overset{s.}{20} : : \text{so}\left\{\begin{array}{l} 20 \text{ to} : \overset{s.}{7}\,\frac{1}{57} \text{ for } A \\ 15 \text{ to} : 5\,\frac{15}{57} \text{ for } B \\ 12 \text{ to} : 4\,\frac{12}{57} \text{ for } C \\ 10 \text{ to} : 3\,\frac{29}{57} \text{ for } D \end{array}\right\}\text{Payment.}$$

Proof 20 0.

But if the Agreement had been, That *A* must pay $\frac{1}{2}$, *B* $\frac{1}{3}$, *C* $\frac{1}{4}$, *D* $\frac{1}{5}$, then the Parts would have exceeded the Whole.

$$\begin{array}{lr} & s.\ \ d. \\ \text{For } \frac{1}{2} \text{ of } 20 \text{ is} & 10 : 0 \\ \text{And } \frac{1}{3} \text{ of } 20 \text{ is} & 6 : 8 \\ \text{And } \frac{1}{4} \text{ of } 20 \text{ is} & 5 : 0 \\ \text{And } \frac{1}{5} \text{ of } 20 \text{ is} & 4 : 0 \\ \hline \text{Sum is} & 25 : 8 \end{array}$$

In this Case work as before, by reducing the Fractions into common Denominators, which will be $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, the Sum of the Numerators is 77: Then,

$$\text{As } 77 : \text{to } \overset{s.}{20} : : \text{so}\left\{\begin{array}{l} 30 : \text{to} : \overset{s.}{7}\,\frac{61}{77} \text{ for } A \\ 20 : \text{to} : 5\,\frac{15}{77} \text{ for } B \\ 15 : \text{to} : 3\,\frac{69}{77} \text{ for } C \\ 12 : \text{to} : 3\,\frac{9}{77} \text{ for } D \end{array}\right\}\text{Payment.}$$

Proof 20 0

FACTORSHIP.

UNDER this Head may be reckoned those Questions which belong to Factorship, such as are these following.

QUEST. I.

A Merchant delivers unto his Factor 50 *l.* and if the Factor put in 30, he will allow him half of the Gain, What is the Factor's Person esteemed at?

Subtract 30 *l.* from 50 *l.* resteth 20 *l.* and so much is the Factor's Person esteemed at. The Reason is evident.

QUEST. II.

A Merchant delivers unto his Factor 60 *l.* and allows him for his Gains $\frac{1}{3}$ of the Gain: What Money must the Factor put in, that he may have equal Gains?

From 60 *l.* take $\frac{1}{3}$, which is 20 *l.* which subtract from 60 leaves 40 *l.* which the Factor must put in to have half the Gains.

QUEST. III.

A Merchant delivers to his Factor 500 *l.* and esteemed his Person at 200 *l.* When they made up their Accounts, they gained 20 *l.* *per Cent.* What is the Factor's Part?

To 500 add 200, the Sum is 700; then say, if 100 *l.* gain 20 *l.* what 700 *l*? *Facit* 140 *l.* then if 700 *l.* gain 140 *l.* what 200? *Facit* 40 *l.* the Factor's Part, and the Merchant must have 100 *l.*

QUEST. IV.

A Factor receives 1000 *l.* from a Merchant, to which he adds 300 *l.* of his own; his Person is esteemed at 260 *l.* What must the Factor have of the Gain?

It is evident the Merchant's Stock is 1000 *l.* and the Factor's own Stock 300 *l.* together with 260 *l.* his personal Value, makes 560 *l.* to which the Factor's Stock is equivalent.

The

The Merchant's Stock	1000
The Factor's Stock	560
Sum	1560.

Then say if 1560 : 1 : : 560, *Facit* $\frac{14}{39}$, for the Factor's Share ; then the Merchant must have $\frac{25}{39}$ for his Share.

Double Fellowship.

IT is called *Double Fellowship* when their Gains are different, not only in respect of their Stocks, but in respect of the time of Continuance in Company. And that you may work any such Question, observe this general Rule.

As the total Sum of the Products of each Man's Money and Time, is to the total Gain ; so the particular Product of each Man's Money and Time, is to each Man's particular Gain.

QUEST. I.

A and *B* put in Money together ; *A* put in 20 *l.* and *B* put in 20 *l.* likewise : But *A*'s Money was in Company 9 Months, and *B*'s but 6 Months ; they gained 60 Pounds, What must each have ?

A's Money multiplied by his Time, is	180
B's Money multiplied by his Time, is	120
The Sum	300

$$\text{Then, as } 300 : 60 :: \begin{cases} 180 : 36 & A\text{'s Gain.} \\ 120 : 24 & B\text{'s Gain.} \end{cases}$$

QUEST. II.

A, *B*, and *C* put in Money together ; *A* put in 20 *l.* for 6 Months ; *B* put in 40 *l.* for 3 Months, and *C*'s Money was 60 *l.* which continued in Company 2 Months ; their common Gain was 36 *l.* What must each have of the Gain?

First,

First, Multiply *A*'s Money . . . 20 Pounds.
By his Time . . . 6 Months.

The Product of *A*'s Money and Time = 120
Multiply *B*'s Money . . . 40 Pounds.
By his Time . . . 3 Months.

The Product of *B*'s Money and Time = 120
Multiply C's Money . . . 60 Pounds.
By his Time . . . 2 Months.

The Product of C's Money and Time = 120

Now seeing the three Products are equal one with another, it is evident every Man must have an equal Share of the Gain: So each Man's Part is 12 *l.*

QUEST. III.

A, *B* and *C* put in Money together; *A* put in 20 *l.* for 3 Months, *B* put in 30 *l.* for 5 Months, and *C* put in 40 *l.* for 7 Months; they Gain'd 60 *l.* What must each have of the Gain?

Multiply every Man's Money by his Time, and the three Products will be found to be, for *A* 60, for *B* 150, and for C 280, whose Sum is 490.

$$\text{Then say, if } 496 : 60 :: \left\{\begin{matrix} 60 \\ 150 \\ 280 \end{matrix}\right\} \textit{Facit} \left\{\begin{matrix} 7.\frac{17}{49} \\ 18.\frac{18}{49} \\ 34.\frac{14}{49} \end{matrix}\right\}$$

Proof 60.00

QUEST. IV.

A and *B* Company: *A* put in the first of *January* 50 *l.* but *B* could not put any Money in till the first of *May*; What must *B* then put in to have an equal Share with *A* at the Year's end?

If I multiply 50 *l.* which is *A*'s Money, by 12 Months, which is *A*'s Time, it will Produce 600. Now it is plain that *B*'s Money can but be in Company 8 Months; it is likewise evident that so much Money is required of *B*,

which multiplied by 8, shall produce 600; divide therefore 600 by 8, quotes 75, which is what *B* must put in.

```
8) 600 (75
   56
   --
    40
    40
    --
     0
```

QUEST. V.

A, *B* and *C* keep Company; *A* put in the first of *March* 60 *l.* *B* put in the first of *May* 160 Yards of broad Cloth, and *C* put in the first of *June* 240 Ducates: On the first of *January* following they accounted their Gain; of which *A* and *B* took up 456 *l.* *B* and *C* took up 431 *l.* and *C* and *A* took up 375 *l.* The Question is, What was Gain'd as well in the Whole as a-part; what *B* valued a Yard of Cloth at, and what was *C*'s Ducate *per* Piece?

If you add the three Numbers together, and take half that Sum, because every Man's Money is there named twice, you will have the whole Gain.

See the Work.

To find the several Gains.

From	631	the whole Gain.
Sub.	431	*B* and *C*'s Gain.
Rest	200	= to *A*'s Gain.
From	631	the whole Gain.
Sub.	375	*C* and *A*'s Gain.
Rest	256	= to *B*'s Gain.

A and *B*'s Gain was	456
B and *C*'s Gain was	431
C and *A*'s Gain was	375
The Sum is =	1262
The half, or whole Gain	631
	l.
Then *C*'s Gain must be	175

To find the Value of a yard of Cloth there are several ways; we shall perform it at two Operations by *the Rule of Three*, which I conceive may be most Beneficial to the Learner, because one Operation will be Inverse.

First, therefore say, if 200 *l.* come from 60, what will 256 *l.* come from? *Facit* 76.8.

If

l. *l.* *l.*
If 200 : 60 :: 256
60

200) 15360 (76.8
1400

1360
1200

1600
1600

0

Then say, if 16 Months come from 76.8; what will 8 Months come from? *Facit* 96. For seeing the time is less, it must come from a greater Stock:

M. *l.* *M.*
If 10 : 76.8 :: 8
10

8) 768.0 (96 = the Value of the whole Cloth.
72

48
48

0

Divide 96 *l.* by 160, gives the Value of a Yard, *viz.* 12 Shillings.

160) 96.0 (.6 = to 12 *s.*
960

0

After the same Method must you find the Value of a Ducate; for first I say, If 200 *l.* which is *A*'s Gain, come from 60 *l.* which is *A*'s Stock; what will 175 *l.* come from, which is *C*'s Gain? *Ans.* from 52 $\frac{1}{2}$, or 52.5.

```
  l.    l.     l.
200 : 60 :: 175
              60
         ------
200) 10500 (52.5
     1000
     ----
       500
       400
       ---
       1000
       1000
       ----
          0
```

Secondly, if 10 Months produce 52.5 *l.* what will 7 Months? *Ans.* 75 *l.* for seeing the time is less, the Money will be more.

```
   M.   l.    M.
If 10 : 52.5 :: 7
            10
          ----
       7) 525.0 (75 l. = to 240 Ducates.
          49
          --
           35
           35
           --
            0
```

Which divided by 240, gives in the Quotient .3125, which is equal to 6 *s.* and 3 *d.* the just Price of a Ducate.

QUEST. VI.

A, *B* and *C* Company, and put in together 3822 *l.* *A*'s Money was in 3 Months; *B*'s Money was in 5 Months, and *C*'s Money was in 7 Months: They Gained 234 *l.* which was so divided, as the $\frac{1}{2}$ of *A*'s Gain was equal to $\frac{1}{3}$ of *B*'s Gain, and $\frac{1}{3}$ of *B*'s Gain was equal to $\frac{1}{4}$ of *C*'s Gain, What did each Merchant gain, and put in?

Suppose

Suppose *A*'s Gain was 4 *l.* then must *B* have 6 *l.* and *C* 8, according to the Tenor of the Question, which Numbers added together make 18, then I say,

$$\text{If } 18 : 234 :: \left\{\begin{matrix}4\\6\\8\end{matrix}\right\} \textit{Facit} \left\{\begin{matrix}52 & A\text{'s Gain.}\\78 & B\text{'s Gain.}\\104 & C\text{'s Gain.}\end{matrix}\right.$$

Next multiply every Man's Gain by his Time, and the Sum of the three Products will be 1274, by which dividing the whole Stock, you will compute a common Multiplicator, by which every Man's Gain and Time multiplied, gives each Man's Stock sought.

See the Work.

```
1274) 3822 (3 Com. Mul.
      3822
      ----
         0

A's Gain and Time = 156
                      3
                    ---
                    468 A's Stock.

B's Gain and Time = 390
                      3
                   ----
                   1170 B's Stock.
```

Then *C*'s Stock must be 2184.

QUEST. VII.

A, *B* and *C* Company. *A* put in the first of *January* 100 *l.* and the first of *May* puts in 150 *l.* more; and on the first of *September* takes out 30 *l.* The Remainder stays in till the Year's End.

B put in the first of *January* 250 *l.* and on the first of *June* 60 *l.* more; and on the first of *November* 100 *l.* more, which continues in till the Year's End.

C put in the first of *January* 300 *l.* and the first of *April* takes out 200 *l.* and on the first of *August* takes out 50 *l.* more; the Remainder stays in till the Year's end; What must each have of the Gain, which was 133 Pound?

It

It is plain that *A* hath 100 *l.* in for 4 Months, and 250 *l.* for 4 Months, and 220 *l.* for 4 Months, which 3 Products will be 2280 for *A*'s whole Money and Time; and it is evident that *B* hath 250 *l.* in for 5 Months; and 310 *l.* for other 5 Months, and 410 *l.* for 2 Months, which 3 Products will be 3620 for *B*'s whole Money and Time. It is likewise evident that *C* hath 200 *l.* in for 3 Months, and 100 *l.* for 4 Months, and 50 *l.* in for 5 Months; so the 3 Products will be 1550 for *C*'s whole Money and Time: And by the Work of the second Question the Parts of the Gain will be found,

$$\text{For}\left\{\begin{array}{ll} A\text{'s Part} & 40.71 \\ B\text{'s} & 64.62 \\ C\text{'s} & 27.67 \end{array}\right\}\text{The Summ } 133.$$

Loss *and* Gain.

BY this Rule we discover what is Got or Lost *per Cent.* in Selling and Buying Goods; and instructs us how to raise or fall the Price of Goods, to Gain or Lose so much *per Cent.* or otherwise, either with or without Time.

This is of excellent Use to most Traders, and there being a great deal of Variety in it we will endeavour to make all plain in the following Questions or Examples.

QUEST. I.

If I buy Yarn for 9 *d.* the Pound, and sell it again for 13 *d.* $\frac{1}{2}$, what is Gained *per Cent.* or in laying out a 100 *l.* at that Price?

Say, If 9 *d.* become 13 *d.* $\frac{1}{2}$, what will 100 *l.* become? *Facit* 150, and 150 − 100, gives 50 *l.* for the Gain.

$$\begin{array}{l} \quad d. \quad d. \\ 9 : 13.5 :: 100 \\ \qquad\qquad\quad l. \\ 9)\ 13500\ (150 \\ \qquad\qquad\ \ 100 \\ \quad 9 \qquad\ \overline{} \\ \overline{} \qquad\quad 50 \text{ the } \textit{Answer}. \\ \ 45 \\ \ 45 \\ \overline{} \\ \ 00 \end{array}$$

QUEST.

QUEST. II.

If I buy Broad Cloth for 11 *s.* 6 *d.* the Yard, how must I sell it to Gain 20 *l. per Cent.*

Say, if 100 become 120, what will 11 *s.* 6 *d.* or 11.5 *s.* become? *Facit* 13.8 or 13 *s.* and 9 *d.* $\frac{1}{2}$; and so much must I sell it for to gain 20 *l. per Cent.*

Here having cut off one Figure for the Decimal, I cut off 2 more instead of dividing by a 100.

```
100 : 120 :: 11.5
            11.5
          ------
            600
           120
          120
          ------        s.
          13.80.0 Ans. 13.8
```

QUEST. III.

If I buy a C. Weight of Tobacco for 4 *l.* 13 *s.* 4 *d.* and sell it again for 11 *d.* the Pound, whether do I Gain or Lose, and what *per Cent*?

First find by Practice what the C. will cost at 11 *d.* the Pound, which by the Work is found to be 5 *l.* 2 *s.* 8 *d.* then say,

```
                                              11.2
                                             ------
If 4.666 : 5.133 :: 100 : 110 Facit           2.8
And 110 l. Minus 100 l. is 10 l. the          1.4
Gain sought.                                  0 933
                                             ------  l.  s.  d.
                                             5.133 = 5 : 2 : 8
```

QUEST. IV.

If a Pack of Yarn weighing 240 *lb.* cost 13 *l.* what must a Pound be sold for to Gain 15 *l.* 10 *s. per Cent*?

Find what a Pound will cost, which in this Case is easie; for a Pack weighing 240 *lb.* as many Pounds as the Pack costeth, so many Pence the Pound will cost: So here a Pound will by that Rule cost 13 *d.* Then say,

If

l. *l.* *d.* *d.*
If 100 : 115.5 :: 13 : *Facit* 15.015
13

3465
1155

15.015 *The Answer.*

QUEST. V.

A *Manchester* Man buyeth Yarn for 6 *s.* the Bundle; which not proving so good as expected, would put it off again; so as but to lose 6 *per Cent.* by it. The Question is, what a Bundle will cost?

l. *l.* *s.*
Say, If 100 : 94 :: 6 *Ans.* 5 *s.* 7 *d.* 2 *q.* $\frac{1}{2}$.
6

Facit 5.64 = to 5 *s.* 7 *d.* $\frac{1}{2}$ and half a Farthing *fere*; and so much he must sell it for.

QUEST. VI.

If I buy Incle for 8 *s.* the Groce, how many Yards may I sell for a Penny to gain 20 *l. per Cent.*

l. *l.* *s.* *s.*
First I say, if 100 : 120 :: what 8 : *Facit* 9.6, or $\frac{96}{10}$, then turn a Groce into Yards thus, multiply 24, the Pieces in a Groce, by 36 the Yards in a Piece, *Facit* 864 Yards.

Then say; If $\frac{96}{10}$ of a Shilling buy $\frac{864}{1}$ of a Yard, what will $\frac{1}{12}$ of a Shilling buy? *Facit* $\frac{8640}{1152}$ = to $\frac{15}{2}$, or 7 Yds. and a half; and so many he may sell for a Penny, and gain 20 *l. per Cent,*

See the Work.

$\frac{96}{10}$: $\frac{864}{1}$ :: $\frac{1}{12}$

$\frac{96}{10}$) $\frac{864}{12}$ ($\frac{8640}{1152}$ = $\frac{15}{2}$, or 7 Yds. $\frac{1}{2}$.

QUEST. VII.

If I sell Incle for 12 *s.* a Groce, wherein is lost after 5 *per Cent.* What did a Yard cost?

Say,

Say, if 95 *l.* come from 100 *l.* what doth 12 *s.* come from? *Facit* 12 $\frac{12}{19}$.

95 : 100 :: 12
12

95) 1200 (12 $\frac{60}{95}$, or $\frac{12}{19}$.
95

250
190

60

QUEST. VIII.

A *Manchester* Chapman going to a Fair, sold Fustians for 11 *s.* 6 *d.* the End, wherein was gained 15 *l.* *per Cent.* and seeing no other Chapman had so good, raiseth them at the latter End of the Fair to 12 *s.* I demand what he gain'd *per Cent.* by this last Sale?

Say, if 11 *s.* 6 *d.* gain 15 *l.* what will 12 *s.* gain? Multiply and Divide, and the Answer will be found to be 15 *l.* 13 *s.* 00 *d.* 2 *qr.*

See the Work.

If 11.5 : 15 :: 12
12

30
15

11.5) 180.000 (15.652 Gain'd by the last Sale.
115

650
575

750
690

600
575

250
230

20

QUEST. IX.

A *Manchester* Man buys 20 Tun of Cheese, with which he went into *Ireland*; it cost him 400 *l.* the Freight and Custom came to 50 *l.* his own Expences and Charges came to 16 *l.* 13 *s.* 4 *d.* how must he sell it *per* Pound to gain 20 *per Cent.* by it?

Collect the Cost and Charges into one Sum, and say, If 100 *l.* become 120 *l.* what will 466 *l.* 13 *s.* 4 *d.*

	l.	*s.*	*d.*
Cost, First Penny - - - - -	400 :	00 :	0
Freight and Custom	50 :	00 :	0
His own Charges - -	16 :	13 :	4
Summ	466 :	13 :	4

See the Work.

l. *l.* *l.* *l.*
If 100 : 120 :: 466.666 : *Facit* 560

Say again, if 44800 Pound of Cheese (and so many are in 20 Tun) cost 560 Pound, what will one Pound cost? *Ans.* 3 *d.* As by the Work appeareth.

lb. *l.* lb.
If 44800 : 560 :: 1
1

44800) 560,0000 (.0125 = to 3 *d.*
44800

112000
89600

224000
224000

0

QUEST.

QUEST. X.

A Merchant selling Corn at 8 *s.* the Bushel, gained 10 *l. per Cent.* but afterwards being by a falling Market forced to sell it for 7 *s.* What did he Gain or Lose *per Cent.* by this last Sale?

Say, if 8 *s.* made 110 *l.* what 7 *s.*? *Facit* 96 $\frac{1}{4}$, whereby he lost 3 *l.* 15 *s. per Cent.* by this Sale.

$$
\begin{array}{llll}
 & 8 : 110 :: 7 & & \\
 & \underline{\quad 7} & & 100 \\
8) & 770\,(96\tfrac{1}{4}. & & 96\tfrac{1}{4}. \\
 & & & 3\tfrac{3}{4} \\
 & \underline{72} & & \\
 & 50 & & \\
 & \underline{48} & & \\
 & 2 & &
\end{array}
$$

QUEST. XI.

If I buy Yarn for 9 *d.* and sell it for 12 *d.* and allow 3 Months for Payment, what do I gain *per Cent. per Annum*?

This Question admits of a double Meaning, and by that means of a double Answer.

For first, it is evident if he on this Sale had received ready Money, he would have gained 33 *l.* $\frac{1}{3}$ *per Cent.* but giving 3 Months for Payment, his Gain must needs be less, by as much as the Rebate of 133 *l.* $\frac{1}{3}$ for 3 Months amounts to. Which by Proposition the Second of Simple Interest, will be found to be 1 *l.* 19 *s.* 5 *d.* which subtract from 33 *l.* 6 *s.* 8 *d.* leaves 31 *l.* 7 *s.* 3 *d.* the Gain in this Case sought.

So he makes his 100 *l.* to be 131 *l.* 7 *s.* 3 *d.*

But some Authors would answer this Question thus: First they say,

$$\overset{d.}{\text{If } 9} : \overset{d.}{12} :: \overset{l.}{100} : \textit{Facit } \overset{l.}{133}\tfrac{1}{3}, \text{ as before;}$$

But instead of making the Gain less by giving Time, they make it vastly more, thus, for they say,

If 3 Months Gain 33 *l.* $\frac{1}{3}$, what will 12 Months Gain? *Facit* 133 *l.* 6 *s.* 8 *d.* the Gain sought.

So they make his 100 *l.* to be 233 *l.* 6 *s.* 8 *d.*

But you may use that which agrees best with your own Reason.

Note, We allow 6 *per Cent.* Simple Interest for the Rebate in the Question aforegoing, that being the Rate allowed by the Statute.

The Rule of ALLIGATION.

ALLIGATION Teacheth how to mix or unite many Simples, or Particulars, into one Mass or Sum, according to any Price or Proportion required.

For the Ease of the Learner, we shall divide this Rule into 4 Varieties, that so when a Question is propounded, it is but considering what Variety it falls under, and the Work will soon be finished.

VARIETY I.

In this Variety we have given the Prices and Quantities of several Simples to be mixed, and the mean Rate or Price of any Part of such Mixture is required.

To find which, the Proportion is,

As the Sum of the Simples to be mixed :
To the total Value thereof : :
So is any Part of the Composition :
To the Value thereof.

EXAMPLE.

A Tobacconist would mix 20 *lb.* of Tobacco at 9 *d.* the Pound, with 60 *lb.* at 12 *d.* the Pound, and with 40 *lb.* at 18 *d.* the Pound, and with 12 *lb.* at 2 *s.* the Pound. The Question is, what a Pound of this Mixture is worth?

Place

Place the Numbers, and their Values as underneath,

lb.		s. d.		l. s. d.
20	at	0 9	*per lb.* will Cost	0 15 0
60	at	1 0	*per lb.* will Cost	3 00 0
40	at	1 6	*per lb.* will Cost	3 00 0
12	at	2 0	*per lb.* will Cost	1 04 0
Sum Simple = 132			Total Value =	7 19 0

Then say, if 132 Pound cost 7 *l.* 19 *s.* what will one? *Facit* 1 *s.* 2 *d.* 2 *q.*

See the Work.

l. *l.* *l.*
132 : 7.95 : : 1

1

132)7.95 (.06022 = to 1 *s.* 2 *d.* $\frac{1}{2}$.
792

300
264

360

QUEST. II.

A Farmer would mix 5 Bushels of Wheat at 6 *s.* the Bushel, with 12 Bushels of Rye at 4 *s.* the Bushel, with 8 Bushels of Beans at 5 *s.* the Bushel, and with 18 Bushels of Barley at 2 *s.* 6 *d.* the Bushel. The Price of one Bushel of this Mixture is demanded.

Place your Numbers and their Values as under.

B.		s. d.		l. s. d.
5	at	6 0	the Bush. will Cost	1 10 0
12	at	4 0	the Bush. will Cost	2 8 0
8	at	5 0	the Bush. will Cost	2 0 0
18	at	2 6	the Bush. will Cost	2 5 0
Numb. Bush. 43			Tot. Val. =	8 3 0

Then

Then say, if 43 Bushels cost 8 *l.* 3 *s.* or 8.15, what will one Bushel? *Facit* 3 *s.* 9 *d.* $\frac{1}{2}$.

See the Work.

B. *l.* *B.*
43 : 8.15 : : 1
1

43) 8.15 (.1895 = to 3 *s.* 9 *d.* $\frac{1}{2}$ *fere.*
43
385
344
410
387
230
215
15

VARIETY II.

In this Variety the Price of the Simples is expressed, but no Quantity given; and it is required how much of each Simple we must take to sell one Quantity or Measure at a mean Rate propounded.

The whole Work of this Variety is in linking the Extreams truly together, and taking the true Differences betwixt them and the Mean; and these Differences are the true Quantities sought.

EXAMPLE.

A Merchant hath Spices, some at 9 *d.* the Pound, some at 12 *d.* some at 24 *d.* and some at 30 *d.* how much of each sort must he take that he may sell a Pound for 20 *d.*?

First, Set down the several Prices of the Spices orderly under one another, with a Line of Connection to the left Hand, as in the Example may be seen.

Note, That 9, 12, 24, 30, I call the Extreams; 9 and 12 being the two lesser Extreams, and 24 and 30 the two Greater:

Greater: Then on the other Hand set down the mean Price, as you see.

That done, I link or join together two Extreams, no matter which, so as one be bigger than the mean, and the other less; and after that other two, till I have finished. So in this Example, I link 9 and 30 together, and 12 and 24, as you see; then take the difference betwixt each Extream and the mean Price, and place it over against its Yoak-fellow: So the difference betwixt 9 and 20 is 11, which place against 30 its Yoak-fellow; the difference betwixt 12 and 20 is 8, which place against 24 its Yoak-fellow; betwixt 24 and 20 is 4, which place against 12 its Yoak-fellow; and lastly the difference betwixt 30 and 20 is 10, which place over against 9 its Yoak-fellow, as you see here done. And the differences here found will be the Answer to the Question; for as oft as he takes 10 *lb.* of 9 *d.* a Pound, he must take 4 of 12 *d*, a Pound, and 8 at 24 *d.* the Pound; that so a Pound may be afforded for 20 *d.* or 1 *s.* 8 *d.*

20	9	10
	12	4
	24	8
	30	11

The Proof thereof is easie by the last Variety, for the Sum of the difference found, multiplied by 20 is equal to the Sum of the Products of the Difference and Extreams. But if at any Time, as here it happens, that the Extreams may be linked more ways than one, then the Question admits of more Answers than one, yet all true; for if in this Question, we link 9 and 24, and 12 and 30, and place the Differences respectively, then we shall have 4 at 9 *d.* a Pound, 10 at 12 *d.* a Pound, 11 at 24 *d.* a Pound, and 8 at 30 *d.* a Pound; and that this is likewise true, may be proved, as in the last, and the mean Price will be found to be 20 *d.*

20	9	4
	12	10
	24	11
	30	8

Again, if we link 9 and 30 together, 12 and 30, and 12 and 24, there will arise a new Method in placing the Differences; for if any Extream have two Yoak-fellows, it will likewise have two Differences: So the Difference be-

20	9	10	10
	12	4.10	14
	24	8	8
	30	11.8	19

twixt 9 and 20 is 11, which place over-against 30, its Yoak-fellow; the difference betwixt 12 and 20 is 8, which because it has two Yoak-fellows, place against them both, to wit against 24 and 30, as you see; the difference betwixt 24 and 20 is 4, which place against 12 its Yoak-fellow; and lastly the difference betwixt 30 and 20 is 10, which place against both its Yoak-fellows, to wit, 12 and 9. Then draw a streight Line, and beyond it the differences; so first I set down 10, then 10 and 4 is 14, which set down; then set down 8, then 11 and 8 is 19, which likewise place down; then as oft as he takes 10 at 9 d. the Pound, he must take 14 at 12 d. the Pound, and 8 at 24 d. the Pound, and 19 at 30 d. the Pound, that so a Pound may be worth 20 d.

And lastly, if we link 9 both with 24 and 30, and 12 both with 24 and 30, and 24 both with 12 and 9, and 30 both with 12 and 9, then the difference betwixt 9 and 20, which is 11, is to be placed against 24 and 30; and also the difference betwixt 12 and 20, which is 8, against 24 and 30, for the same reason; and the difference betwixt 24 and 20, which is 4, against 12 and 9, they being thereunto linked; and lastly the difference betwixt 20 and 30, which is 10, place against 12 and 9, for the same reason; then on the other side the Line, place the Sum of the differences against their respective Numbers, as you see in the example: So by this *Alligation*, you will find, he must take 14 *lb.* at 9 *d.* and 12 *d.* the Pound, and 19 *lb.* at 24 *d.* and 30 *d.* to make a Pound worth 20 *d.*

20	9	4.10	14
	12	4.10	14
	24	11.8	19
	30	11.8	19

I shall not name any more ways of linking these Numbers, these being sufficient to understand how to link them any way, and leave the rest to the Scrutiny of the Learner, to exercise himself with: For I have been the larger in these Examples, that I may not trouble my self any more with the Method of linking in the Examples following; only Note, that if at any time you have but one Extream, either lesser or bigger than the Mean, it will but admit of one way of linking, and the Question will have but one Answer, as in the following Example may be seen.

EXAM-

EXAMPLE.

A Merchant hath Wines; Canary at 24 *d.* the Quart, Sherry at 16 *d.* the Quart, and Mallaga at 12 *d.* the Quart, how much of each sort must he take to sell a Quart for 18 *d.* This Question you see admits but of one way of linking, and so but of one Answer; and observing the Directions before given, you will find 8 of Canary, 6 of Sherry, and 6 of Mallaga must be mixed together, that so a Quart may be sold for 18 *d.*

18	24	2.6	8
	16	6	6
	12	6	6

VARIETY III.

In this Variety we have the Price of all the Simples, and the Quantity of one given to find the Quantity of all the rest, so as one Measure or Quantity may bear a mean Rate or Price propounded; which to do, observe the Proportion following.

As the Difference standing against the Quantity given:
To the rest of any of the Differences besides: :
So the Quantity given :
To the Quantities sought. Each to its respective Difference.

EXAMPLE.

A Tobacconist hath 30 *lb.* of the best Tobacco at 2 *s.* or 24 *d. per* Pound, which he would mix with some at 12 *d.* some at 9 *d.* and some at 7 *d.* and he would know how much of each sort of the said less Prices, must be mixed with the 30 *lb.* of the best, that he may sell it for a Penny the Ounce, or 16 *d.* the Pound.

16	24	4.7.9	20.30
	12	8	8
	9	8	8
	7	8	8

Having by the last Variety set down the Numbers, linked them, and found the Differences, as in the Example; then say, as 20, the Difference against the Quantity given : to 8 : : the next difference, so 30 : to 12, the Quantity required at 12 *d.* the Pound; and seeing the

the other Differences are equal, it will require 12 Pound of each; so that he must take off the less Prices at 12 *lb.* a piece, to mix with the 30 *lb.* of the best, that so a Pound may be afforded for 16 Pence. The Proof of this, and the following, are by the first *Variety*.

EXAMPLE II.

A *Goldsmith* hath 20 Ounces of Gold at 20 Carracts fine, and would mix it with some at 22 Carracts fine, and some at 24 Carracts fine; how much of 22 and 24 Carracts fine, and how much Alloy must he mix with the 20 Ounces of 20 Carracts fine, so as an Ounce, and consequently the whole Mass, may be 18 Carracts fine?

Note, That Alloy is a sort of course Silver, or Copper, or some base Metal, with which *Goldsmiths* mix Gold or Silver to abate the fineness thereof.

An Ounce of Gold is divided into 24 parts, called Carracts, and an Ounce of Silver into 22 parts, called Penny-Weights; therefore to distinguish fineness of Metals, such Gold as will abide the Fire without loss, is accounted 24 Carracts fine; if it lose 2 Carracts in trial, it will then be 22 Carracts fine, &c.

Silver is valued in Ounces, and a Pound of Silver which loseth nothing in trial, is called 12 Ounces fine; but if it lose 2 Penny-weight, it is then said to be 11 Ounces, 18 Penny-weight fine.

First set down the Values in Order as usual, with the mean Value, and in the Place of the Alloy, because it is not accounted of any Value, place a Cypher, then take the Differences, which by the linking you may see, will all be the same, except only in the place of the Alloy. Then say,

If : 18 : 18 :: 20 : 20
If : 18 : 18 :: 20 : 20
If : 18 : 12 :: 20 : $13\frac{1}{3}$

18	20	18	18
	22	18	18
	24	18	18
	0	2.4.6	12

Thus you see, That with the 20 Ounces of 20 Carracts fine, there must be mixed 20 of 22 Carracts fine, and 20 of 24 Carracts fine, and 13 Ounces and $\frac{1}{3}$ of Alloy, that so an Ounce would bear 18 Carracts fine.

By

By changing the two last Terms of the Proportion in the aforegoing Rule, which is the same thing in effect, we may work any of these Questions by the contracted way in *Fellowship*; and if there be any Fractions bring them up Decimally.

If the Terms be changed, the Proportion stands.

As the Difference against that Price whose Quantity is given :

Is to the Quantity given : :

So any other Difference :

To its Quantity sought.

EXAMPLE III.

A Chapman hath Yarn at several Rates, and would mix 40 Pound, at 24 Pence the Pound, with some at 20 Pence the Pound, with some at 14 Pence the Pound, with some at 9 Pence the Pound, and some at 7 Pence the Pound; How much of each sort must he mix with the 40 Pound, at 24 Pence the Pound, that he may sell a Pound for 16 Pence?

16	24	7.9	16
	20	2	2
	14	4	4
	9	8	8
	7	8	8

Having placed your Numbers and linked them, and taken the Differences, as in the Margin.

Then the Proportion is, as 16 to 40, what 2, what 4, what 8, what 8; instead of these Operations divide 40 by 16, the Quotient multiplied by every Difference, gives every particular Quantity sought.

First, Mult. 2.5   *See the Work.*   16) 40 (2.5
By 2   32
5.0 at 20 *d.* the Pound.   80
80
0

2*ly*, Mult. 2.5
By 4
10.0 at 14 *d.* the Pound.

3*ly*, Mult. 2.5
By 8
20.0 at 14 *d.* and 7 *d.* the Pound;

So you see with the 40 at 24 *d.* the Pound, he must mix 5 at 20 *d.* the Pound, and 10 at 14 *d.* the Pound, and 20 at 9 *d.* and 7 *d.* the Pound; and so a Pound will be worth 16 Pence.

VARIETY IV.

In this *Variety* the Prices of every Simple is expressed, and the mean Rate or Price; and it is required to find how much of each sort must be taken to make up a certain Quantity propounded, agreeable to the mean Rate given?

Which to do, observe the Proposition following.

As the total Sum of the Differences:
To the total Quantity given ::
So any particular Difference:
To its particular Quantity sought.

EXAMPLE.

A *Grocer* hath 4 sorts of Currants, one at 4 Pence the Pound, another at 6 Pence, another at 9 Pence, and the best at 11 Pence the Pound: The worst would not sell, and the best were too dear, and he concludes to mix 240 Pound, and so much of each sort, as to sell a Pound for 8 Pence; how much of each sort must he take?

8	4	3	72
	6	1	24
	9	2	48
	11	4	96
		10	240

Having placed your Numbers with the mean Price, linked them, and taken the Difference, as here, divide 240, by 10: quotes 24, multiplied by every Difference, gives 72, for the Quantity of 4 *d.* the Pound; and 24 for the Quantity at 6 *d.* the Pound; and 48 for the Quantity at 9 *d.* the Pound; and 96 for the Quantity at 11 *d.* the Pound, the Sum making 240, is the Proof.

Note, If he hath a desire to put off more of his worst sort, he may alter the Quantities by some other way of linking, as was shewed in the second *Variety*.

QUEST.

QUEST. II.

A *Goldsmith* hath several sorts of Gold, some of 24 Carracts fine, some of 22 Carracts, some of 18 Carracts, some of 16 Carracts fine, and is desirous to melt of all these sorts, so much together, as may make a Mass of 60 Ounces of 21 Carracts fine. How much of each sort must he take?

The Numbers being placed linked and differenced, as hath been shewed, and is here expressed. I say,

21	24	5	25
	22	3	15
	18	1	5
	16	3	15
		12	Proof 60

1 As 12 : 60 : : 5 : 25
2 As 12 : 60 : : 3 : 15
3 As 12 : 60 : : 1 : 5
4 As 12 : 60 : : 3 : 15

Or if you will use the contracted way, divide 60 by 12, quotes 5; by which multiplying each Difference, gives the same Quantities. So I conclude, that 25 Ounces of 24 Carracts fine, 15 Ounces of 22 Carracts fine, 5 Ounces of 18 Carracts fine, and 15 Ounces of 16 Carracts fine, will produce a mass of Gold of 60 Ounces, and 21 Carracts fine.

QUEST. III.

How many Gallons of Water must be mixed with Wine, at 3 Shillings the Gallon, to fill a Vessel of 100 Gallons, so as a Gallon may be afforded for 2 *s.* 6 *d*?

First set down the Value of a Gallon of Wine, and the Water being of no Value, put a Cypher; then having set down the mean Rate, and linked, and taken the Difference; say, If 3, the Sum of the Difference, give 100, what will 2.5? *Facit* 83.333, or 83 and $\frac{1}{3}$ of Wine, which subtracted from 100, leaves 16 $\frac{2}{3}$ for the quantity of Water.

	s.	
2.5	3	2.5
	0	.5
		3

QUEST.

QUEST. IV.

A *Vintner* hath two Veſſels, one will hold 50 Gallons, and the other 30; and would know how much Water he muſt mix with Wine at 4 Shillings the Gallon, to fill the bigger Veſſel, that every Gallon drawn may be worth 3 Shillings the Gallon; and with Wine at 2 Shillings and 6 Pence the Gallon, to fill the leſs Veſſel, that a Gallon may be worth 2 Shillings.

The Quantity of Water is demanded.

Prop. I.

$$3\begin{cases}4\\0\end{cases}\!\!\begin{matrix}]3\\]1\end{matrix}$$

If 4 : 50 :: 3 : 37.5

Facit $37\frac{1}{2}$ Gallons of Wine, then there muſt be $12\frac{1}{2}$ of Water in the greater Veſſel.

Prop. II.

$$2\begin{cases}2.5\\0\end{cases}\!\!\begin{matrix}]2.\\].5\end{matrix}$$

If 2.5 : 30 :: 2 : 24

Facit 24 Gallons of Wine, then there muſt be 6 of Water in the leſſer Veſſel.

The Rule of FALSE.

THIS RULE is more for Recreation and Delight, than for any ſolid Uſe; but becauſe it is an ingenious Rule, and may exerciſe the Wits of Youth, we ſhall here inſert it.

The Rule of Falſe is ſo named, not from the Falſity of it, but becauſe we, by ſuppoſed Numbers, taken at Adventure, and by them working the Queſtion according to the Nature thereof, do, by thoſe falſe ſuppoſed Numbers find the true Numbers ſought.

This Rule is divided into two Parts, commonly called the *Single Rule*, and *Double Rule*.

The

The Single Rule of FALSE.

IN the *Single Rule* we need but to use one Supposition, as may be seen in the Questions following.

QUEST. I.

A certain Sum of Money put out at 6 *per Cent.* Simple Interest, at the end of 10 Years amounts to 20 *l.* what was the Stock? *Answer* 12 *l.* 10 *s.*

Here I suppose any Number, as 10 Pound, then according to the nature of the Question.

What will 10 *l.* amount to, forborn 10 Years? Which by the Table of *Simple Interest*, or by the *Double Rule of Three*, will be found to be 16 *l.* which should have been 20 *l.* if I had guessed right.

Now I say, If 16 *l.* come from 10 *l.* my supposed Number, what will 20 *l.* come from? *Answer* 12 *l.* 10 *s.* the Stock sought.

See the Work.

If 16 : 10 : : 20
20
———
16) 200 (12.5
16
———
40
32
———
80
80
———
0

QUEST. II.

A *Schoolmaster* being asked how many Scholars he had, Answered, If I had as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$, or 1 quarter as many, I should have 99; How many had he? *Answer* 36.

Suppose

Suppose he had any Number, as 40, then as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many would make 110, which should have been 99. Then I say,

If 110 come from 40, what will 99 come from? *Answ.* 36, the number of Scholars sought.

See the Work.

If : 110 : 40 :: 99

$$\begin{array}{r} 99 \\ 40 \\ \hline 110)\ 3960\ (36 \\ 330 \\ \hline 660 \\ 660 \\ \hline 0 \end{array}$$

QUEST. III.

There is a Cistern with 3 unequal Cocks, containing 60 Gallons of Water; and if the greatest Cock be opened, the Cistern will be empty in one Hour; if the second Cock be opened, it will be empty in three Hours. Now I demand in what time it will be empty if all run together?

Suppose in $\frac{1}{2}$ an Hour, or 30′, then must there empty at the greatest Cock 30 Gallons or $\frac{1}{2}$, and by the second Cock 15 Gallons or $\frac{1}{4}$, and by the least Cock 10 Gallons, or $\frac{1}{6}$, which added together, make 55, which should have been 60. Now say,

If 55 Gallons run in 30 Minutes, what will 60 Gallons run in? *Answ.* 32′ $\frac{8}{11}$, or 3.2′ .727, the Time sought.

QUEST. IV.

Three Merchants, *A*, *B*, *C*, put in Money together, and gain'd 100*l.* of which *A* took up a certain Sum; *B* took up twice as much as *A*, or double to *A*, and *C* took up thrice as much as *B*, or treble to *B*; what did each take up apart?

Suppose

Suppose *A* took up 3 Pound, then *B* must have 6 Pound, and *D* 18 Pound, which makes 27 Pound, which should have been 100 Pound.

Then say, If 27 *l.* should be 100 *l.* what $\left\{\begin{matrix}3\\6\\18\end{matrix}\right\}$ *Facit* $\left\{\begin{matrix}11.111111 = A\\22.222222 = B\\66.666666 = C\end{matrix}\right.$

Proof .99.999999

See the Work.

```
27) 100 (3.703703
    81
    ---
    190
    189
    ---
      100
       81
      ---
      190
      189
      ---
        100
         81
        ---
         19
```

This Quotient multiplied by 3 for *A*, by 6 for *B*, and by 18 for C, produceth the former Numbers. See the contracted way in *Fellowship*.

Thus may any Question of these Natures be wrought; so I shall forbear mentioning any more of *single Position*, only Note, that if there be no Partition in Numbers to make a Proportion, you must use the *double Rule*, which now we shall begin with.

Double Rule of False.

In the *Double Rule* we use two Suppositions; and if with either we find the Number that satisfies the Question, there is no more to be done; but if, as commonly it happens, we err in both Suppositions, see whether they be greater or lesser than the Solution requires, which mark with +, *plus*, or —, *minus*; and over against either Supposition its respective Error, then observe this general Rule.

As the difference of Errors if alike, or Sum if unlike:
Is to the difference of Suppositions:
So is either Error to a fourth Number; which, added to, or subtracted from, the Supposition overagainst it, gives the Number sought. *See the Examples.*

QUEST. I.

Good Morrow, good Fellow, with your 20 Geese; nay, says he, I have not 20, but if I had as many, $\frac{1}{2}$ as many, 2 Geese and $\frac{1}{2}$, then had I 20. I demand how many he had?

First, Suppose 6, then as many, $\frac{1}{2}$ as many, two Geese and $\frac{1}{2}$, would make 17 and a $\frac{1}{2}$, which would be 20; the Error therefore is— 2 $\frac{1}{2}$, which mark as in the Margin.

6—2.5
9+5

Secondly, Suppose he had 9, then as many, $\frac{1}{2}$ as many, 2 $\frac{1}{2}$ would make 25, which should be 20, the Error therefore is +5, which put down under the other, as you see done. Then because the Errors are unlike, that is, one *plus*, the other *minus*. I say, as the Sum of the Errors 7.5, to the difference of the Suppositions 3 : : So either Error; suppose the first 2.5 to 1, which because the first Supposition was *minus*, according to the Rule added to 6, makes 7, the number of Geese sought.

QUEST. II.

A Gentleman had two Horses of good Value, and a Saddle worth 50 *l.* which set on the back of the first Horse, makes his Value double the second; but if set on the Back of the second Horse, makes his Worth treble the first Horse. The Price of each Horse is demanded.

Suppose the Price of the first Horse be 20 Pound, which, with the Saddle makes 70 *l.* then seeing this is double the Price of the second Horse, the second will be worth 35 *l.* which with the Saddle would be 85, which should be 60, 3 times 20, the Price of the first Horse; the Error therefore is— 25, which put down as you see.

Suppose again the first Horse worth 25 *l.* which, with the Saddle, would be 75 *l.* then the second would be worth 37 *l.* 10 *s.*

20—25
25—12.5

l.

or 37.5, which, with the Saddle, would amount to 87 *l.* 10 *s.*

or

or 87.5, which should be 75 = to 3 times 25, the Price of the first Horse; the Error therefore is — 12.5, which place under the other Error, and say,

As the Difference of Errors, because alike 12.5 :

To the Difference of Suppositions 5 : :

So either Error, suppose the First 25 : to 10, which added to the Supposition over-against it, because — makes 30, the Price of the first Horse; and by Consequence the Price of the Second will be 40. And if you had taken the Second Error 125, the fourth Number, would have been 5, which added to 25, makes 30, as before.

Proof. First Horse, 30
Saddle = 50

Sum = 80

$\frac{1}{2}$ = 40 = Second Horse.
Saddle = 50

Sum = 90
$\frac{1}{3}$ = 30 = First Horse.

QUEST. III.

A stealing Apples was taken by *B*, and to appease him gives him half he had, and *B* gives him back 10; and going further met with *C*, and was forced to give him half of what he had left, and he returns him back 4; and going further meets *D*, and gives him half he had, and he returns him back 1; and getting safe away, finds he had 13 left, what had he at first?

Suppose first 80, and working according to the Nature of the Question, he had 15 $\frac{1}{2}$ left, which is + 2.5.

Suppose again he had 40, and working as before, he will have 10 $\frac{1}{2}$ left, which is — 2.5.

Then working by the general Rule, you will find he had 60 Apples at the first,

80 + 2.5
40 — 2.5

But the Number sought in this Question may more quickly be found: For *Note*, That if at any Time, as here it happens, that the Errors are the same in Quantity, and

unlike in Quality, half the Sum of the Suppositions is the Number sought; and the Sum of the Suppositions is 120, half of which is 60 as before.

See the Work both ways.

As 5 : 40 : : 2.5   80
40   40

5) 10.00 (20+40=60.   Sum 120
10   [or 20—80=60.   60

00

QUEST. IV.

Three Men, as *A*, *B* and *C*, bought a Ship for 200 *l.* *A* says to *B* give me half your Money, and I will pay for the Ship; *B* says to *C*, give me $\frac{1}{3}$ of your Money, and I will pay for the Ship; *C* says to *A*, give me $\frac{1}{4}$ of your Money, and I will pay for the Ship, what Sum of Money had each? *Ans.* *A*, 128, *B*, 144, *C*, 168.

1 Sup. 120—50
2 Sup. 130+12.5

Proof $\begin{cases} 128+\frac{1}{2}\ 144=200 \\ 144+\frac{1}{3}\ 168=200 \\ 168+\frac{1}{4}\ 128=200 \end{cases}$

QUEST. V.

Three Men, as *A*, *B*, and *C*, thus discoursed of their Money: *A* saith to *B* and *C* give me half your Money, and I shall have a 100 *l.* *B* saith to *C* and *A*, give me one third of your Money, and I shall have 100 *l.* *C* saith to *A* and *B*, give me one fourth of your Money, and I shall have 100 *l.* what had each?

This Question will require more Suppositions than two, before it can well be wrought by this Rule; which may convince some that affirm, if a Question require more Suppositions than two, it will not be wrought by the Rule of *False*; but the Contrary may be seen in the following Work.

Let the first general Supposition for *A*, be 20 *l* then he wanted 80 *l.* which is the half of *B* and *C*'s Money; then they must have 160 *l.* whereof we suppose *B* had 40 *l.* then *C* must have 120 *l.* Now *B* will have of *C* and *A* $\frac{1}{3}$ of their Money, which is 46 *l.* $\frac{2}{3}$, which added to *B*'s Money 40 *l.* makes

makes 86 $\frac{2}{3}$, which should be 100 *l.* so we have supposed too little for *B* by 13 $\frac{1}{3}$. 20 — 13 $\frac{1}{3}$ *B.*

Suppose again, *B* had 70 *l.* then *C* must have 90 *l.* Now *B* will have of *C* and *A* $\frac{1}{3}$ of their Money, which is 36 $\frac{2}{3}$, which added to *B*'s Money 70 *l.* makes 106 $\frac{2}{3}$, which should be a 100 *l.*; here we have supposed too much for *B* by 6 $\frac{2}{3}$.

Now say according to the Rule,
As 20 : 30 :: 6 $\frac{2}{3}$: 10, and 10 Sub. S. 1 = 40 — 13 $\frac{1}{3}$ *B.*
tract from 70, resteth 60 for *B*; then S. 2 = 70 + 6 $\frac{2}{3}$ *B.*
if *A* had 20, *B* had 60, *D* 100.

But the Question saith, *C* will have of *A* and *B* $\frac{1}{4}$ of their Money, which is 20 *l.* which added to 100 *l.* of *C*'s Money, makes 120, which should be 100 *l.* Therefore our supposition for *A* is too little by 20 *l.* —— S. 1 = 20 — 20 *A.*

Let the second general Supposition for *A* be 30 *l.* then he wanted 70 *l.* therefore 70 *l.* is the half of B's and C's Money; then they must have 140 *l.* whereof we suppose *B* had 30 *l.* then must *C* have 110 *l.* now *B* will have of *C* and *A*, one third of their Money, which is 46 $\frac{2}{3}$, which added to 30 *l.* of *B*'s Money, makes 76 $\frac{2}{3}$, which should be 100 *l.* here we have supposed too little for *B* by 23 $\frac{1}{3}$.

S. 1 = 30 — 23 $\frac{1}{3}$ *B.*

Suppose again, *B* had 50 *l.* then must *C* have 90 *l.* now *B* will have of *C* and *A* one third of their Money, which is 40 *l.* which added to *B*'s Money 50 *l.* makes 90 *l.* which should be 100. Here we have supposed too little for *B* by 10 *l.* S. 2 = 50 — 10 *B.*

S. 1 = 30 — 23 $\frac{1}{3}$ *B.*
S. 2 = 50 — 10 *B.*

Then say, if 13 $\frac{1}{3}$: 20 :: 10 : 15, and 15 added to 50, makes 65 for *B*. Then if *A* had 30, *B* would have 65, and *D* 75.

But the Question saith, *C* will have of *A* and *B* $\frac{1}{4}$ of their Money, which is 23 $\frac{3}{4}$, which added to 75 of *C*'s Money, makes 98 *l.* $\frac{3}{4}$, which should be 100 *l.* therefore our second Supposition for *A*, is too much by 1 $\frac{1}{4}$. S. 2 = 30 + 1 $\frac{1}{4}$ *A.*

S. 1 *A* = 20 — 20
S. 2 *A* = 30 — 1 $\frac{1}{4}$.

Now

Now I ſay, if $21\frac{1}{4} : 10 :: 20 : 9\frac{7}{17}$, and $9\frac{7}{17}$ added to 20, gives 29 *l.* $\frac{7}{17}$, for the true ſhare of *A*.

Seeing then that *A* hath 29 *l.* $\frac{7}{17}$, he wanteth 70 *l.* $\frac{10}{17}$ which is half *B*'s and *C*'s Money; then they muſt have $141\frac{3}{17}$, whereof ſuppoſe *B* had 30 *l.* then *C* had 111 *l.* $\frac{3}{17}$ Now *B* will have of *C* and *A* $\frac{1}{3}$ of their Money, which is $46\frac{44}{51}$, which added to his own Money 30 *l.* makes $76\frac{44}{51}$ which ſhould be 100 *l.* here we have ſuppoſed too little for *B* by $23\frac{7}{51}$. $S.\ 1 = 30 - 23\frac{7}{51}\ B$.

Suppoſe again, *B* had 65 *l.* then *C* had $76\frac{3}{17}$, now *B* will have of *C* and *A* one third of their Money, which is $35\frac{10}{51}$, which added to his own Money 65 *l.* makes $100\frac{10}{51}$, which is too much by $\frac{10}{51}$. $S.\ 2 = 65 + 0\frac{10}{51}\ B$.

$$S.\ 1 = 30 - 23\tfrac{7}{51}\ B.$$
$$S.\ 2 = 65 + 0\tfrac{10}{51}\ B.$$

Then ſay, as $23\frac{17}{51}$: to 35 :: ſo is $\frac{10}{51}$ to $\frac{5}{17}$, which ſubtracted from 65, leaves $64\frac{12}{17}$, for *B*'s true ſhare, then muſt *C* have $76\frac{8}{17}$.

$$\text{Their Parts}\begin{cases} A = 29\tfrac{7}{17} \\ B = 64\tfrac{12}{17} \\ C = 76\tfrac{8}{17} \end{cases} \!\!l.\qquad \text{In Decimals}\begin{cases} A = 29.4117647 \\ B = 64.7058824 \\ C = 76.4705882 \end{cases} \!\!l.$$

$$\text{Proof}\begin{cases} 29\tfrac{7}{17} + \tfrac{1}{2}\left\{\begin{matrix} B\ 64\tfrac{12}{17} \\ C\ 76\tfrac{8}{17} \end{matrix}\right\} = \text{to } 70\tfrac{10}{17}, = 100 \text{ for } A. \\ 64\tfrac{12}{17} + \tfrac{1}{3}\left\{\begin{matrix} C\ 76\tfrac{8}{17} \\ A\ 29\tfrac{9}{17} \end{matrix}\right\} = \text{to } 35\tfrac{5}{17}, = 100 \text{ for } B. \\ 76\tfrac{8}{17} + \tfrac{1}{4}\left\{\begin{matrix} A\ 29\tfrac{7}{17} \\ B\ 64\tfrac{12}{17} \end{matrix}\right\} = \text{to } 23\tfrac{9}{17}, = 100 \text{ for } C. \end{cases}$$

This Queſtion is not capable of an exact Anſwer in *Engliſh* Coin, as you may ſee: But if you would have an Anſwer in Integers, you muſt make the common Sum in this Queſtion 100 *l.* ſome multiple of 17; or if you reduce their Shares into improper Fractions, then

A will have $\frac{500}{17}$

B will have $\frac{1100}{17}$

C will have $\frac{1300}{17}$

And

And seeing the Denominators are Equal, neglect them, and the Numerators will be Proportional Numbers for *A*, *B*, *C*, which you abbreviate into lesser, by cutting two Cyphers from each; then will *A* have 5 *l.* *B* 11 *l.* and *C* 13 *l.* then if *A* have half of *B* and *C*'s Money, he will have 17 *l.* If *B* have one third of *C*'s and *A*'s Money, he will have 17 *l.* And if *C* have one fourth of *A*'s and *B*'s Money, he will 17 *l.* So this Question consisteth all of Integers; and 17 *l.* falls into the Place of 100 *l.*

I have been the longer upon this Question, that the Learner may observe the Variety of Work, that may proceed from such like Questions.

Here I have annex'd two or three more, with their Answers, which may serve for the Learner's Exercise; and so conclude this Rule.

Questions in the Rule of False.

QUEST. I.

What Number is that, which multiply'd by 20, and divided by 6, gives 140 in the Quotient? *Facit* 42.

QUEST. II.

What Number is that, which added to its half and its one fourth, and to three more, makes 108? *Facit* 60.

QUEST. III.

A Vessel that holdeth 60 Gallons, hath 4 Cocks; and being fill'd with Water or any other Liquor; if they all be set open at once the Liquor will run out in 24 Hours. Now the second Cock will empty twice as much as the first during the same time; and the third will empty three times as much as the first in the same time; and the fourth will empty 5 times as much as the first. What Number of Gallons doth each Cock empty.

Facit the First, 5 $\frac{5}{11}$ Gallons; the Second, 10 $\frac{10}{11}$ Gallons, the Third, 16 $\frac{4}{11}$ Gallons; the Fourth 27 $\frac{3}{11}$ = 60 Gall.

QUEST. IV.

A Gentleman hired a Workman for 40 Days and agreed for every Day he worked, he should have 8 Pence; and every Day he play'd, he should return back 4 Pence. At the

the End of 40 Days, the Labourer received 10 *s.* 5 *d.* How many Days did he play? *Facit* 16 Days, and one Quarter.

QUEST. V.

A young Man coming into a Garden, saith, Bless you all you 10 Fair Maids! Sir, you mistake your self, saith one, for we are not 10; but if we were twice as many as we are, we should be as many above 10, as now we are less. How many were there? *Facit* 5 Maids, or rather Maidens.

QUEST. VI.

What Numbers are they, whose $\frac{5}{8}$ of the one, suppose *A*, is equal to $\frac{3}{7}$ of the other *B*? *Facit A* 72, *B* 105.

QUEST. VII.

Suppose there are two Numbers, *A* and *B*, the lesser of which, to wit *A* hath such Proportion to the Greater, to wit *B*, as 2 $\frac{4}{5}$ to 6, and the Sum of the said Numbers hath such Proportion to the Sum of the Square of the same Numbers, as 5 $\frac{1}{2}$ to 68 $\frac{1}{2}$. I demand each Number? *Facit* 7 for *A*, and 15 for *B*.

QUEST. VIII.

Divide 45 into two such Parts, that the Greater may be in triple Proportion to the Less, What are those Parts? *Facit* 11 $\frac{1}{4}$, and 33 $\frac{3}{4}$.

QUEST. IX.

Divide 10 into two such Parts, as if the Greater be divided by the Lesser, the Quotient may be 20, what are both Parts? *Facit* $\frac{10}{21}$, and 9 $\frac{11}{21}$.

QUEST. X.

A Vessel of 63 Gallons was fill'd with *French* Wine of two sorts; the one at 2 *s.* the Gallon, and the other at 2 *s.* 6 *d.* the Wine in the Hogshead thus fill'd did cost 7 *l.* 4 *s.* How much was there of each sort? *Facit* 27 Gallons of 2 *s.* and 36 of 2 *s.* 6 *d.*

QUEST. XI.

A Gentleman bought a House with a Garden, and a good Horse in the Stable, for 500 *l.* Now he paid 4 times the Price of the Horse for the Garden, and 5 times the Price of the Garden for the House: What did the House, what did the Garden, and what did the Horse cost?

Answer, The House 400 *l.* the Garden 80 *l.* the Horse 20 *l.*

Note, Some of the foregoing Questions may be wrought by the *Single Rule of False*; though notwithstanding they may be wrought by two Suppositions, as you may try at your Leisure. And to Conclude, any Question whatsoever, if not impossible, may be resolved; if by Comparing, Adding, Subtracting, or Proportion, you could prove your Question, if the true Resolution was given; for otherwise the Question cannot be resolved, because you cannot come to know what the Errors were at the Positions; but then they must be wrought by *Algebra.*

In the next Place we shall proceed to *Logarithmical Arithmetick*; and in that shall be very concise: Their chiefest Use in Arithmetick being in resolving Questions concerning *Compound Interest* and *Annuities.*

Logarithmical Arithmetick.

Logarithms are artificial Numbers which differ equally, fitted to the Natural, for Ease in Calculation.

The first Figure, called the *Index,* or *Characteristick,* shews how many Figures the answering Number consists of, which are always more by one than the Index, if the same be whole.

So the Index of any Number under 10, is (0); betwixt 10 and 100, (1); betwixt 100 and 1000 is (2) *&c.*

The Logarithm of a Fraction, or Decimal Number, is all one as an Integer, only with this Difference for the Index,

dex,

dex, that if the first Figure of the Decimal to the Left-hand be significant, the Index is (.9), if there be one Cypher before it, the Index is (.8), if two before it, the Index is (.7), &c.

EXAMPLE.

W. Numb.	*Logarithms.*	*Defect. Numbers.*	*Logarithms.*
2345	3.3701428	.2345	.9.3701428
234.5	2.3701428	.02345	.8.3701428
23.45	1.3701428	.002345	.7.3701428
2.345	0.3701428	.0002345	.6.3701428

Thus may you see the Logarithms are the same, but the Index thereof differeth, according as the first Figure thereof is removed from Unity.

Construction of Logarithms.

Their Construction, according to the common Rules, given by many Extractions of Roots, is tedious; the best way yet known is this which follows.

To make a Table of Logarithms.

First, Put for the Logarithm of 1, a Cypher for the Index, and a competent number of Cyphers for the Logarithm, according to the number of Places you would have your Logarithms consist of; for 10 an Unit, with the same number of Cyphers; for 100, 2, with as many Cyphers; for 1000, 3, with as many Cyphers, &c.

Secondly, Find the difference between some two Logarithms above 1000, or rather above 10000, that differ by Unity; thus, Multiply the two Numbers together, and that Product you must multiply again by 43429448190325182896, which last Product divided by the Arithmetical Mean between both Numbers, the Quotient is the difference sought.

Suppose we would find the difference between the Log. 10000, and 10001, the Product of these two Numbers is 100010000, which multiplied by 4343 produceth 434343430000; this divided by 10000.5, quotes 4343. Now if to

the

the Logarithm of 10000, which is 4.0000000, you add the difference before found, to wit, 434, the Sum 4.0000434 is the true Logarithm of 10001 to 7 Places.

Thirdly; Having thus found the difference of any two Logarithms difference by Unity, and consequently some of the Logarithms by dividing the Difference found by the Arithmetical Mean, between any two Numbers difference by Unity, you shall have the difference of the Logarithm of those two Numbers.

Thus to find the difference betwixt the Logarithm of 274, and 275; divide 4343, the difference of the Logarithm of 10000, and 10001 by 274.5, the Quotient 15821, is the difference sought.

Fourthly, Having by this means found a few of the prime Logarithms, the rest are made by Addition and Subtraction; and having made the Canon upward, above 1000 to 10000, by Consequence it's made for all inferiour Numbers.

The prime Numbers to which Logarithms must be found, in the first Place, are these, 2 . 3 . 7 . 11 . 13 . 17. 19 . 23 . 29 . 31 . 37 . 41 . 43 . 47 . 53 . 59 . 61 . 67 . 71 . 73 . 79 . 89 . 97, &c. or the same Numbers with Cyphers. There being several Tables of Logarithms, we shall only explain those, which in this Place we have made use of, which are Mr. *Oughtred's* in his *Trigonometry*, they being of as good, if not a better Character than any extant; and the Logarithms extended to 7 Places after the Index; of the same sort are Mr. *Gunter's*, Mr. *Norwood's*, Mr. *Leybourn's*, &c.

The Logarithm of any Number under 10000, is found by Inspection; so the Logarithm of 1234 is 3.0913151; but if your Number given consists of 5 or 6 Places, then you must use Proportion: Thus seek for the first 4 as before, noting the difference betwixt that Logarithm and the next greater; then say, As 10, if your Number consists of 5 Places, or as 100, if of 6 Places, to the said difference : : So the Figure, or Figures wanting to the Part Proportional, which added to the Logarithm before found, the Sum (when a true Characteristick or Index is fitted) is the Logarithm of the Number sought.

So the Logarithm of 12345 is 4.0914910 } &c.
And of 12356 is 5.0915121 }

To find the Number answering a Logarithm given, is but the Converse; the first four Figures are found by Inspection; but if you want for 5 or 6 Places, do thus; seek the Logarithm next less, and against it are the four first Figures, then seek the difference 'twixt that and the next greater, as likewise betwixt the given Logarithm and the next less, and say, As the first difference to 10, if for 5 Figures, or to 100, if for 6 Places; So the other difference to the remaining Figure or Figures sought.

So the Number answering 5.0915121 is 123456, &c.

Addition of Logarithms.

In *Addition* take this general Rule.

A General Rule.

If your Indices be Affirmative, add them as usual, and you have the true Sum.

But if they be Negative, add them as before; only Note; That if the Sum of the Indices be under 10, add 10; if just 10, add Unity; if above 10, cast 10 away, the Sum, or Remainder will be Negative.

But if the Indices be of different kinds, that is, one Affirmative and the other Negative, add them also.

If the Sum be 10, or above, cast away 10, the Remainder is Affirmative; if under 10, Negative.

EXAMPLES.

I. Unto 2.2671717
Add 3.1414498
Sum = 5.4086215

II. Unto .3.2671717
Add .5.1414498
Sum = 18.4086215

III. Add 2.2671717
.9.1414498
.8.8750613
Sum = 0.2836828

IV. Add .9.1414498
.7.2671717
.8.8750613
Sum = 5.2836828

More

More EXAMPLES.

V. Unto 2.2671717
Add .8.1414498

Sum = 0.4086215

VI. Unto .9.2671717
Add 3.1414498

Sum = 2.4086215

In the adding of these Logarithms there is no difficulty, excepting in the second, which may appear abstruse, where the Sum of the Indices is .18, which shews there is 11 Cyphers before the first significant Figure, the 10 of which signifies 10 Cyphers, and the 8 being defective, always is the sign of one Cypher before the first significant Figure, as was noted before.

And now we shall proceed to *Subtraction.*

Subtraction of Logarithms.

A General Rule.

If your Indices be Affirmative, and the higher the greater, then as usual.

If one or both be Negative, observe if the Index of the higher be smaller than the lower, if it be, add 10 to it; and if the higher be of greater Value, the Remains are Affirmative; if not, they are Negative.

EXAMPLES.

I. From 3.1414498
Subtract 2.2671717

Rest 0.8742781

II. From 2.2671717
Subtract 3.1414498

Rest .9.1257219

III. From .9.2671717
Subtract 3.1414498

Rest .6.1257219

IV. From 3.1414498
Subtract .9.2671717

Rest 3.8742781

V. From

V. From .8.8750613		VI. From .9.1414498
Subtract .9.1414498		Subtract .8.8750613
Rest .9.7336115		Rest 0.2663885

In these there is nothing obscure.

Multiplication of Logarithms.

To multiply one Number by another, is nothing but to add their Logarithms together, their Sum is the Logarithm of their Product.

EXAMPLES.

I. Multiply	144	Log.	2.1583625	Add.
By	12	Log.	1.0791812	
Product	1728	Log.	3.2375437	
II. Mult.	1385	Log.	3.1414498	Add.
By	185	Log.	2.2671717	
Product	256225	Log.	5.4086215	
III. Mult.	1.385	Log.	0.1414498	Add.
By	.0185	Log.	8.2671717	
Prod.	.0256225	Log.	8.4086215	
IV. Mult.	138.5	Log.	2.1414498	Add.
By	18.5	Log.	1.2671717	
Prod.	2562.25	Log.	3.4086215	

And so of any other.

Division in Logarithms.

To divide one Number by another, is nothing but to Subtract the Logarithm of the Divisor from the Logarithm of the Dividend, the Remainder is the Logarithm of the Quotient.

EXAM-

EXAMPLES.

I. Divide	1728	Log. 3.2375437	} Subtract.
By	12	Log. 1.0791812	
Quotes	144	Log. 2.1583625	
II. Divide	256225	Log. 5.4086215	} Subtract.
By	185	Log. 2.2671717	
Quotes	1385	Log. 3.1414498	
III. Divide	.0256225	Log. .8.4086215	} Subtract.
By	1.385	Log. 0.1414498	
Quotes	.0185	Log. .8.2671717	
IV. Divide	256.225	Log. 2.4086215	} Subtract.
By	138.5	Log. 2.1414498	
Quotes	1.85	Log. 0.2671717	

And thus of any other.

The Golden Rule in Logarithms.

In this Rule we have 3 Numbers given, to find a Fourth; wherefore if your Question be direct, work thus: Add the Logarithms of the Second and Third, and from that Sum subtract the Logarithm of the First, the Remainder is the Logarithm of the fourth Proportional sought.

EXAMPLE.

If 13 Groce of Incle cost 7 *l.* 12 *s.* what will 65½ Cost?

See the Work.

If 13 Groce, Log. - - - - - - - - - - - -	1.1139433
l. Cost 7.6, Log. - - - - - - - - - - - -	0.8808136
What 65.5 Groce, Log. - - - - - - - -	1.8228216
	2.7036352
Answer, 38.8769. Log. =	1.5896919

This

This may be performed by Addition, thus; Add the Arithmetical Complement of the Logarithm of the First, unto the Logarithm of the Second and Third, the Sum is the Logarithm of the Fourth.

Arith. Comp.	13 .	8.8860567
	7.6 .	0.8808136
	66.5 .	1.8228216
	38.8769 .	1.5896919

The Arithmetical Complement is only the Remainder of every Figure to 9, and the last to 10. So the Arithmetical Comp. of 0.8808136 is 9.1191864, of 2.0000000 is 8.0000000.

Here you may see the Answer is the same as it was before.

But if your Question be Inverse, work thus; Add the Logarithms of the First and Second together, and from that Sum subtract the Logarithm of the Third, the Remainder is the Logarithm of the fourth Proportional sought.

EXAMPLE.

If 12 Men do a piece of Work in 20 Days, in how many Days will 60 Men do the same Work?

The Operation.

If 12 Men, Log.	1.0791812
Require 20 Days, Log.	1.3010300
The Sum	2.3802112
What will 60 Men require? Log. Sub.	1.7781512
Answer, 4 Days. Log.	0.6020600

This may likewise be performed by Addition, by adding the Arithmetical Complement of the Logarithm of the Third, to the Logarithm of the First and Second; the Sum is the Logarithm of the Fourth.

Thus, If 12 Men, Log.		1.0791812
Require 20 Days, Log.	Add.	1.3010300
What will 60 Men require, *Arith. Comp.*		8.2218488
Answer, 4 Days, as before, Log.		0.6020600

And so in any other.

Whence

Whence you may observe, that in Multiplication, instead of adding the two Logarithms together, you may subtract the Arithmetical Complement of the Logarithm of the one, from the Logarithm of the other, the Remainder is the Logarithm of the Product.

Likewise in Division, instead of subtracting one from the other, you may add the Arithmetical Complement of the Logarithm of the Divisor to the Logarithm of the Dividend, the Sum is the Logarithm of the Quotient.

Extraction of the Square Root.

Though Extraction of Roots by natural Numbers, be one of the difficultest Parts of Arithmetick, yet by artificial Numbers or Logarithms, nothing is more easie and plain, as may be seen in the Practice thereof.

To Extract the Square Root of any Numbers, is perform'd by parting or halving its Logarithm; the said half is the Logarithm of the Root sought.

EXAMPLE I.

What is the Square Root of 144? Log.	2.1583625
Half is	1.0791812

Which is the Logarithm of 12, the Root sought.

EXAMPLE II.

Let it be required to find the Square Root of 160.

Log. of 160, is	2.2041200
Half is the Logarithm of 12.6491	1.1020600

Which is the side of a square Acre, and true to 4 Places of Decimals; which is exact enough for common Use, 160 being a Surd Number, its true Root is inexpressible.

Note, If the Number whose Root is sought be a Decimal, add 10 to the Index, and halve it, as in this

EXAMPLE III.

What is the Square Root of .225? Log.	19.3521825
Half is	9.6760912

Which is the Logarithm of .4743, the Root sought.

And seeing halving the Logarithm of any Number gives the Logarithm of its Root, then it follows, that multiplying the Logarithm of any Logarithm by 2, gives the Square thereof; as may be seen in this Example.

EXAMPLE IV.

What is the Square of 11826? Its Log.	4.0728378
Multiply by	2
Which is the Logarithm of 139854276 =	8.1456756
Proof is its Root, *viz.* 11826, $\frac{1}{2}$ =	4.0728378

This Number 139854276 is a very remarkable Number: *First*, It's a square Number; *Secondly*, It contains 9 Places, and they are the 9 Digits; and I think there is not another that does.

Extraction of the Cube Root.

As the Square Root was found by Bi-partition, or halving its Logarithm, so the Cube Root is found by Tri-partition, or taking one third Part of its Logarithm, which will be the Logarithm of the Cube Root sought.

EXAMPLE I.

What is the Cube Root of 1728?

The Logarithm of 1728, is	3.2375437
One Third of this is	1.0791812

Which is the Logarithm of 12, the Cube Root sought.

EXAMPLE II.

What is the Cube Root of 123456?

Its Logarithm is	5.0915121
One third thereof is	1.6971707

Which is the Logarithm of 49.7932, the Cube Root sought.

If your Number be a Decimal, add 20 to its Index, and take $\frac{1}{3}$ as before.

So the Cube Root, of .256, its Log. $\frac{1}{3}$	29.4082400
will be .6348	9.8026800

Hence

Hence if you have a mind to Cube any Number, you must multiply its Number by 3, and you have the Logarithm of its Cube; so the Cube of 9 will be found to be 729,

	9 Log. 0.9542425
	3
	2.8627275

Proportions in Logarithms,

To find a mean Proportional between two Numbers.

RULE.

Add the Logarithm of two Numbers into one Sum, the $\frac{1}{2}$ of which is the Logarithm of the mean Proportional sought.

EXAMPLE.

Let the two Numbers be 16 and 144, and let a mean Proportional be required.

Log. of 16 is	1.2041200
Log. of 144 is	2.1586525
Their Sum	3.3624825
Half of which is	1.6812412

which is the Logarithm of 48, the mean Proportional sought.

Note, If one be a Decimal, if the Sum of the Indices be 10, or above, cast away 10, and then halve it; if it be not 10, add 10 to it, and then halve it.

So a mean Proportional betwixt 12 Log.	1.0791812
And .25 Log.	9.3979400
The Sum	0.4771212
Will be 1.732	0.2385606

Between two Numbers given, to find any Number of mean Proportionals desired.

RULE,

Subtract the Logarithm of the less Number out of the Logarithm of the greater; the Remainder divide by a Number greater by one than the number of Means sought; this

Quotient added to the Logarithm of the less Number, the Sum is the Logarithm of the first Mean; to which adding again the said Quotient, the Sum is the Logarithm of the second Mean, and so forward as far as you have occasion.

EXAMPLE.

Betwixt 16 and 64 find five mean Proportionals.

Log. of 64 is	1.8061800
Log. of 16 is	1.2041200
Divide, the Difference is	0.6020600
$\frac{1}{6}$ part for 5 Means is	0.1003433
To which add the Log. of 16	1.2041200
The Sum is the first Mean 20158	1.3044633
To which add again	1003433
The Sum is the Log. of the 2d Mean 25398	1.4048066
To which add again	1003433
The Log. of the 3d Mean 32	1.5051499
To which add	1003433
The Log. of the 4th Mean 40.317	1.6054932
Unto which add again	.1003433
The Log. of the last Mean 50.796	1.7058365

This Proposition is of excellent Use in the Calculation of Tables belonging to compound Interest, as shall be shewn in due Place.

Having three Numbers given to find a fourth in a duplicated Proportion.

RULE.

Double the difference of the Log. of those two Numbers, which have the same denomination; then according as the first Term is less, or greater than the second, add or subtract the double Difference to, or from, the Logarithm of the other Number; this done, the Sum or Remainder is the Logarithm of the 4th Number sought.

EXAM-

EXAMPLE.

If the Content of a Circle, whose Diameter is 7 Inches, be 38.484, what is the Content of that Circle, whose Diameter is 21? *Answer* 346.361.

See the Work.

Diameter 7 Inches,	Log.	0.8450980	
Diameter 21 Inches,	Log.	1.3222193	
Difference is.		0.4771213	
Difference doubled		0.9542426	} Add.
Content given, 38.484	Log.	1.5852802	
Content required, 346.3561	Log.	2.5395228	

By this Proposition we find the Proportion betwixt like Superficies, which by *Euclid* the 6th, and 19th and 20th, is proved to be in duplicate Proportion of their Homologous Sides.

So if a Field measured by a Statute Perch, contain 36 Acres, it would, if measured by a *Cheshire* Perch, of 24 Feet to the Pole, be found to contain but 17 Acres, and $\frac{1}{64}$th Part.

Having three Numbers given, to find a fourth, in a Triplicated Proportion.

RULE.

Triple the difference of the Logarithm of those two Numbers, which have the same Denomination; then according as the first Term is lesser or greater than the 2d, add or subtract the tripled Difference to, or from the Logarithm of the other Number; this done, the Sum or Remainder is the Log. of the 4th Number sought.

EXAMPLE.

If a Bullet, whose Diameter is 9 Inches, do weigh 72 Pound, what will a Bullet of the same Metal weigh, whose Diameter is 6 Inches?

Diameter

Diameter 9 Inches,		Log.	0.9542425
Diameter 6 Inches,		Log.	0.7781512
Difference is			0.1760913
Difference tripled			5282739 } Sub;
Weight given	72 *lb.*	Log.	1.8573325 }
Weight required	21 $\frac{1}{3}$		1.3290586

By this Propofition we find the Proportion betwixt like Solids; for as like Superficies do hold in a duplicated Proportion; fo like Solids do hold in a triplicated Proportion of their Homologous Sides, Diameters, *&c.*

In the next Place we fhall give you a few Propofitions in *Military Orders*, and then proceed to our chief Defign, which is *Compound Intereft.*

Military Orders by the Logarithms.

PROP. I.

Any number of Soldiers being given, to place them in a fquare Battalia of Men.

RULE.

One half of the Logarithm of the number of Soldiers given, will be the Logarithm of the number of Men, both in Rank and File, to make a fquare Battalia of Men.

EXAMPLE.

Let 1764 Men be given, and let it be required to frame them into a fquare Battalia.

Log. of 1764 is	3.2464986
$\frac{1}{2}$ is	1.6232493

which is the Logarithm of 42, which is the number of Men that muft be placed both in Rank and File, to make a fquare Battalia of Men.

PROP.

PROP. II.

Any number of Men being given, to order them into a double Battalia; that is, that shall have twice as many in Rank as File.

RULE.

Half the Logarithm of ½ the number of Men, is the Logarithm of the number of Men to be placed in File, and that number doubled, is the number to be placed in Rank.

EXAMPLE.

Let the number of Men be 8450, and let it be required to make a double Battalia of them.

Half the given number of Men is 4225, Log. 3.6258267
The half of which is 1.8129133

which is the Logarithm of 65, and so many must be placed in File, which doubled, makes 130, which is the number of Men to be placed in Rank, for 65 times 130, is equal to 8450, the number of Men given.

PROP. III.

Any number of Soldiers given, to order them into a quadruple Battalia of Men; that is, that shall have 4 times as many in Rank as File.

RULE.

Half the Logarithm of one quarter of the number of Soldiers given, is the Logarithm of the number of Men to be placed in File, which Number multiplied by 4, is the Number to be placed in Rank.

EXAMPLE.

Let the given number of Men be 4900 to be ordered into a quadruple Battalia.

One quarter of which Number is 1225. Log. 3.0881361
The half of which is 1.5440680

which is the Logarithm of 35, the number to be placed in File, which multiplied by 4, gives 140, which must be the number to be placed in Rank.

PROP.

PROP. IV.

Any number of Soldiers given, to order them into three equal square Battalions.

RULE.

Half the Logarithm of $\frac{1}{3}$ part of the number of Soldiers given, is the Logarithm of the Number to be placed both in Rank and File in every Battalion.

EXAMPLE.

Let the given number of Soldiers be 6075, and let it be required to form 3 square Battalions of them that shall have an equal number of Men, both in Rank and File.

One third of 6075 is 2025 its Log.	3.3064250
The half of which is	1.6532125

which is the Logarithm of 45, the number of Men in each Battalion that must be placed both in Rank and File.

PROP. V.

Any number of Soldiers given, to place them in Rank and File, according to the Proportion of any two Numbers given.

RULE.

From the Sum of the Logarithms of the number of Soldiers given, and the Proportional Number for the Rank, subtract the Logarithm of the Proportional Number of File, half the Remainder is the Logarithm of the Men to be placed in Rank, and the same Logarithm subtracted from the Logarithm of the whole number of Soldiers, leaves the Logarithm of the number to be placed in File.

EXAMPLE.

Let 3600 Soldiers be so placed, that the number in File may be to those in Rank, as 5 to 9.

The

The Log. of 3600 is	3.5563025
The Log. of 9, the Prop. of Rank is	0.9542425
The Sum is	4.5105450
Log. of 5, Prop. for File is	0.6989700
Difference is	3.8115750
Half of which is	1.9057875

which is the Logarithm of 80, nearest the number in Rank, and the last Log. subtracted from the first, leaves the Log. of the File, to wit, 1.6505150, the Log. of 44, nearest the number in File.

PROP. V.

Any number of Soldiers given, with their distance in Rank and File, to order them into a square Battalia of Ground.

RULE.

From the Sum of the Log. of the number of Soldiers, and of the distance in Rank, subtract the Logarithm of their distance in File, half of the Remainder is the Number in File; which Logarithm subtract from the whole number of Soldiers, the remainder is the Logarithm of the number of Soldiers to be placed in Rank.

EXAMPLE.

Let 3600 Soldiers be ordered into a square Battalia of Ground, so that the distance in File may be 7 Foot, and in Rank 3 Foot, so that the Ground they stand upon may be a true Square.

The Logarithm of 3600 is	3.5563025
Log. of 3, the distance in Rank is	0.4771212
The Sum is	4.0334237
Log. of 7, the distance in File is	0.8450980
Difference is	3.1883257
Half of which is	1.5941628

which is the Logarithm of 31 nearest for the number of Men in File, and the last Logarithm subtracted from the first Logarithm leaves 1.9621397, which is the Logarithm of 91 nearest the number of Men in Rank.

More might be added, but these are sufficient.

Compound INTEREST.

WHen a Sum of Money is lent, and the Interest, when due, is not paid, but kept in the Borrower's Hands, and by that means become a Part of the Principal, then it is called *Compound Interest*.

As if *A* lend to *B* an 100 *l.* at the Rate of 6 *per Cent.* for a Year, then it is evident that at the End of one Year, *B* is got into *A*'s Debt 106 *l.* and if this be continued in *B*'s Hand till the End of the second Year, there will then be due to *A* the increase of 106, *viz.* 112 *l.* 7 *s.* 2 *d.* which will be a new Stock for the third Year, if not paid at the second Year's End.

Whereby it is plain, that if it be Lawful to take Interest at all, it is Lawful to take Compound Interest: For if *A* had received this Interest Annually as it became due, he had the Advantage of putting out those Annual Payments at the same Rate, and none would have stiled it Compound Interest. This will yet appear more plain, by supposing *A* lay'd out his 100 *l.* in purchasing an Annual Rent of 6 *l.* clear Value, which Annual Rent may be made Use of to his best Advantage, and none call him an Extortioner.

And lastly it will appear, that for any time under a Year, Compound Interest is more easie than Simple; for he that takes 3 *l.* for the Use of an 100 *l.* for one half Year, takes too much, which may be proved thus; For as Simple Interest was performed by a rank of Numbers Arithmetically proportional; so Compound Interest is perform'd by a rank of Numbers Geometrically proportional.

And it is to be known, that if three Numbers be in Geometrical Proportion, the Product of the two Extreams is equal to the Square of the Mean, by the 20*th* of the 7*th* of *Euclid*. So on the contrary, if the Rectangle contained under the Extreams of any three Numbers, be equal to the Square of the Mean, then those three Numbers are in Geometrical Proportion.

Now

Now if 3 *l.* be the Interest of 100 *l.* for a half Year, or six Months, then these three Numbers 100, 103, 106, should be in Geometrical Proportion; but it may be proved by the aforesaid Proposition they are not: For the Rectangle of 100 and 106 is but 10600, and the Square of the Mean 103, is 10609. But if the Square Root of 10600 be sought, it will be found to be 102.9561. So that the true Proportional Interest of 100 *l.* for six Months, or half a Year, is but 2 *l.* 19 *s.* 1 *d.* ½ *fere.*

In the Solution of Questions of Compound Interest, four Things are to be consider'd.

First, The Principal, or Money lent.

Secondly, The Time of Forbearance, in Years, or Parts of a Year:

Thirdly, The Rate of Interest *per Cent.* by the Year, half Year, or Quarter, *&c.* Equal to 1.06, 1.08, 1.10, *&c.*

Fourthly, The Amount of the said Principal for the said Rate and Time.

Any three of these being given to find the Fourth, as in the four Propositions following.

PROPOSITION I.

Principal, Rate, and Time given, to find the Amount.

RULE.

Unto the Logarithm of the Rate multiply'd by the Time, add the Logarithm of the Principal, the Sum is the Logarithm of the Amount.

EXAMPLE.

What will 20 *l.* amount to forborn 7 Years, at 6 *per Cent.* Compound Interest?

l.
Principal 20 *l.* Rate 1.06, Time 7 Years.

See the Work.

Log. of 1.06, the Rate is		0.0253059
Multiply by the Time		7
Product of the Rate and Time,	Log.	.1771413
Add the Log. of 20 the Principal		1.3010299
The Sum is		1.4781712

Which is the Log. of 30 *l.* 1 *s.* 5 *d.* ¼, the Amount sought.

EXAMPLE II.

What will 365 *l.* 15 *s.* 6 *d.* amount to, forborn 11 Years and a Quarter, at 5 *per Cent.* Compound Interest?

l. *l.* Y.
Principal 365.775, Rate 1.05, Time 11.25.

The Work.

Log. of 1.05 the Rate, is		0.0211893
Multiply by the Time		11.25
Product of the Rate and Time,	Log.	0.2383796
Add the Log. of 365.771, the Principal		2.5632140
The Sum is		2.8015936

Which is the Log. of 633 *l.* 5 *s.* 6 *d.* ¼, the Amount.

PROPOSITION II.

Amount, Rate, and Time given, to find the Principal.

RULE.

From the Logarithm of the Amount, subtract the Logarithm of the Rate, multiply by the Time, the Remainder is the Logarithm of the Principal, or present Worth.

EXAMPLE I.

What present Money will pay a Debt of 20 *l.* due 7 Years hence, at 5 *per Cent. per Ann.* Compound Interest?

l. *l.* Y.
20 the Amount, 1.05 the Rate, and 7 the Time.

Log.

Log. of 20 *l.* the Amount 1.3010300
Product of the Rate and Time fub. 0.1483251

The Remainder is 1.1527049
Which is the Log. of 14 *l.* 4 *s.* 3 *d.* ¼, the ready Money fought.

EXAMPLE II.

A Gentleman left his Son 150 *l.* to be paid at the Age of 21 Years, of which 7 Years were fpent at the faid time; the Executors defire to pay ready Money, fo they may have Rebate allow'd after the Rate of 6 *per Cent. per Ann.* Compound Intereft, the Queftion is, What ready Money will pay this Debt?

Amount 150, Rate 1.06, Time 14 Years.

Log. of the Rate 1.06, is	0.0253059
Multiply by the Time	14
	1012236
	253059
Product of the Rate and Time	3542826
Which Sub. from the Log. of 150, viz.	2.1760913
Reft	1.8218087

Which is the Log. of 66.345, or 66 *l.* 6 *s.* 11 *d.* which is the Anfwer to the Queftion.

PROPOSITION III.

Principal, Amount, and Rate given, to find the Time.

RULE.

From the Logarithm of the Amount fubtract the Logarithm of the Principal, that divided by the Logarithm of the Rate, gives the Time.

EXAMPLE I.

In what time will 20 *l.* amount to 40 *l.* at 6 *per Cent. per Ann.* Compound Intereft.

Prin-

Principal 20 *l.* Amount 40 *l.* Rate 1.06.

The Log. of 40 *l.* the Amount is	1.6020600
The Log. of 20 *l.* the Principal is	1.3010300
Diff.	.3010300

0253059) 3010300 (11.895
253059
479710
253059
2266510
2024472
2420380
2277531
1428490

Ans. 11 Years, 11 Months, 2 Weeks, and 5 Days ; dividing a Year as in Simple Interest.

EXAMPLE II.

In what time will 15 *s.* amount to 15 *l.* at 10 *per Cent. per Ann.* Compound Interest?

Principal .75, Amount 15, Rate 1.10.

Log. of 15 the Amount, is	1.1760913
Log. of .75 the Principal is	9.8750613
Difference is,	1.3010300

0413927) 1.3010300 (31.4313
1241781
592490
413927
1785630
1655708
1299220
1241781

Answer, In 31 Years, 2 Weeks, and 3 Days.

P R O P.

PROPOSITION IV.

Principal, Time, and Amount given, to find the Rate.

RULE.

From the Logarithm of the Amount, ſubtract the Logarithm of the Principal, the Remainder divided by the Time, quotes the Logarithm of the Rate.

EXAMPLE I.

At what Rate of Compound Intereſt will 20*l.* amount to 30.072, or 30*l.* 1 *s.* 5 *d.* 1 *q.* in 7 Years?

l.

Principal 20*l.* Amount 30.072, Time 7 Years.

Amount 30.072, Log.	1.4781712
Principal 20 *l.* Log.	1.3010300
	0.1771412
$\frac{1}{7}$ =	0.0253058

Equal to the Logarithm of 1.06, the Rate ſought.

EXAMPLE II.

At what Rate of Compound Intereſt will 51 *l.* 15 *s.* amount to 70 *l.* 18 *s.* in 5 Years?

Principal 51.75, Amount 70.9, Time 5 Years.

l.	
Amount 70.9, Log.	1.8506462
Principal, 51.75, Log.	1.7139103
	0.1367359
$\frac{1}{5}$	0.0273488

Equal to the Logarithm of 1.065, which is 6 *l.* 10 *s.* *per Cent. per Ann.* the Rate ſought.

The two firſt Propoſitions being often uſed, we have, as in *Simple Intereſt*, annexed Tables fitted thereto, at the Rates of 5 and 6 *per Cent.* and to continue for 31 Years.

TABLE

Years	TABLE I. Shewing the Amount of one Pound for 31 Years, at 5 and 6 per Cent. Compound Interest.		TABLE II. Shewing the Rebate of One Pound for 31 Years, at 5 and 6 per Cent, Compound Interest.	
	5	6	5	6
1	1.050000	1.060000	.952381	.943396
2	1.102500	1.123600	.907029	.889996
3	1.157625	1.191016	.863838	.839619
4	1.215506	1.262477	.822703	.792093
5	1.276281	1.338225	.783526	.747258
6	1.340096	1.418519	.746215	.704960
7	1.407100	1.503630	.710681	.665057
8	1.477455	1.593848	.676839	.627412
9	1.551328	1.689479	.644609	.591898
10	1.628895	1.790848	.613913	.558394
11	1.710339	1.898298	.584679	.526787
12	1.795856	2.012196	.556837	.496969
13	1.885649	2.132928	.530321	.468839
14	1.979932	2.260904	.505068	.442301
15	2.078928	2.396558	.481017	.417265
16	2.182874	2.540352	.458111	.393647
17	2.292018	2.692773	.436296	.371364
18	2.406619	2.854339	.415520	.350343
19	2.526950	3.025599	.395734	.330513
20	2.653298	3.207135	.376889	.311804
21	2.785962	3.399564	.358942	.294155
22	2.925261	3.603537	.341849	.277505
23	3.071524	3.819750	.325571	.261797
24	3.225100	4.048935	.320067	.246978
25	3.386355	4.291871	.295302	.232998
26	3.555673	4.549383	.281240	.219810
27	3.733456	4.822346	.267848	.207368
28	3.920129	5.111687	.255093	.195630
29	4.116135	5.458388	.242946	.184556
30	4.321942	5.743491	.231377	.174110
31	4.538039	6.088101	.220359	.164254

The

The Construction and Uses of the foregoing Tables.

For the Construction of these two Tables are several Methods used, we shall only mention that which is most Easie and Expeditious, which is by the Logarithms.

For the first Table thus; Seek by the first Proportion aforegoing, the Amount of one Pound for 31 Years, and betwixt that Log. and the Log. of the Rate find 30 Geometrical mean Proportionals, as before taught, which shall be the Log. of the Number in the first Table; which is nothing else but the continual Addition of the Log. of the Rate to it self, and to its last Sum: As if we add the Log. of the Rate to its self, the Sum is the Log. of the Number belonging to the second Year, and to that Sum add again the Log. of the Rate, gives the Log. of the Number belonging to the third Year; and thus you may do till you have finish'd: Or if you multiply the Log. of the Rate by 1, 2, 3, 4, 5, 6, *&c.* gives the Log. of the Numbers answering those respective Years.

And for the Numbers in the second Table, take the Arithmetical Complements of the Log. of the Numbers in the first Table, and you will have the Log. of the Numbers in the Second. *Now for their Uses.*

These Tables are to be Used in the same Manner as those in *Simple Interest*, and so need but few Examples.

Take an Example for the Use of the first Table.

What will 20 *l.* amount to forborn 7 Years at 6 *per Cent.* Compound Interest?

In the first Table under 6 *per Cent.* and over-against 7 Years is 1.50363

Which multiply by 20

The Product is 30.07260

Which is Equal to 30 *l.* 1 *s.* 5 *d.* 1 *q.*

Take another Example for the Use of the second Table,

What ready Money will pay a Debt of 36 *l.* 10 *s.* Due 21 Years hence, at 5 *per Cent.* Compound Interest?

The Operation.

In the 2d. Table, and under 5 *per Cent.* and overagainſt 21 Years is,

21 Years is,	.358942
Which multiplied by	36.5
	1794710
	2153652
	1076826
Produceth	13.1013830

which is equal to 13 *l.* 2 *s.* The Anſwer.

SECTION II.

In the Solution of Queſtions of *Compound Intereſt* concerning Annuities in arrear, we may conſider it under theſe four Particulars, (*viz.*)

Firſt, The Annuity, or Penſion.
Secondly, The Time of Forbearance in Years, or Parts of a Year.
Thirdly, The Rate of Intereſt. And
Fourthly, The Amount of the ſaid Annuity, for the ſaid Rate and Time.

Any three of theſe being given to find the fourth, as in theſe four *Propoſitions* following.

PROP. I.

Annuity, Rate and Time given, to find the Amount.

RULE.

Firſt you find a correſpondent Principal in this manner; As the Intereſt to its Principal: So the given Annuity to its correſpondent Principal. Next multiply the Logarithm of the Rate by the Time, to which add the Logarithm of the correſpondent Principal, the Sum is the Logarithm of a Number; from which, ſubtract the correſpondent Principal, leaves the Amount.

EXAM-

EXAMPLE.

An Annuity of 20 *l. per Annum* is forborn 7 Years, what will then be due at 6 *per Cent.* Compound Intereſt.

First ſay, If 6 *l.* : 100 *l.* : : 20 *l.*
20

2000

$\frac{1}{6}$ 333 $\frac{1}{3}$ = the Correſpondent Principal

Log. of the Rate	0.0253059
Multiplied by the Time	7
Log. of the Rate × by the Time	0.177143
Log. of 333$\frac{1}{3}$ the Correſp. Principal add.	2.5228788
Equal to the Log. of 501.210	2.7000201
Cor. Princ. ſubtract. 333.333	

Reſt 167.877 = to 167 *l.* : 17 *s.* : 6$\frac{1}{2}$ *d.* *fere*, the Anſwer.

EXAMPLE II.

There is an Annuity of 50 *l. per Annum*, payable by Quarterly Payments, (*viz.*) 12.5 *l. per* Quarter; this Annuity is forborn to the end of 11 Years and Half; the Queſtion is, What will then be due at 6 *per Cent.* Compound Intereſt?

Note, If the Interval betwixt any Payments be leſs than a Year, as ſuppoſe half Yearly, Quarterly, Monthly, Weekly, Daily, *&c.* then you muſt divide the Logarithm of the Rate, by ſuch part; as by 2 for half Yearly Payments, 4 for Quarterly Payments, by 52 for Weekly Payments, and by 365 for Daily Payments; and your Quotient will be a Proportional Rate, whereby to find your correſpondent Principal; for if the abſolute Number anſwering that Quotient, be made leſs by an Unit, it will be a new Diviſor; by which dividing your half Yearly, Quarterly, *&c.* Payment, your Quotient, will be a correſpondent Principal; then may you work as before.

See underneath the Logarithm of 1.05 and 1.06, so divided with their natural Numbers placed overagainst them.

	Logarithms.		*Natural Numb.*
Log. of 1.05	0.0211893	=	1.05
$\frac{1}{2}$	0.0105946	=	1.0246738
$\frac{1}{4}$	0.0052973	=	1.0122722
$\frac{1}{12}$	0.0017658	=	1.0040741
$\frac{1}{52}$	0.0004075	=	1.0009387
$\frac{1}{365}$	0.0000581	=	1.0001336
Log. of 1.06	0.0253058	=	1.06
$\frac{1}{2}$	0.0126529	=	1.0295630
$\frac{1}{4}$	0.0063264	=	1.0146738
$\frac{1}{12}$	0.0021088	=	1.0048675
$\frac{1}{52}$	0.0004866	=	1.0011219
$\frac{1}{365}$	0.0000693	=	1.0001596

So in the last Question, the Payments being Quarterly, I take the natural Number answering $\frac{1}{4}$ part of the Logarithm of the Rate 1.06, which made less by Unity, is .014673, by which dividing the Quarterly Payment, 12 *l.* 10 *s.* Quotes 851.9048, the Correspondent Principal.

See the Work.

.014673) 12.5000000 (851.9048

117384

76160
73365

27950
14673

132770
132057

71300
58692

12608

Log.

Log. of the Rate 1.06 is	0.0253058
Multiply by the Time	11.5
	1265290
	253058
	253058
Log. of the Rate and Time	2910167
Log. of the Corresp. Principal, add.	2.9303911
The Sum is the Log. of 1665.2053	= 3.2214078
Sub. the Cor. Principal, 851.9048	

Rest 813.3005 = to 813 *l.* 6 *s.* which is the Answer.

PROP. II.

Amount, Rate and Time given, to find the Annuity.

RULE.

Suppose an Annuity at Pleasure, and by the last *Proposition* find the Amount or Arrearages; then you may say: As the Amount to the supposed Annuity; so the Amount given, to the Annuity required.

EXAMPLE.

What Annuity at 6 *per Cent.* Compound Interest, will raise a Stock of 167.877 *l.* in 7 Years?

Suppose 3 *l.*

Then if 6 *l.* : 100 *l.* :: 3 *l.* : *Facit* 50 *l.* a correspondent Princip.

Log. of the Rate	
Multiply by the Time	0.0253059
	7
	0.1771413
Log. of Correspond. Principal, add.	1.6989700
Equal to the Log. of 75.181	1.8761113
Cor. Princip. subtr. 50.000	
25.181	

Then

Then say, If 25.181 : 3 :: 167.877

```
                 3
          -------
25.181) 503.631 (20 l.
        50362
        -------
           11
```

Answer 20 *l.*

PROP. III.

Annuity, Rate, and Amount given, to find the Time.

RULE.

Find a correspondent Principal, and add it to the given Amount, and from the Logarithm of that Sum, subtract the Logarithm of the correspondent Principal, the Remainder, divided by the Logarithm of the Rate, quotes the Time.

EXAMPLE.

In what Time will 20 *l. per Annum* raise a Stock of 167.877 Compound Interest, being computed at 6 *per Cent. per Annum*.

First, If 6 : 100 :: 20 : *Facit* 333 $\frac{1}{3}$ Corresp. Principal.

```
      Given Stock  167.877
    Corresp. Princ.  333.333
                     -------
           The Sum  501.210
```

```
Log. of 501.21 is                    2.7000197
Log. of Correspondent Principal is   2.5228788
                                     ---------
               0253059) 1771409 (7
                        1771413
                        -------
```

Answer in 7 Years.

PROP. IV.

The Annuity, Amount and Time given, to find the Rate of Interest.

RULE

RULE.

To anſwer this, we will uſe Approximation, it being the moſt conciſe and quickeſt Method we can uſe: Wherefore make two or three Trials, till you get the Anſwer bounded betwixt two of the neareſt Reſults; then the Work may be performed by Proportion, as may be ſeen in the Work of the following *Example.*

EXAMPLE.

An Annuity of 20 *l. per Annum* is offered to be let for *l.* 180.5, or 180 *l.* 10 *s.* to be paid at the end of the ſaid Term, what Intereſt is allow'd in this Bargain?

Intereſt of Money being ſeldom above 10 *l.* and under 5 *per Cent.* Wherefore I make a Suppoſition at 8 *per Cent.* and by the firſt *Propoſition* I find the Amount at that Rate, to be 178.456, which is too little by *l.* 2.044.

Wherefore becauſe I ſee I am near, I make my ſecond Trial at 8 *l.* 10 *s. per Cent.* and working as before, I find the Amount to be 181.21, whereby I ſee I have overſhot the Truth by .71, and I ſee the Anſwer is bounded betwixt 8 *l. per Cent.* and 8 *l.* 10 *s. per Cent.*

Wherefore, as in the *Rule of Falſe*, by theſe two Suppoſitions, and their reſpective Errors, I find the Rate as under.

Firſt Suppoſition	8	the Error	2.044	—
Second Suppoſit.	8.5	the Error	0.71	+
Supp. Difference	.5	Sum	2.754	

Then ſay, As 2.754 : .5 :: 2.044 : to .371, which added to the firſt *Suppoſition* 8, gives 8.371, or 8 *l.* 7 *s.* 5 *d.* the Rate of Intereſt ſought.

The firſt Propoſition being of good and frequent Uſe, we have adjoyned a Table fitted thereto, and Calculated at the Rates of 5 and 6 *per Cent.* Compound Intereſt, and to continue for 31 Years.

The

A TABLE, Shewing the Amount of one Pound Annuity forborn for 31 Years, or under, at 5 and 6 per Cent. Compound Interest.

Years	5	6
1	1.000000	1.000000
2	2.050000	2.060000
3	3.152500	3.183600
4	4.310125	4.374616
5	5.525621	5.637093
6	6.801913	6.975318
7	8.142008	8.393837
8	9.649108	9.897467
9	11.026564	11.491315
10	12.577892	13.180794
11	14.206787	14.971642
12	15.917126	16.869940
13	17.712982	18.882137
14	19.598631	21.015065
15	21.578563	23.275969
16	23.657491	25.672527
17	25.840366	28.212879
18	28.132384	30.905651
19	30.539003	33.759990
20	33.065954	36.715590
21	35.719251	39.992725
22	38.505214	43.392289
23	41.430475	46.995826
24	44.501999	50.815575
25	47.727099	54.864510
26	51.113453	59.156381
27	54.669126	63.705763
28	58.402583	68.528109
29	62.322712	73.639796
30	66.438847	79.058184
31	70.760790	84.801675

The Construction and Use thereof, here follow.

Its Construction.

The Logarithmical differences of .05 or .06 (being the Rates here used *Minus* Unity) and the Numbers in the Table shewing the Amount of one Pound at 5 and 6 *per Cent.* for 31 Years *minus* Unity, are the Logarithms of the Numbers in this Table.

Take an Example or two for the Use.

What will an Annuity of 3 *l.* 15 *s.* 6 *d.* amount to forborn 21 Years, compound Interest being computed at 6 *per Cent.*

Tabular Number answering 21 Years, and under 6 *per Cent.* is 39.992725
Multiply by 3.775

```
     39.992725
         3.775
  ------------
     199963625
    279949075
   279949075
  119978175
  ------------
  150.972536875
```

Facit 150 *l.* 19 *s.* 5 *d.* $\frac{1}{4}$.

E X A M-

EXAMPLE II.

What will an Annuity of 50 *l. per Annum* amount to, forborn 7 Years, at 5 *per Cent.* compound Interest?

Tabular Number under 5 *per Cent.* and against 7 Years is	8.142008
Multiplied by	50
Answer 407 *l.* 2 *s.*	407.100400

In the Solutions of Questions of Compound Interest, relating to many equal Payments, at many equal Times, as in the Buying or Purchasing Annuities, Pensions, or Leases in Reversion, we may consider it under these 4 Particulars:

First, The Annuity, or Pension to be sold.
Secondly, The Time of Continuance, either considered as Yearly, half-Yearly, or Quarterly Payments,
Thirdly, The Rate of Interest, And
Fourthly, The present Worth of the whole, paid at one entire Payment, or equally reduced to such.

Any three of these being given, to find the fourth, as in the four *Propositions* following.

PROB. I.

Annuity, Rate, and Time given, to find the present Worth.

RULE.

Find a correspondent Principal as before taught; then multiply the Logarithm of the Rate by the Time, which subtract from the correspondent Principal, the Remainder is the Logarithmical Difference of the Principal and Worth, and so by Consequence the Worth is given.

EXAMPLE.

There is an Annuity of 20 *l. per Annum*, payable by Yearly Payments, and to continue 7 Years, to be sold for ready Money: What is it Worth, compound Interest being allowed the Purchaser, at 5 *per Cent*?

See the Work.

If 5 *l.* : 100 *l.* : : 20 *l.* : *Facit* 400 *l.* Corresp. Princip.

Log. of 1.05 the Rate is	0.0211893
Multiply by	7
Log. of the Rate and Time	0.1483251
Log. of 400 the Corresp. Princip.	2.6020600
The Difference is	2.4537349

Equal to the Logarithm of 284.2725, which subtracted from 400 *l.* leaves 115 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$, the present Worth sought.

EXAMPLE II.

An Annual Rent of 365 *l.* paid Yearly, and to continue 12 Years, is to be sold for present Money; what is it worth at 5 *per Cent.* compound Interest?

See the Work.

If 5 *l.* : 100 *l.* : : 365 : *Facit* 7300 Corresp. Princip.

Log. of 1.05 the Rate is	0.0211893
Multiply by the Time	12
	0423786
	211893
Logarithm of the Rate and Time	2542716
Log. of 7300 the Corresp. Princip.	3.8633228
Difference is	3.6090512

which is the Logarithm of 4064.913, or 4064 *l.* 18 *s.* 3 *d.* which subtracted from 7300 *l.* leaves 3235 *l.* 1 *s.* 9 *d.* the Worth sought.

EXAM-

EXAMPLE III.

But if the aforenamed Annuity were to be paid by Quarterly Payments, (*viz.*) 91 *l.* 5 *s.* *per* Quarter, what would be the present Worth, allowing the same Rate of Interest as before.

Proportional Interest for Quarterly Payments at 5 *per Cent.* is = to 1.0122722 — 1 = 0122722, by which dividing the Quarterly Rent, quotes the correspondent Principal.

.0122722) 91.2500 (7435.5046 Cor. Principal.

Log. of the Rate 1.05, is	0.0211893
Multiply by the time	12
	423786
	211893
Log. of the Rate and Time	.2542716
Log. of 7435.5046, the Cor. Principal	3.8713104
The Difference is	3.6170388

Which is the Logarithm of 4140.4671, which subtracted from 7435.5046, leaves 3295.0375, Equal to 3295 *l.* and 9 *d.* the present Worth; by which you may perceive, that Quarterly Payments in this Annuity, raiseth the Value 59 *l.* 19 *s.*

Notwithstanding, in the Purchasing of Annuities, very few Persons will value a Lease the more for being paid Quarterly.

EXAMPLE IV.

But if the said Annuity were to be paid by daily Payments, *viz.* 20 *s.* *per* Day, what would be the present Worth, keeping the same Rate of Interest still?

Proportional Interest for daily Payments is .0001336, when Unity is subtracted, by which dividing the daily Rent, quotes the Correspondent Principal.

.0001336) 1.0000 (7485.03 Cor. Principal.

Log. of 7485.03, is	3.8741935
The Log. of the Rate and Time, is	0.2542716
Difference is	3.6199219

Which is the Logarithm of 4167.9446, which subtracted from 7485.03, will leave 3317.0854, Equal to 3317 *l.* 1 *s.* 8 *d.* ½. So the Difference of the present Worth of this Annuity, according to Yearly and Daily Payments, is 82 *l.*

EXAMPLE V.

An Annuity of 24 *l. per Ann.* to begin after the End of 6 Years, whereby the first Rent will not be received till after the Expiration of 7 Years, and to continue 21 Years, is to be sold for present Money, what is it worth on this Condition, allowing the Purchaser 6 *per Cent.* Compound Interest?

If this Annuity were to begin presently, the Worth by this Proposition would be found to be 382.3378, or 382 *l.* 6 *s.* 9 *d.*

But seeing it begins not till the End of 6 Years, you must by the second Proposition of the first Section of this Chapter, find what ready Money would pay a Debt of 382.3378, Due 6 Years hence; which will appear to be 269.5328, or 269 *l.* 10 *s.* 8 *d* which is the present Worth of this Annuity according to the Condition aforesaid.

PROPOSIT. II.

Present Worth, Rate, and Time given to find the Annuity.

RULE.

Suppose an Annuity at pleasure, and find the Worth by the last; then the Proportion runs,

As the Worth found : to the supposed Annuity ::
So the Worth given : to the Annuity required.

EXAM-

EXAMPLE.

What Annuity to continue 7 Years will be Purchased for 120 *l.* at 6 *per Cent.* Compound Interest?

Suppose 15 *l.* and by the last Proposition, the present Worth will be found to be 83.7357.

Then say, if 83.7357 : 15 :: 120 : *Facit* 21.4962, or 21 *l.* 9 *s.* 11 *d.* 1 *q.* the *Answer.* And thus of any other.

PROPOSIT. III.

Annuity, present Worth, and Rate of Interest given, to find the Time of Continuance.

RULE.

Find a correspondent Principal, subtract the Debt out of the correspondent Principal, and the Logarithm of their difference out of the Logarithm of the correspondent Principal; this last Difference divided by the Logarithm of the Rate, shews the Time.

EXAMPLE I.

In what time will 20 *l. per Ann.* pay a Debt of 115.7275, or 115 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$, at 5 *per Cent.* Compound Interest?

First, if 5 : 100 :: 20 : *Facit* 400 Cor. Prin.

	l.
From the Cor. Prin.	400
Subtract the Debt	115.7275
The Difference is	284.2725

Log. of the Cor. Prin. 400 *l.* is	2.6020600
Log. of 284.2725, is	2.4537349
	0211893) 1483251 (7
	1483251

Answer, in 7 Years.

EXAM-

EXAMPLE II.

A Borrows of B 1728 *l.* and at the same time delivers up to B an Annuity of the clear Value of 240 *l.* *per Ann.* which he is to enjoy till he be fully satisfy'd for his 1728 *l.* The Question is, How long B must enjoy the Premises, Compound Interest being computed at 6 *per Cent. per Ann.*

First I say, if 6 : 100 :: 240. *Facit* 4000 Cor. Prin.

From the Cor. Principal	4000
Subtract the Debt	1728
Difference is	2272
Log. of 4000, the cor. Principal, is	3.6020600
Log. of the said Difference 2272, is	3.3564083
Difference is	2456517

```
0253059) 2456517 (9.70729
         2277531
         -------
          1789860
          1771413
          -------
            1844700
            1771413
            -------
              732870
              506118
              ------
              2267520
```

Ans. In 9 Years, 9 Months, 0 Weeks, and 5 Days; And so long B must enjoy the Premises.

EXAMPLE III.

A lends to B 600 *l.* and B is willing to pay a Quarterly Rent of 15 *l. per* Quarter' till A be satisfied for his 600 *l.* How many Quarters Rent must B receive, Compound Interest

terest being computed at 5 *per Cent.* And what will the last Payment be?

.0122722) 15.0000000 (1222.2747 Cor. Prin.

From the Cor. Principal	1222.2747
Subtract the Debt	600
Difference is	622.2747

Log. of the Cor. Prin. 1222.2747, is		3.0871688
Log. of the Difference 622.2747, is		2.7939821
Their Difference is		.2931867

$\frac{1}{4}$ Log. Rate }

.0052973) .2931867 (55.3464 = to 55 whole Quarters, and something above $\frac{1}{3}$ of a Quarter. *Answer,* He must receive 55 Quarters Rent; and the last Payment will be 5 *l.* 3 *s.* 11 *d.* 1 *q.*

PROPOSIT. IV.

The Annuity, Present Worth, and Time of Continuance given, to find the Rate of Interest.

This Proposition is best performed by Approximation, for by two or three Tryals (but they must be near the Truth) you will have the Answer bounded betwixt two Numbers; as in the last Proposition of the last Section.

An Annuity of 20 *l. per Ann.* to continue for 7 Years, is sold for 100 *l.* ready Money, what Rate of Compound Interest hath the Purchaser for his Money?

Interest of Money being seldom above 10, or under 5 *per Cent.* I make my first Supposition at 9 *per Cent.* and by the first Proposition of this Section, the present Worth of 20 *l.* *per An.* to continue 7 Years, will be found to be *l.* 100.659056, which should have been 100 *l.* wherefore the Error is .659056. And seeing the Supposition was short, I place it and the Error, as here,

9 — .659056
9.25 + .18

But

But seeing I am pretty near, I make my next Supposition at 9 *l.* 5 *s.* and by the same Proposition, the said Annuity for the same time will be worth 99.82, which should have been 100, by which I see I have supposed too much; and the Error is .18, which Supposition and Error I place under the other, and say,

As .839, the Sum of the Errors to .25, the Difference of Suppositions, so .18 the latter Error to .0536, which Subtract from the latter Supposition, because it was too great, leaves 9.1964, or 9 *l.* 3 *s.* 11 *d.* the Rate sought.

And though this be Mathematically true, and Demonstrable; and that by delivering up of an Annuity of 20 *l.* *per Ann.* to continue for 7 Years, for 100 *l.* paid in Hand, he allows 9 *l.* 3 *s.* 11 *d.* *per Cent. per Ann.* yet he will never be able to make that Interest by his Annuity, unless he can find such a Fool as will take his Annual Payments as they become due, and give him 9 *l.* 3 *s.* 11 *d.* *per Cent.* Compound Interest; which will be hard to do, when any responsible Man may be fitted for 6, nay in most Places for 5 *per Cent.*

Wherefore Mr. *Martindale* was in the Right, according to the Intent and Import of his Proposition; and that he can but make 7 *l.* 13 *s.* 7 *d.* $\frac{1}{3}$, supposing every Payment be taken off his Hands at 6 *per Cent.* Compound Interest; and this will be something difficult to do. And if some of his Annual Rents, or all of them should not be Improved, which is no impossible thing, he will not be able to make 6 *per Cent.* by his Annuity; So that I had rather put forth my Hundred Pound at 6 *per Cent.* Compound Interest for 7 Years, than stand to the Venture of the Improvement of the Annuity. But this by the By.

The two first Propositions being of good Use, we have annexed Tables fitted thereto for 31 Years, at 5 and 6 *per Cent.* Compound Interest.

TABLE

Years.	TABLE I. *Shewing the preſent Worth of one Pound Annuity to continue for 31 Y. at 5 and 6 per Cent. Comp. Int.*		TABLE II. *Shewing what Annuity to continue for 31 Y. one Pound will Purchaſe, at 5 and 6 per Cent. Comp. Intereſt.*	
	5	6.	5	6
1	0.952381	0.943396	1.050000	1.060000
2	1.859410	1.833392	.537805	.543637
3	2.723248	2.673012	.367208	.374110
4	3.545950	3.465105	.282012	.288591
5	4.329477	4.212363	.230952	.237396
6	5.075692	4.917324	.197017	.203 63
7	5.786373	5.582381	.172820	.179135
8	6.463212	6.209792	.154722	.161036
9	7.107821	6.801691	.140690	.147022
10	7.721734	7.360086	.129505	.135868
11	8.306414	7.886873	.120389	.126793
12	8.863251	8.383843	.112825	.119272
13	9.393572	8.852682	.106456	.102960
14	9.898640	9.294983	.101023	.107585
15	10.379658	9.712248	.096342	.102963
16	10.837769	10.105894	.092270	.098952
17	11.274065	10.477258	.088699	.095445
18	11.689586	10.827602	.085546	.092356
19	12.085320	11.158115	.082745	.089621
20	12.462209	11.469920	.080242	.087184
21	12.821152	11.764075	.077996	.085004
22	13.163002	12.041580	.075970	.083045
23	13.488573	12.303377	.074137	.081278
24	13.798641	12.550356	.072741	.079679
25	14. 93944	12.783354	.070952	.078227
26	14.375184	13.003164	.069564	.076904
27	14.643033	13.210531	.068292	.075697
28	14.898127	13.406162	.067122	.074592
29	15.141073	13.500719	.066045	.073579
30	15.372450	13.764829	.065051	.072649
31	15.592810	13.929984	.064132	.071792

 The

The Construction of the foregoing TABLES.

If from the Logarithms of the Numbers in that Table under Section the Second, you Subtract the Logarithms of the Numbers in Table the First, Section the First, the Remainders are the Logarithms of the Numbers in the first Table here.

And their Complements Arithmetical are the Logarithms of the Numbers in the Second Table.

Their USE.

There is no Difference betwixt the Use of these Tables, and those going before, as may be seen in the following Examples.

Examples in the Use of the First TABLE.

I. An Annuity of 20 *l. per Ann.* clear Value, is to be sold for 7 Years, what ready Money is it worth, at 5 *per Cent.* Compound Interest?

Multiply the Tabular Number under 5 *per Cent.* and overagainst 7 Years, *viz.* 5.786373
By 20, the given Annuity 20

115.727460

Gives the *Answer*, *viz.* 115 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$.

EXAMPLE II.

There is a Lease of Lands worth 32 *l. per Ann.* more than the Rent paid to the Lord; of which Land there is yet a Lease in being for 7 Years; and the Lessee is desirous to take a Lease in Reversion for 21 Years, to begin when his old Lease is expir'd. What Sum of Money is to be paid for this Lease, allowing Interest at the Rate of 6 *per Cent. per Annum.*

First, See what this Rent of 32 *l.* is worth for 7 Years, which will be 178 *l.* 12 *s.* 9 *d* *feré.*

Secondly, Add 7 Years to 21 Years, which makes 28 Years; then see what 32 *l.* to continue 28 Years is worth, which will be 428 *l.* 19 *s.* 11 *d.* $\frac{1}{4}$.

Lastly,

Lastly, Subtract the present Worth for 7 Years from the present Worth for 28 Years, the Difference is the Answer to the Question, to wit, 250 *l.* 7 *s.* 2 *d.* $\frac{1}{2}$.

The Work for 7 Years.	*The Work for 28 Years.*
5.582381	13.406162
32	32
11164762	26812324
16747143	40218486
178.636192	428.997184

From 428.997184
Sub. 178.636192

250.360992 The *Answer.*

Examples in the Use of the Second TABLE.

I. What Annuity, to continue 9 Years, will 34 *l.* Purchase, Compound Interest being Computed at 5 *per Cent?*

Tabular Number in Table the Second, under 5 *per Cent.* and overagainst 9 Years, is .140690
Multiply by 34

562760
422070

Answer, 4 *l.* 15 *s.* 8 *d.* *Facit* 4.783460

EXAMPLE II.

What Annuity to continue 21 Years will 365 *l.* purchase, Compound Interest being computed at 6 *per Cent. per Ann.*

Tabular Number in Table the Second, under 6 *per Cent.* and overagainst 21 Years, is .085004
Multiply by 365

425020
510024
255012

Ans. 31 *l.* 0 *s.* 6 *d.*

31.026460

SECTION IV.

Now in the last Place, we shall treat of Compound Interest, as it relates to the Purchasing of Freehold Estates, to be Bought or Sold for Ever.

This by several is called Compound Interest infinite, because it relates to divers equal Payments at divers equal Times; but the number of those equal Times are Infinite: As in Purchasing an Estate in Fee-Simple for Ever.

And this may be consider'd under these three Particulars.

First, The Annuity paid by Yearly or Quarterly Payments.

Secondly, The Price, or present Worth.

Thirdly, The Rate of Interest.

Any two of these being given, to find the third; as in the three Propositions following.

PROP. I.

The Annuity and Rate of Interest given, to find the present Worth.

RULE.

The Annual (half Yearly, or Quarterly) Payment divided by the Rate of Interest, *minus* Unity, quotes the present Worth.

EXAMPLE.

There is an Estate to be sold of the clear Value of 20 *l. per Ann.* what Sum of ready Money is this Estate worth, Compound Interest being allow'd the Purchaser at 6 *per Cent*?

.06) 20.000 (333.333 = 333 *l.* 6 *s.* 8 *d.*

18

20
18

20
18

20
18

2

Ans. 333 *l.* 6 *s.* 8 *d.*

EXAM-

EXAMPLE II.

But if the said Annuity were paid by Quarterly Payments; *viz.* 5 *l.* *per* Quarter; what would be the present Worth, Holding still to the same Rate of Interest?

Quarterly Rate,
minus Unity, is .014674) 5.000000 (340.7387

44022

Ans. 340 *l.* 14 *s.* 9 *d.* 59780
58696

Quarterly Payment raiseth 108400
the Worth 7 *l.* 8 *s.* 1 *d.* 102718

PROP. II.

Present Worth, or Purchase Money, together with the Rate of Interest being given, to find the Annuity.

RULE.

Multiply the Purchase Money by the Rate of Interest, *minus* Unity, the Product shall be the Annual Rent.

EXAMPLE.

A Gentleman hath a Desire to lay out 333 *l.* $\frac{1}{3}$ on a Freehold Estate, provided he meet with a Bargain as shall bring him in 6 *per Cent.* Compound Interest for his Money, What Annual Rent must this be?

333.333
.06

19.99999 *Facit* 20 *l.*

PROP. III.

The Annuity and present Worth given, to find the Rate of Interest.

RULE.

The Annual Rent divided by the present Worth, or Sum demanded, quotes the Rate, *minus* Unity.

EXAM-

EXAMPLE.

There is an Estate to be sold of the Yearly Value of 20 *l.* for 333 *l.* $\frac{1}{3}$, what Rate of Compound Interest will the Purchaser have for his Money.

$$333.333)\ 20.000000\ (.06$$
$$19999999$$
$$0$$

Ans. 6 *per Cent.*

EXAMPLE II.

There is a Freehold Estate to be sold for 1600 *l.* the Yearly Rent being 128 *l.* What Rate of Compound Interest shall the Purchaser have for his Money?

$$1600)\ 128.000\ (.08$$
$$12800$$

Ans. 8 *per Cent.*

Lastly, If it be inquired, how many Years Purchase any Annuity is worth, work thus; Divide Unity by the Rate, *minus* Unity, the Quote exhibits the number of Years.

EXAMPLE.

There is a Freehold Estate to be sold, how many Years Purchase is it worth at 5 *per Cent. per Ann.* Compound Interest.

$$.05\ \ 1.00\ (20$$
$$10$$
$$00$$

Ans. 20 Years Purchase.

What is it worth at 6 *per Cent* ?

$$.06)\ 1.0\ (16.666$$
$$6$$
$$40$$
$$36$$
$$4,\ \&c.$$

Ans. 16 Years and $\frac{2}{3}$.

Likewise, if an Estate be offer'd at any Number of Years Purchase, and the Rate of Interest be demanded, do thus, Divide Unity by the Number of Years propos'd, and the Quote gives the Rate, *minus* Unity.

EXAM-

EXAMPLE.

An Estate is offer'd at 20 Years Purchase, what is the Rate of Interest?

20) 1.00 (.05
1.00 — *Ans.* 5 *per Cent.*

o

Here follow divers Questions of Interest to exercise the Learner, both Simple and Compound; and so we will conclude *Logarithmical Arithmetick.*

QUEST. I.

A doth owe unto *B* 800 *l.* to be paid in 4 Years, that is at the End of every two Years 400 *l.* *B* doth owe unto *A* 900 *l.* to be paid in 6 Years, that is at the End of every two Years 300 *l.* They agree to clear their Debts, and allow each other 8 *per Cent.* Compound Interest. Which must pay Money, and how much?

Ans. B must pay unto *A*, 29 *l.* 16 *s.* 3 *d.* 1 *q.*

QUEST. II.

A doth owe unto *B* 455 *l.* to be paid in 14 Years; that is, at the End of every two Years 65 *l.* He would agree with his Creditor to pay him in 7 Years, *viz.* Each Year one equal Payment, which *B* agrees to; and they conclude Compound Interest shall be allow'd at 6 *per Cent.* What will this equal Payment be? *Ans.* 52 *l.* 10 *s.* 8 *d.*

Found by seeking the present Worth of the 7 Payments, paid each two Years, which will be 293 *l.* 5 *s.* 2 *d.* Then seek what Annuity, to continue 7 Years 293 *l.* 5 *s.* 2 *d.* will Purchase; which will be found to be 52 *l.* 10 *s.* 8 *d.* The *Answer* sought.

QUEST. III.

A Merchant hath owing to him 10000 *l.* to be paid in five Years, *viz.* at the End of every Year 2000 *l.* and agrees with his Debtor, that if he will pay him 5000 *l.* ready Money, he will take the Remainder in 21 Years by an equal Annual Payment, Compound Interest being computed at 6 *per Cent.* to which his Debtor assents. The Question is, what will this equal Annual Payment be?

Ans. 291.11725 *l.*, or 291 *l.* 2 *s.* 4 *d.*

Found

Found by ſeeking the preſent Worth of 2000 *l.* to continue 5 Years, which will be 8424 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$, from which ſubtracting 5000, reſt 3424 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$, then find what Annuity to continue 21 Years 3424 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$ will purchaſe, *viz.* 291 *l.* 2 *s.* 4 *d.* and that is the *Anſwer.*

Q U E S T. IV.

There is an Annuity of 64 *l.* 10 *s.* to continue 120 Years, to be ſold for ready Money; whether is it better to Purchaſe this Annuity, at 6 *per Cent.* Simple Intereſt or at 6 *per Cent.* Compound Intereſt; likewiſe what is the Difference, and laſtly what is its Value taken as a Freehold Eſtate?

The Work according to Simple Intereſt.

120		.120	120
.12		.06	120
240		7.20	240
120			120
14.40	16.4) 70743.6 (4313.6341		14400
2.	656		.06
16.40	514		864.00
	492		240
	223		1104
	164		7.2
	596		1096.8
	492		64.5
	1040		54840
	984		43872
	560		65808
	492		70743.60
Anſ. 4313 *l.* 12 *s.* 8 *d.*	680		
According to Simple	656		
Intereſt.	24		

The

The Work according to Compound Interest.

First, if 6 : 100 :: 645 ? *Facit* 1075 Cor. Principal.

6) 64500 (1075
6
045
42
30
30

Ans. 1074 *l.* 00 *s.* 3 *d.* according to Comp. Interest.

Log. of the Rate	.0253059
Multiply by the Time	120
	506118
	253059
Log. of the Rate and Time	3.0367080
Log. of the Cor. Principal 1075, is	3.0314084
Log. of the Rate and Time, is	3.0367080
Difference is	.9.9947004

Which is the Logarithm of .98788, which subtracted from the Correspondent Principal, leaves the present Worth, *viz.* 1074.01212 *l.*, which is Equal to 1074 *l.* 0 *s.* 3 *d.*

The Work as a Freehold Estate at 6 *per Cent.* Compound Interest.

.06) 6450 (1075
6
045
42
30
30
0

Ans. 1075 *l.* as a Freehold Estate.

	l.	s.	d.
The preſent Worth at 6 *per Cent.* Simp. Int. is	4313	12	8
The preſent Worth at 6 *per Cent.* Comp. In. is	1074	0	3
Difference of the Worths is	3239	12	5

The preſent Worth as a Freehold Eſtate, at 6 *per Cent.* Compound Intereſt, is 1075 *l.*

By which you may ſee it is better to purchaſe it at Compound Intereſt, by 3239 *l.* 12 *s.* 5 *d.* which is a very great Difference, being more than the Eſtate is Worth for Ever.

And though the preſent Worth of this Eſtate to continue 120 Years at 6 *per Cent.* Compound Intereſt, comes ſo near the Worth of the ſame Eſtate to continue for ever, the ſame Rate of Intereſt being computed; yet if this Eſtate were to continue 200 Years, nay double that Time, yet it would not reach 1075 *l.* which ſhews the Agreement of the Rules: For if it were otherwiſe, it would be found better to purchaſe an Annuity for Ever, than for a certain number of Years; which would be a Paradox.

QUEST. V.

A Gentleman pays 350 *l.* for a Leaſe in Reverſion, to Commence at the End of 13 Years and a Quarter, and to continue for 21 Years and 3 Quarters; what Quarterly Rent may he lett the Premiſes for, after he comes to be in Poſſeſſion thereof, ſo as to gain 8 *per Cent.* Compound Intereſt for his Money?

The Log. of 350 *l.*		2.5440680
Worth of 1 *l.* forborn 53 Quarters	=	0.4428653
Log. of the Increaſe of 350 *l.*	=	2.9869333
Worth of 1 *l.* for 87 Quarters	=	1.6214203
The Log. of 23.201		1.3655130

Anſwer, 23 *l.* 4 *s.*

Problems or *Questions*, IN ALGEBRA.

1. TO find a Number, which being multiply'd by 3, subtracting 5, from the Product, and the remainder divided by 2, if the Number sought be added to the Quotient, that the Sum may be 40.

2. To find a Number, which being multiplied by 12, and 48 added to the Product, as much may be produc'd, as if the same Number sought were multiplied by 18.

3. To find a Number, to which if 11 be added and 7 subtracted from the same Number [*viz.* the first] the Sum of the Addition may be double the remainder.

4. To find a Number, to which if its double [treble, quadruple, *&c.*] be added, the Square of the same Number may be produc'd.

5. To find a Number, which if added to it self, and the Sum multiply'd by the same; and the same Number still subtracted from the Product: And lastly, the remainder divided by the same, that it may produce 13.

6. To divide the Number 16 into 2 parts, so that the Square of the greater part may exceed the Square of the less by 32.

7. To divide the Number 36 into two parts, so that if 12 be added to the first, and 6 to the second, the former may be the double of the latter.

8. Let the Line AB (of 70 parts) be divided any how in C [so that AC may be 42, BC 48] it is required to divide the same Line again in another Point: *For Example,* in D, so that the Rectangle ADC may be equal to the Square DB. Let the Segment CD be inquir'd [which being obtain'd, AD, DB will be known.]

9. Let the Line E F be divided any how in G, [so that EG may be 6, GF 4] it is requir'd to produce this right line EF [for Example unto H] so that the Rectangle EHF may be equal to the Square GH; the length of FH is requir'd,

10. A General disposing his Army into a Square Battle, finds he has 284 Soldiers over and above; but increasing each side with one Soldier, he wants 25 Soldiers to fill up the Square: How many Soldiers had he?

11. A certain Captain sends out $\frac{1}{3}$ of his Soldiers + 10, there remain $\frac{1}{2}$ + 15. How many Soldiers had he?

12. There is an Army to which if you add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of it self and take away 5000, the Sum total will be 100000. What was the Number of the Army?

13. In the Rectangle ABCD, the difference of the greater side AB, and of the lesser side BC, is 12, but the difference of the Squares of the sides 1680. What are the sides of the Rectangle ABCD?

14. The length DE of the Rectangle DEFG, is twice the breadth EF; and the Sum of the Squares of the length and breadth is ten times the Sum of the two Sides DE, EF. What are the Sides of the Rectangle DEFG?

15. To find two Numbers in the proportion of 2 to 3, whose Product, if they be multiplied by one another, shall be 54.

16. To find two Numbers whose *Ratio* is to one another as 4 to 5: and the Sum of the Squares of both, is 2624.

17. To

17. To find the side of a Square, whose *Area* is to the Sum of the sides in a Given *Ratio*, as 45 to 12.

18. To find the side of a Cube, whose *Superficies* is to the Solidity in a Given *Ratio*, as 6 to 11.

19. A certain Man hires a Labourer, on this condition, that for every Day he work'd he should receive 12 Pence, but for every Day he was idle, he should be mulct'd 8 Pence. When 390 Days were past, neither of them were indebted to one another. How many Days did he work; and how many was he idle?

20. A certain Gentleman hires a Servant, and promises him 24 Pounds yearly Wages, together with a Cloak. At 8 Months End the Servant obtains leave to go away, and instead of his Wages receives a Cloak + 13 Pounds. How much did the Cloak cost?

21. A Person being ask'd how Old he was, answer'd, If I quadruple $\frac{2}{3}$ of my Years, and add $\frac{1}{2}$ of them + 50 to the Product; the Sum will be so much above 100, as the number of my Years is now below 100.

22. One being ask'd what hour of the Day it was, answered, The Day at this time is 16 hours long; if now $\frac{1}{2}$ of the hours past be added to $\frac{2}{3}$ of the remainder, you will have the hour desir'd reckoning from Sun-rising.

23. From *Norimberg* to *Rome* are 140 Miles: A Traveller sets out at the same time from each of the two Cities, one goes 8 Miles a Day, the other 6. In how many Days from their first setting out will they meet one another, and how many Miles did each of them go?

24. A certain Messenger goes 6 Miles every Day: 8 Days after, another follows him, and he goes 10 Miles a Day. In what number of Days will he come up to the first?

25. A certain Messenger goes 6 Miles a Day: And after he has gone 56 Miles, another follows him who goes 8 Miles a Day. In how many Days will he come up to him?

26. One bought 3 Books, whose Prices were in proportion, as 12, 5, 1: If the Price of the first be doubled, of the second trebled, of the third quadrupled; the Sum of these Products will as much exceed 10 Crowns, as the Sum of the Prices of the greatest and middle is below 5. How much did the said Books cost?

27. Suppose the number 50 were to be divided into two parts, so that the greater part being divided by 7, and the less multiply'd by 3, the Sum of this Product and the former Quotient may make the same Number proposed, which was 50.

28. Let the Number 20 be divided into two parts, so that the Square of the less part, being taken out of the Square of the greater, may leave the very number propos'd, which was 20 [or may leave the double, treble, &c. of the Number propos'd.]

29. If a Man gains 30 Crowns a Week; how much must he spend a Week to have 500 Crowns, together with the Expence of 4 Weeks remaining at the Years end?

30. A Labourer, after 40 Weeks in which he had been at work, lays up 28 Crowns — the pay of three Weeks; and finds that he had expended 36 Crowns + the pay of eleven Weeks. What Pay did he receive a Week?

31. In

31. In the Rectangle ABC, is given the basis AB = 9 and the difference of the other Sides, that is the Segment BD, = 3. Requir'd, the Sides AC, BC?

32. In the Rectangle Triangle ABC, is given the Basis AB = 5, and the Sum of the other Sides AC + BC = 25. Required, the Sides AC, BC severally?

33. Suppose two Towers, AB 180 feet high, and CD 240, at the distance AC 360 feet. A Ladder is to be set upon the line AC, at some point, suppose in E, of such a length, as from thence it may reach the top of both the Towers. We require the point E in the line of distance, as also the length of the Ladder E B, E D?

34. In the Triangle ABC, the several sides AB = 13, AC = 14, BC = 15 are given; and the perpendicular BD being drawn. Requir'd the segments of the Basis AD, DC?

35. In the obtusangle Triangle DEF, the several sides are given, *viz.* DE 11, EF 13, DF 20; and the perpendicular FG, being let fall upon the Basis produc'd. Requir'd the Prolongation of the Basis EG.

36. In the Rectangle ABCD, is given the difference between the Length AB and the Diagonal BD, that is DE = 2; and likewise the difference between the Breadth AD and the Diagonal BD, that is, FB = 9. Requir'd the sides of the Rectangle AB, AD?

37. In a Rectangle DEFG, the right line DK is drawn from the Angle D to the opposite side, cutting the Diagonal EG at right Angles in H: And there is given the segment HK = 2, and HE = 16. Requir'd the sides of the Rectangle?

38. Let

38. Let there be a Circle, whose Diameter is AB, which another less Circle whose is AC, Diameter touches within in A: and from the Center of the greater Circle D, draw the radius DE at right Angles to AB, cutting the Periphery of the lesser Circle in F. Now there is given BC [the difference of the Diameters] = 9, with the Segment EF = 5. Requir'd the Diameters AB, AC of the said Circles?

39. Two Companions have got a parcel of Guinea's; says A to B, if you will give me one of your Guinea's, I shall have as many as you will have left. Nay, replies B, if you will give me one of your Guinea's, I shall have twice as many as you will have left. How many Guinea's had each of them?

40. A certain Person bought two Horses, with the Trappings, which cost 100 Pounds; which Trappings if laid on the first Horse A, both the Horses will be of equal value: But if the said Trappings be laid on the other Horse, he will be double the value of the first. How much did the said Horses cost?

41. A Vinter has two sorts of Wine, *viz.* A and B: which if mix'd in equal Parts, a Flaggon of mix'd will cost 15 Pence; but if they be mix'd in a *sesqui-alter* proportion, as if you should take 2 Flaggons of A as often as you take 3 of B, a Flaggon will cost 14 Pence. Requir'd the Price of each Wine singly?

42. A Son ask'd his Father how Old he was; his Father answered him thus, If you take away 5 from my Years, and divide the remainder by 8, the Quotient will be $\frac{1}{3}$ of your Age: but if you add 2 to your Age, and multiply the whole by 3, and then substract 7 from the Product, you will have the number of the Years of my Age. What was the Age of the Father and the Son?

43. To

43. To find out two Numbers, to the Sum whereof if you add 6, the whole shall be double the greater; and if you subtract 2 from their difference, the remainder will be half of the least.

44. To find two Numbers, the Product whereof is 240, and the triple of the greater divided by the less is 5.

45. Two Men have a mind to purchase a House rated at 1200 Pounds; says *A* to *B*, if you give me $\frac{2}{3}$ of your Money, I can purchase the House alone; but says *B* to *A* if you will give me $\frac{3}{4}$ of yours, I shall be able to purchase the House. How much Money had each of them?

46. Some young Men and Maids had a reckoning of 37 Crowns to pay for a Treat, and this was their Conditions, that every young Man should pay 3 Crowns, and every Maid 2. Now, if there had been as many young Men as there were Maids, observing the same conditions, the reckoning would have come to 4 Crowns less than it did. How many young Men and Maids were there?

47. A General, who had fought a Battle, upon reviewing his Army, whose Foot was thrice the number of his Horse, finds that before the Battle $\frac{1}{12}-120$ of his Foot had deserted, and of his Horse $\frac{1}{20}+120$, besides $\frac{1}{4}$ of his whole Army were sent into Garrisons (reckoning the Sick and Wounded), and $\frac{3}{8}$ of his Army remain'd; the rest, who were wanting, being either slain or taken Prisoners; now, if you add 3000 to the number of the slain, the Sum will be equal to half the Foot he had at the beginning. What were the numbers of each?

48. To divide 100 twice into two parts, so that the *major* part of the first division may be treble the *minor* part of the second division; and the *major* part of the second may be double the *minor* part of the first.

49. To divide 30 twice into two parts, ſo that the *major* part of the firſt diviſion with the *minor* of the ſecond may be 33; and the Sum of the *minor* parts ſubtracted from the Sum of the *major*, may leave 14 remaining.

50. A Man, his Wife, and his Son's Ages make up 96 Years, ſo that the Husband's and Son's Years together make the Wife's + 15; but the Wife's and the Son's make the Husband's + 2. What was the Age of each?

51. Three Merchants from three different Fairs, meet together at an Inn, where they reckon up their Gains, and find them the Sum of 780 Crowns. Moreover, if you add the Gain of the firſt and ſecond, and ſubtract the Gain of the third from the Sum, there remains the Gain of the firſt + 82 Crowns; but if you add the Gain of the ſecond and third, and from the Sum ſubtract the Gain of the firſt, there remains the Gain of the third — 43 Crowns. What was the Gain of each?

52. Three Perſons, *A*, *B*, *C*, owe a certain Sum of Money, ſo that *A* and *B* together owe 210 Crowns; *B* and *C* 290, and *C* and *A* 400. What did each of them owe?

53. To find three Numbers, ſo that the firſt and half of the remainder, the ſecond and $\frac{1}{3}$ of the remainder, and the third and $\frac{1}{4}$ of the remainder, may always make 34.

54. Let a Square be divided into 9 ſmall Squares: We are to find and diſpoſe the Numbers through the ſeveral ſmall *Areas*, ſo that the Sum of every three, taken either laterally or diagonally, may be always 15.

55. [Theorem]

55. [Theorem] Let any Numbers whatſoever be given, if you ſubtract every leſs Number from that which is the next greateſt: I ſay that the Sum of theſe differences is equal to the difference of the greateſt and leaſt Numbers.

56. To find a Number, which being multiply'd by 6, and the product ſubtracted from the Square of the Number to be found, the remainder will be 280.

57. To find a Number which being multiply'd by 8, and the Product added to the Square of the Number to be found, the Sum will be 660.

58. To divide 140 into two parts, ſo that the Product of thoſe parts may = the Square of 56, that is 3136.

59. Let 969 Soldiers be drawn up into an oblong Battle, ſo that the difference of the greater and leſs ſides is 40. Requir'd the Number of the Soldiers of each rank in length and breadth?

60. Again, let 480 Soldiers be drawn up into an oblong Battle, ſo that the Sum of the greater and leſs ſides is 52. Requir'd the Number of the Soldiers of each rank in length and breadth?

61. In the Square ABCD is given the difference of the Diagonal and the ſide, that is EC = 6. Requir'd the ſide of the Square?

62. The Rectangle EK is added to the Square DF, [being of the same heighth ;] whose breadth EL is given = 2, and also the *Area* of the whole compound rectangle DK, = 60. Requir'd the side of the Square?

63. A Man buys some Ells of Cloth for 70 Crowns; and finds that if he had 4 Ells more, he had then bought every Ell, 2 Crowns cheaper. How many Ells did he buy?

64. A set of boon Companions dining at an Inn, the Reckoning in all came to 175 Shillings : But, before the Bill was paid off, two of them slunk away, and then the Club of those that remain'd came to 10 Shillings a Man more. How many were there in Company?

65. To divide the Number 21 into two parts, so that if the greater be divided by the less; and again the less by the greater; and then the first Quotient being multiply'd by 4, and the latter by 25, the Numbers produc'd may be equal.

66. Let the line AB be divided in C, so that AC may be 8, and CD 6: We are to divide the same line AB in D, so that the Rectangle under AD and DC may be equal to the Rectangle under AC and CB, or to the Product from 8 and 6, which is 48. Requir'd the Segment CD?

67. Let there be a Rectangular Garden ABCD, the length of which AB is thrice the breadth AD: and reckoning 18 Perches from B towards A, that is BE, and drawing EF parallel to AD; let the *Area* of the remaining Rectangle ED be given = 120 square Perches. What was the length and breadth of the said Garden?

68. Let 600 Soldiers be dispos'd into an oblong Battle: Which the Colonel willing to make broader, finds that if he takes away 10 ranks from the length, he shall augment the breadth with two ranks. What was the Number of his Soldiers through every rank in length and breadth?

69. A Man buys a Horse, which he sells again for 56 Crowns, and gains as many Crowns in 100 as the Horse cost him. How much did he give for the Horse?

70. A certain Linnen Draper buys two sorts of Linnen for 30 Crowns, one finer, the other courser: An Ell of the finest cost as many Crowns as he had Ells: And also 28 Ells of the coursest at such a Price, that 8 Ells cost as many Crowns as one Ell of the finest. How many Ells of the finest Linnen did he buy and what Price did he give for them both?

71. In a certain rectangular Garden, the length of which AD is 22 Perches, and the breadth AD is 10, the walk DG is to be made, in a situation parallel to the sides of the Figure, so that the *Area* of the said Walk or Gnomon DG may be equal to the remaining Rectangle FC, or that the Gnomon DG may be half of the whole Figure ABCD propos'd. Requir'd the breadth of the said Gnomon DE, BG?

72. Of

72. Of three proportional Numbers there is the middle Term given = 12, and the difference of the Extremes = 10. Requir'd the Extremes?

73. Of three proportional Numbers there is given the Sum of the first and second = 10, and the difference of the second and third = 24. Requir'd the several Numbers?

74. Of four proportional Numbers there is given the third = 12, also the Sum of the first and second = 8; besides the second Number being subtracted from its Square, the remainder is to be the fourth. Requir'd the said Numbers?

75. Of four Numbers in continu'd proportion there is given the Sum of the means = 24, and likewise the Sum of the Extremes = 56. Requir'd the said Numbers [supposing that the first is the least of all?]

76. Two Country-women A and B, carry 100 Eggs, together to Market, and in the sale of them, one took as much Mony as the other: but A [who had the largest, and consequently the best Eggs] says to B, had I carry'd as many Eggs as you, I should have had 18 Pence for them; B replies, if I had brought as many Eggs as you, I should have had but 8 Pence for them. How many Eggs had each?

77. Two Country-men A and B sell their Corn at different Prices: A sells 20 Bushels; and B receiv'd for one Bushel as many Crowns as he sold Bushels: A perceives that if he had sold as many Bushels as B receiv'd Crowns, he should then have receiv'd 252 Crowns; but both together receiv'd 176 Crowns. How many Bushels did B sell and what Price had A?

78. Two

78. Two Merchants sell 21 Ells of Cloth: The first sells 1 Ell for as many Crowns, as is $\frac{1}{5}$ of the number of Ells that the second had; and the second sells 1 Ell for as many Crowns, as is $\frac{1}{3}$ of the number of the Ells that the first had. The Sale being over, they had taken 48 Crowns in all. How many Ells did each sell, and at what Price?

79. Two Merchants have a parcel of Silk; the first 40 Ells, the second 90: The first sells for a Crown $\frac{1}{3}$ of an Ell more than the second: When the Sale was over, they had taken between them 42 Crowns. How many Ells did each of them sell for a Crown?

80. To find a Number, to the quadruple of which if you add 91, the whole shall be to the Square of the Number sought, as 3 to 4.

81. To find a Number, from the double of which if you subtract 12, the Square of the remainder less 1, will be nine times the Number sought.

82. To divide the Number 19 into two parts, so that the Sum of the Squares of the parts will be 193.

83. To divide 7 into two parts, so that the difference of the Squares, which are made from the treble of the less part, and the double of the greater, may be 17.

84. A Man buys a Piece of Linnen, and by selling it again, he gains 12 Crowns — $\frac{1}{10}$ of what he bought it for: And finds by this means that he had gain'd as much for 100 Crowns, as the Linnen cost him. What price was the Linnen bought and sold at?

85. A

85. A Man buys 18 Ells of Cloth of different sorts and colour, suppose red and black; what he bought of each cost 40 Crowns: And he pays for every Ell of red Cloth 1 Crown more than for the black. How many Ells of each sort did he buy?

86. A Man buys 120 Pounds of Pepper, and as many of Ginger: and receiv'd for a Crown one Pound of Ginger more than of Pepper. So that the whole Price of the Pepper came to 6 Crowns more than the Price of Ginger. How many Pounds of each did he buy for a Crown?

87. A Man buys 80 Pounds of Pepper and 36 Pounds of Saffron, so that for 8 Crowns he had 14 Pounds of Pepper more than he had of Saffron for 26 Crowns, and what he laid out amounted to 188 Crowns. How many Pound of Pepper had he for 8 Crowns, and how many of Saffron for 26?

87. A and B between them owe 174 Pounds. A pays 8 Pounds a Day, and B pays the first Day 1 Pound, the second 2, the third 3, and so on. In how many Days will they clear the Debt, and how much did each of them owe?

89. A certain Man intends to Travel as many Days as he has Crowns: It happens that every following Day of his Journey he had as many Crowns as he had the Day before, besides two Crowns over and above; and when he came to his Journey's end he finds he had in all 45 Crowns. How many Crowns had he at first?

90. A certain Traveller goes 9 Miles a Day, three Days after another follows him, who the first Day Travels 4 Miles, the second 5, the third 6, and so on, gaining a Mile every Day. In what time will he overtake the former?

91. Two

91. Two Travellers set out at the same time from two Cities, the one from A, and the other from B, which are 70 Miles distant one from another; one of them goes 6 Miles every Day; and the other 2 Miles the first Day, $2\frac{1}{2}$ the second, three the third, and so on, adding $\frac{1}{2}$ a Mile to every Day's Journey: In what time will they meet with one another?

92. Again, Two Travellers set out at the same time from two Cities, the one from A, the other from B, which are 120 Miles distant from one another; the first goes 5 Miles a Day, and the other 3 Miles less than the Number of Days in which they meet. When will they meet?

93. A Post sets out from A towards B, who Travels 8 Miles a Day: after he had gone 27 Miles, another sets out from B to meet him, who goes every Day $\frac{1}{20}$ of the whole Journey or distance of the Places A and B, and meets the first Post after so many Days as is $\frac{1}{20}$ of the said distance. Requir'd the distance of A and B.

94. Two Merchants A and B go Partners, B brings 420 Crowns, and A receives out of the gains 52 Crowns, and the Sum of both their Shares is 854 Crowns. How much did A bring, and how much did B receive out of the Gains?

95. A Son asks his Father how Old he was? his Father replies thus; If you take 4 from my Age, the Remainder will be thrice the Number of your Years: But if you take 1 from your Age, Half the Remainder will be the Square Root of my Age. Requir'd the Age of the Father and Son?

96. To find two Numbers, the Sum of whose Squares may be 317, and the Product, if they be multiply'd by one another, 154.

97. To find two Numbers, the Product of which may be 108, and the Difference of the Squares 63.

98. Two Farmers sell two sorts of Corn: A sells 6 Bushels; B receives in all for his 20 Crowns: Now, says B to A, if we add the number of my Bushels to the number of your Crowns, the Sum will be 28: Says A to B, and if I add the Square of my Crowns to the Square of your Bushels, the Sum will be 424. How many Bushels did B sell, and how many Crowns did A receive?

99. To find two Numbers, the first of which + 2 multiply'd into the second — 3, may produce 110: and on the contrary the first — 3, multiply'd by the second + 2, may produce 80.

A TABLE

A TABLE OF LOGARITHMS FOR NATURAL NUMBERS,

From 1, to 10.000.

A Table of Logarithms.

N.	Logarith.	N.	Logarith.	N.	Logarith.
1	0.0000000	34	1.5314789	67	1.8260748
2	0.3010300	35	1.5440680	68	1.8325089
3	0.4771212	36	1.5563025	69	1.8388491
4	0.6020600	37	1.5682017	70	1.8450980
5	0.6989700	38	1.5797836	71	1.8512583
6	0.7781512	39	1.5910646	72	1.8573325
7	0.8450980	40	1.6020600	73	1.8633229
8	0.9030900	41	1.6127839	74	1.8692317
9	0.9542425	42	1.6232493	75	1.8750613
10	1.0000000	43	1.6334685	76	1.8808136
11	1.0413927	44	1.6434527	77	1.8864907
12	1.0791812	45	1.6532125	78	1.8920946
13	1.1139433	46	1.6627578	79	1.8976271
14	1.1461280	47	1.6720979	80	1.9020900
15	1.1760913	48	1.6812412	81	1.9084850
16	1.2041200	49	1.6901961	82	1.9138138
17	1.2304489	50	1.6989700	83	1.9190781
18	1.2552725	51	1.7075702	84	1.9242793
19	1.2787536	52	1.7160033	85	1.9294189
20	1.3010300	53	1.7242759	86	1.9344984
21	1.3222193	54	1.7323938	87	1.9395192
22	1.3424227	55	1.7403627	88	1.9444827
23	1.3617278	56	1.7481880	89	1.9493900
24	1.3802112	57	1.7558748	90	1.9542425
25	1.3979400	58	1.7634280	91	1.9590414
26	1.4149733	59	1.7708520	92	1.9637878
27	1.4313638	60	1.7781512	93	1.9684829
28	1.4471580	61	1.7853298	94	1.9731278
29	1.4623980	62	1.7923917	95	1.9777236
30	1.4771212	63	1.7993405	96	1.9822712
31	1.4913617	64	1.8061800	97	1.9867717
32	1.5051500	65	1.8129133	98	1.9912261
33	1.5185139	66	1.8195439	99	1.9956352
34	1.5314789	67	1.8260748	100	2.0000000

N.	Logarith.	N.	Logarith.	N.	Logarith.
101	2.0043214	134	2.1271048	167	2.2227165
102	2.0086002	135	2.1303338	168	2.2253093
103	2.0128372	136	2.1335389	169	2.2278867
104	2.0170333	137	2.1367206	170	2.2304489
105	2.0211893	138	2.1398791	171	2.2329961
106	2.0253059	139	2.1430148	172	2.2355284
107	2.0293838	140	2.1461280	173	2.2380461
108	2.0334238	141	2.1492191	174	2.2405492
109	2.0374265	142	2.1522883	175	2.2430380
110	2.0413927	143	2.1553360	176	2.2455127
111	2.0453230	144	2.1583625	177	2.2479733
112	2.0492180	145	2.1613680	178	2.2504200
113	2.0530784	146	2.1643528	179	2.2528530
114	2.0569048	147	2.1673173	180	2.2552725
115	2.0606978	148	2.1702617	181	2.2576786
116	2.0644580	149	2.1731863	182	2.2600714
117	2.0681859	150	2.1760913	183	2.2624511
118	2.0718820	151	2.1789769	184	2.2648178
119	2.0755470	152	2.1818436	185	2.2671717
120	2.0791812	153	2.1846914	186	2.2695129
121	2.0827854	154	2.1875207	187	2.2718416
122	2.0863598	155	2.1903317	188	2.2741578
123	2.0899051	156	2.1931246	189	2.2764618
124	2.0934217	157	2.1958996	190	2.2787536
125	2.0969100	158	2.1986571	191	2.2810334
126	2.1003705	159	2.2013971	192	2.2833012
127	2.1038037	160	2.2041200	193	2.2855573
128	2.1072100	161	2.2068259	194	2.2878017
129	2.1105897	162	2.2095150	195	2.2900346
130	2.1139433	163	2.2121876	196	2.2922561
131	2.1172713	164	2.2148438	197	2.2944662
132	2.1205739	165	2.2174839	198	2.2966652
133	2.1238516	166	2.2201081	199	2.2988531
134	2.1271048	167	2.2227165	200	2.3010300

N.	Logarith.	N.	Logarith.	N.	Logarith.
201	2.3031961	234	2.3692159	267	2.4265113
202	2.3053514	235	2.3710679	268	2.4281348
203	2.3074960	236	2.3729120	269	2.4297523
204	2.3096302	237	2.3747483	270	2.4313638
205	2.3117539	238	2.3765770	271	2.4329693
206	2.3138672	239	2.3783979	272	2.4345609
207	2.3159703	240	2.3802112	273	2.4361626
208	2.3180633	241	2.3820170	274	2.4377506
209	2.3201463	242	2.3838154	275	2.4393327
210	2.3222193	243	2.3856063	276	2.4409091
211	2.3242824	244	2.3873898	277	2.4424798
212	2.3263359	245	2.3891661	278	2.4440448
213	2.3283796	246	2.3909351	279	2.4456042
214	2.3304138	247	2.3926969	280	2.4471580
215	2.3324385	248	2.3944517	281	2.4487063
216	2.3344537	249	2.3961993	282	2.4502491
217	2.3364597	250	2.3979400	283	2.4517864
218	2.3384565	251	2.3996737	284	2.4533183
219	2.3404441	252	2.4014005	285	2.4548449
220	2.3424227	253	2.4031205	286	2.4563660
221	2.3443923	254	2.4048337	287	2.4578819
222	2.3463530	255	2.4065402	288	2.4593925
223	2.3483049	256	2.4082400	289	2.4608978
224	2.3502480	257	2.4099331	290	2.4623980
225	2.3521825	258	2.4116197	291	2.4638930
226	2.3541084	259	2.4132998	292	2.4653828
227	2.3560259	260	2.4149733	293	2.4668676
228	2.3579348	261	2.4166405	294	2.4683473
229	2.3598351	262	2.4183013	295	2.4698220
230	2.3617278	263	2.4199557	296	2.4712917
231	2.3636120	264	2.4216039	297	2.4727564
232	2.3654880	265	2.4232459	298	2.4742163
233	2.3673559	266	2.4248816	299	2.4756712
234	2.3692159	267	2.4265113	300	2.4771212

N.	Logarith.	N.	Logarith.	N.	Logarith.
301	2.4785665	334	2.5237465	367	2.5646661
302	2.4800069	335	2.5250448	368	2.5658478
303	2.4814426	336	2.5263393	369	2.5670264
304	2.4828736	337	2.5276299	370	2.5682017
305	2.4842998	338	2.5289167	371	2.5693739
306	2.4857214	339	2.5301997	372	2.5705429
307	2.4871384	340	2.5314789	373	2.5717088
308	2.4885507	341	2.5327544	374	2.5728716
309	2.4899585	342	2.5340261	375	2.5740313
310	2.4913617	343	2.5352941	376	2.5751878
311	2.4927604	344	2.5365584	377	2.5763413
312	2.4941546	345	2.5378191	378	2.5774918
313	2.4955443	346	2.5390761	379	2.5786392
314	2.4969296	347	2.5403295	380	2.5797836
315	2.4983105	348	2.5415792	381	2.5809250
316	2.4996871	349	2.5428254	382	2.5820634
317	2.5010593	350	2.5440680	383	2.5831988
318	2.5024271	351	2.5453071	384	2.5843312
319	2.5037907	352	2.5465427	385	2.5854607
320	2.5051500	353	2.5477747	386	2.5865873
321	2.5065050	354	2.5490033	387	2.5877110
322	2.5078559	355	2.5502283	388	2.5888317
323	2.5092025	356	2.5514500	389	2.5899496
324	2.5105450	357	2.5526682	390	2.5910646
325	2.5118834	358	2.5538830	391	2.5921768
326	2.5132176	359	2.5550944	392	2.5932861
327	2.5145477	360	2.5563025	393	2.5943925
328	2.5158738	361	2.5575072	394	2.5954962
329	2.5171959	362	2.5587086	395	2.5965971
330	2.5185139	363	2.5599066	396	2.5976952
331	2.5198280	364	2.5611014	397	2.5987905
332	2.5211381	365	2.5622929	398	2.5998831
333	2.5224442	366	2.5634811	399	2.6009729
334	2.5237465	367	2.5646661	400	2.6020600

N.	Logarith.	N.	Logarith.	N.	Logarith.
401	2.6031444	434	2.6374897	467	2.6693169
402	2.6042260	435	2.6384893	468	2.6702458
403	2.6053050	436	2.6394865	469	2.6711728
404	2.6063814	437	2.6404814	470	2.6720979
405	2.6074550	438	2.6414741	471	2.6730209
406	2.6085260	439	2.6424645	472	2.6739420
407	2.6095944	440	2.6434527	473	2.6748611
408	2.6106602	441	2.6444386	474	2.6757783
409	2.6117233	442	2.6454223	475	2.6766936
410	2.6127839	443	2.6464037	476	2.6776069
411	2.6138418	444	2.6473830	477	2.6785184
412	2.6148972	445	2.6483600	478	2.6794279
413	2.6159500	446	2.6493349	479	2.6803355
414	2.6170003	447	2.6503075	480	2.6812412
415	2.6180481	448	2.6512780	481	2.6821451
416	2.6190933	449	2.6522463	482	2.6830470
417	2.6201360	450	2.6532125	483	2.6839471
418	2.6211763	451	2.6541765	484	2.6848454
419	2.6222140	452	2.6551384	485	2.6857417
420	2.6232493	453	2.6560982	486	2.6866363
421	2.6242821	454	2.6570558	487	2.6875290
422	2.6253124	455	2.6580114	488	2.6884198
423	2.6263404	456	2.6589648	489	2.6893089
424	2.6273659	457	2.6599162	490	2.6901961
425	2.6283889	458	2.6608655	491	2.6910815
426	2.6294096	459	2.6618127	492	2.6919651
427	2.6304279	460	2.6627578	493	2.6928469
428	2.6314438	461	2.6637009	494	2.6937269
429	2.6324573	462	2.6646420	495	2.6946052
430	2.6334685	463	2.6655810	496	2.6954817
431	2.6344773	464	2.6665180	497	2.6963564
432	2.6354837	465	2.6674529	498	2.6972293
433	2.6364879	466	2.6683859	499	2.6981005
434	2.6374897	467	2.6693169	500	2.6989700

N.	Logarith.	N.	Logarith:	N.	Logarith.
501	2.6998377	534	2.7275413	567	2.7535831
502	2.7007037	535	2.7283538	568	2.7543483
503	2.7015680	536	2.7291648	569	2.7551123
504	2.7024305	537	2.7299743	570	2.7558749
505	2.7032914	538	2.7307823	571	2.7566361
506	2.7041505	539	2.7315888	572	2.7573960
507	2.7050080	540	2.7323938	573	2.7581546
508	2.7058637	541	2.7331973	574	2.7589119
509	2.7067178	542	2.7339993	575	2.7596678
510	2.7075702	543	2.7347998	576	2.7604225
511	2.7084209	544	2.7355989	577	2.7611758
512	2.7092700	545	2.7363965	578	2.7619278
513	2.7101174	546	2.7371926	579	2.7626786
514	2.7109631	547	2.7379873	580	2.7634280
515	2.7118072	548	2.7387806	581	2.7641761
516	2.7126497	549	2.7395723	582	2.7649230
517	2.7134905	550	2.7403627	583	2.7656686
518	2.7143298	551	2.7411516	584	2.7664128
519	2.7151674	852	2.7419391	585	2.7671559
520	2.7160033	553	2.7427251	586	2.7678976
521	2.7168377	554	2.7435098	587	2.7686381
522	2.7176705	555	2.7442930	588	2.7693773
523	2.7185017	556	2.7450748	589	2.7701153
524	2.7193313	557	2.7458552	590	2.7708520
525	2.7201593	558	2.7466342	591	2.7715875
526	2.7209857	559	2.7474118	592	2.7723217
527	2.7218106	560	2.7481880	593	2.7730547
528	2.7226339	561	2.7489629	594	2.7737864
529	2.7234557	562	2.7497363	595	2.7745170
530	2.7242759	563	2.7505084	596	2.7752463
531	2.7250945	564	2.7512791	597	2.7759743
532	2.7259116	565	2.7520484	598	2.7767012
533	2.7267272	566	2.7528164	599	2.7774268
534	2.7275413	567	2.7535831	600	2.7781512

N.	Logarith.	N.	Logarith.	N.	Logarith.
601	2.7788745	634	2.8020893	667	2.8241258
602	2.7795965	635	2.8027737	668	2.8247765
603	2.7803173	636	2.8034571	669	2.8254261
604	2.7810369	637	2.8041394	670	2.8260748
605	2.7817554	638	2.8048207	671	2.8267225
606	2.7824726	639	2.8055009	672	2.8273693
607	2.7831887	640	2.8061800	673	2.8280151
608	2.7839036	641	2.8068580	674	2.8286599
609	2.7846173	642	2.8075350	675	2.8293038
610	2.7853298	643	2.8082110	676	2.8299467
611	2.7860412	644	2.8088859	677	2.8305887
612	2.7867514	645	2.8095597	678	2.8312297
613	2.7874605	646	2.8102325	679	2.8318698
614	2.7881684	647	2.8109043	680	2.8325089
615	2.7888751	648	2.8115750	681	2.8331471
616	2.7895807	649	2.8122447	682	2.8337844
617	2.7902852	650	2.8129134	683	2.8344207
618	2.7909885	651	2.8135810	684	2.8350561
619	2.7916906	652	2.8142476	685	2.8356906
620	2.7923917	653	2.8149132	686	2.8363241
621	2.7930916	654	2.8155777	687	2.8369567
622	2.7937904	655	2.8162413	688	2.8375884
623	2.7944880	656	2.8169038	689	2.8382192
624	2.7951846	657	2.8175654	690	2.8388491
625	2.7958800	658	2.8182259	691	2.8394780
626	2.7965744	659	2.8188854	692	2.8401063
627	2.7972675	660	2.8195439	693	2.8407332
628	2.7979596	661	2.8202015	694	2.8413595
629	2.7986506	662	2.8208580	695	2.8419848
630	2.7993405	663	2.8215135	696	2.8426092
631	2.8000294	664	2.8221681	697	2.8432328
632	2.8007171	665	2.8228216	698	2.8438554
633	2.8014037	666	2.8234742	699	2.8444772
634	2.8020893	667	2.8241258	700	2.8450980

N	Logarith.	N.	Logarith.	N.	Logarith.
701	2.8457180	734	2.8656961	767	2.8847954
702	2.8463371	735	2.8662873	768	2.8853612
703	2.8469553	736	2.8668778	769	2.8859263
704	2.8475727	737	2.8674675	770	2.8864907
705	2.8481891	738	2.8680564	771	2.8870544
706	2.8488047	739	2.8686444	772	2.8876173
707	2.8494194	740	2.8692317	773	2.8881795
708	2.8500333	741	2.8698182	774	2.8887410
709	2.8506462	742	2.8704039	775	2.8893017
710	2.8512583	743	2.8709888	776	2.8898617
711	2.8518696	744	2.8715729	777	2.8904210
712	2.8524800	745	2.8721563	778	2.8909796
713	2.8530895	746	2.8727388	779	2.8915375
714	2.8536982	747	2.8733206	780	2.8920946
715	2.8543060	748	2.8739016	781	2.8926510
716	2.8549130	749	2.8744818	782	2.8932067
717	2.8555191	750	2.8750613	783	2.8937618
718	2.8561244	751	2.8756399	784	2.8943161
719	2.8567289	752	2.8762178	785	2.8948696
720	2.8573325	753	2.8767950	786	2.8954225
721	2.8579353	754	2.8773713	787	2.8959747
722	2.8585372	755	2.8779469	788	2.8965262
723	2.8591383	756	2.8785218	799	2.8970770
724	2.8597386	757	2.8790959	790	2.8976271
725	2.8603380	758	2.8796692	791	2.8981765
726	2.8609366	759	2.8802418	792	2.8987252
727	2.8615344	760	2.8808136	793	2.8992732
728	2.8621314	761	2.8813847	794	2.8998205
729	2.8627275	762	2.8819550	795	2.9003671
730	2.8633229	763	2.8825245	796	2.9009131
731	2.8639174	764	2.8830934	797	2.9014583
732	2.8645111	765	2.8836614	798	2.9020029
733	2.8651040	766	2.8842288	799	2.9025468
734	2.8656961	767	2.8847954	800	2.9030900

N.	Logarith.	N.	Logarith.	N.	Logarith.
801	2.9036325	834	2.9211660	867	2.9380191
802	2.9041744	835	2.9216865	868	2.9385197
803	2.9047155	836	2.9222063	869	2.9390198
804	2.9052560	837	2.9227254	870	2.9395192
805	2.9057959	838	2.9232440	871	2.9400181
806	2.9063350	839	2.9237620	872	2.9405165
807	2.9068735	840	2.9242793	873	2.9410142
808	2.9074114	841	2.9247960	874	2.9415114
809	2.9079485	842	2.9253121	875	2.9420080
810	2.9084850	843	2.9258276	876	2.9425041
811	2.9090208	844	2.9263424	877	2.9429996
812	2.9095560	845	2.9268567	878	2.9434945
813	2.9100905	846	2.9273704	879	2.9439889
814	2.9106244	847	2.9278834	880	2.9444827
815	2.9111576	848	2.9283958	881	2.9449759
816	2.9116901	849	2.9289077	882	2.9454686
817	2.9122220	850	2.9294189	883	2.9459607
818	2.9127533	851	2.9299296	884	2.9464523
819	2.9132839	852	2.9304396	885	2.9469433
820	2.9138138	853	2.9309490	886	2.9474337
821	2.9143431	854	2.9314579	887	2.9479236
822	2.9148718	855	2.9319661	888	2.9484130
823	2.9153998	856	2.9324738	889	2.9489018
824	2.9159272	857	2.9329808	890	2.9493900
825	2.9164539	858	2.9334873	891	2.9498777
826	2.9169800	859	2.9339932	892	2.9503648
827	2.9175055	860	2.9344984	893	2.9508514
828	2.9180303	861	2.9350031	894	2.9513375
829	2.9185545	862	2.9355073	895	2.9518230
830	2.9190781	863	2.9360108	896	2.9523080
831	2.9196010	864	2.9365137	897	2.9527924
832	2.9201233	865	2.9370161	898	2.9532763
833	2.9206450	866	2.9375179	899	2.9537597
834	2.9211660	867	2.9380191	900	2.9542425

N.	Logarith.	N.	Logarith.	N.	Logarith.
901	2.9547248	934	2.9703469	967	2.9854265
902	2.9552065	935	2.9708116	968	2.9858753
903	2.9556877	936	2.9712758	969	2.9863238
904	2.9561684	937	2.9717396	970	2.9867717
905	2.9566486	938	2.9722028	971	2.9872192
906	2.9571282	939	2.9726656	972	2.9876663
907	2.9576073	940	2.9731278	973	2.9881128
908	2.9580858	941	2.9735896	974	2.9885589
909	2.9585639	942	2.9740509	975	2.9890046
910	2.9590414	943	2.9745117	976	2.9894498
911	2.9595184	944	2.9749720	977	2.9898946
912	2.9599948	945	2.9754318	988	2.9903388
913	2.9604708	946	2.9758911	989	2.9907827
914	2.9609462	947	2.9763500	980	2.9912261
915	2.9614211	948	2.9768083	981	2.9916690
916	2.9618955	949	2.9772662	982	2.9921115
917	2.9623693	950	2.9777236	983	2.9925535
918	2.9628427	951	2.9781805	984	2.9929951
919	2.9633155	952	2.9786369	985	2.9934362
920	2.9637878	953	2.9790929	986	2.9938769
921	2.9642596	954	2.9795484	987	2.9943171
922	2.9647309	955	2.9800034	988	2.9947569
923	2.9652017	956	2.9804579	989	2.9951963
924	2.9656720	957	2.9809119	990	2.9956352
925	2.9661417	958	2.9813655	991	2.9960736
926	2.9666110	959	2.9818186	992	2.9965117
927	2.9670797	960	2.9822712	993	2.9969492
928	2.9675480	961	2.9827234	994	2.9973864
929	2.9680157	962	2.9831751	995	2.9978231
930	2.9684829	963	2.9836263	996	2.9982593
931	2.9689497	964	2.9840770	997	2.9986952
932	2.9694159	965	2.9845273	998	2.9991305
933	2.9698816	966	2.9849771	999	2.9995655
934	2.9703469	967	2.9854265	1000	3.0000000

N.	Logarith.	N.	Logarith.	N.	Logarith.
1001	3.0004341	1034	3.0145205	1067	3.0281644
1002	3.0008677	1035	3.0149403	1068	3.0285712
1003	3.0013009	1036	3.0153597	1069	3.0289777
1004	3.0017337	1037	3.0157787	1070	3.0293838
1005	3.0021661	1038	3.0161973	1071	3.0297895
1006	3.0025980	1039	3.0166155	1072	3.0301948
1007	3.0030295	1040	3.0170333	1073	3.0305997
1008	3.0034605	1041	3.0174507	1074	3.0310043
1009	3.0038912	1042	3.0178677	1075	3.0314089
1010	3.0043214	1043	3.0182843	1076	3.0318123
1011	3.0047511	1044	3.0187005	1077	3.0322157
1012	3.0051805	1045	3.0191163	1078	3.0326188
1013	3.0056094	1046	3.0195317	1079	3.0330214
1014	3.0060379	1047	3.0199467	1080	3.0334238
1015	3.0064660	1048	3.0203613	1081	3.0338257
1016	3.0068937	1049	3.0207755	1082	3.0342273
1017	3.0073209	1050	3.0211893	1083	3.0346284
1018	3.0077478	1051	3.0216027	1084	3.0350293
1019	3.0081742	1052	3.0220157	1085	3.0354297
1020	3.0086002	1053	3.0224284	1086	3.0358298
1021	3.0090257	1054	3.0228406	1087	3.0362295
1022	3.0094509	1055	3.0232524	1088	3.0366289
1023	3.0098756	1056	3.0236639	1089	3.0370279
1024	3.0102999	1057	3.0240750	1090	3.0374265
1025	3.0107239	1058	3.0244857	1091	3.0378247
1026	3.0111474	1059	3.0248960	1092	3.0382226
1027	3.0115704	1060	3.0253059	1093	3.0386201
1028	3.0119931	1061	3.0257154	1094	3.0390173
1029	3.0124154	1062	3.0261245	1095	3.0394141
1030	3.0128372	1063	3.0265333	1096	3.0398105
1031	3.0132587	1064	3.0269416	1097	3.0402066
1032	3.0136797	1065	3.0273496	1098	3.0406023
1033	3.0141003	1066	3.0277572	1099	3.0409977
1034	3.0145205	1067	3.0281644	1100	3.0413927

N.	Logarith.	N.	Logarith.	N.	Logarith.
1101	3.0417873	1134	3.0546130	1167	3.0670708
1102	3.0421816	1135	3.0549958	1168	3.0674428
1103	3.0425755	1136	3.0553783	1169	3.0678145
1104	3.0429691	1137	3.0557604	1170	3.0681859
1105	3.0433623	1138	3.0561423	1171	3.0685569
1106	3.0437551	1139	3.0565237	1172	3.0689276
1107	3.0441476	1140	3.0569048	1173	3.0692980
1108	3.0445398	1141	3.0572856	1174	3.0696681
1109	3.0449315	1142	3.0576661	1175	3.0700379
1110	3.0453230	1143	3.0580462	1176	3.0704073
1111	3.0457140	1144	3.0584260	1177	3.0707765
1112	3.0461048	1145	3.0588055	1178	3.0711453
1113	3.0464952	1146	3.0591846	1179	3.0715138
1114	3.0468852	1147	3.0595634	1180	3.0718820
1115	3.0472749	1148	3.0599419	1181	3.0722499
1116	3.0476642	1149	3.0603200	1182	3.0726175
1117	3.0480532	1150	3.0606978	1183	3.0729847
1118	3.0484418	1151	3.0610753	1184	3.0733517
1119	3.0488301	1152	3.0614525	1185	3.0737183
1120	3.0492180	1153	3.0618293	1186	3.0740847
1121	3.0496056	1154	3.0622058	1187	3.0744507
1122	3.0499928	1155	3.0625820	1188	3.0748164
1123	3.0503797	1156	3.0629578	1189	3.0751818
1124	3.0507663	1157	3.0633334	1190	3.0755470
1125	3.0511525	1158	3.0637086	1191	3.0759118
1126	3.0515384	1159	3.0640834	1192	3.0762762
1127	3.0519239	1160	3.0644580	1193	3.0766404
1128	3.0523091	1161	3.0648322	1194	3.0770043
1129	3.0526939	1162	3.0652061	1195	3.0773679
1130	3.0530784	1163	3.0655797	1196	3.0777312
1131	3.0534626	1164	3.0659530	1197	3.0780942
1132	3.0538464	1165	3.0663259	1198	3.0784568
1133	3.0542299	1166	3.0666985	1199	3.0788192
1134	3.0546130	1167	3.0670708	1200	3.0791812

N.	Logarith.	N.	Logarith.	N.	Logarith.
1201	3.0795430	1234	3.0913151	1267	3.1027766
1202	3.0799045	1235	3.0916669	1268	3.1031192
1203	3.0802656	1236	3.0920185	1269	3.1034616
1204	3.0806265	1237	3.0923697	1270	3.1038037
1205	3.0809870	1238	3.0927206	1271	3.1041455
1206	3.0813473	1239	3.0930713	1272	3.1044871
1207	3.0817073	1240	3.0934217	1273	3.1048284
1208	3.0820669	1241	3.0937718	1274	3.1051694
1209	3.0824263	1242	3.0941216	1275	3.1055122
1210	3.0827854	1243	3.0944711	1276	3.1058507
1211	3.0831441	1244	3.0948204	1277	3.1061909
1212	3.0835026	1245	3.0951693	1278	3.1065308
1213	3.0838608	1246	3.0955180	1279	3.1068705
1214	3.0842187	1247	3.0988664	1280	3.1072100
1215	3.0845763	1248	3.0962146	1281	3.1075491
1216	3.0849336	1249	3.0965624	1282	3.1078880
1217	3.0852906	1250	3.0969100	1283	3.1082266
1218	3.0856473	1251	3.0972573	1284	3.1085650
1219	3.0860037	1252	3.0976043	1285	3.1089031
1220	3.0863598	1253	3.0979511	1286	3.1092410
1221	3.0867156	1254	3.0982975	1287	3.1095785
1222	3.0870712	1255	3.0986437	1288	3.1099159
1223	3.0874264	1256	3.0989896	1289	3.1102529
1224	3.0877814	1257	3.0993353	1290	3.1105897
1225	3.0881361	1258	3.0996806	1291	3.1109262
1226	3.0884905	1259	3.1000257	1292	3.1112625
1227	3.0888446	1260	3.1003705	1293	3.1115985
1228	3.0891984	1261	3.1007151	1294	3.1119343
1229	3.0895519	1262	3.1010593	1295	3.1122698
1230	3.0899051	1263	3.1014033	1296	3.1126050
1231	3.0902580	1264	3.1017471	1297	3.1129400
1232	3.0906107	1265	3.1020905	1298	3.1132747
1233	3.0909631	1266	3.1024337	1299	3.1136091
1234	3.0913151	1267	3.1027766	1300	3.1139433

N	Logarith	N.	Logarith.	N.	Logarith.
1301	3.1142773	1334	3.1251558	1367	3.1357685
1302	3.1146110	1335	3.1254813	1368	3.1360861
1303	3.1149444	1336	3.1258064	1369	3.1364034
1304	3.1152776	1337	3.1261314	1370	3.1367206
1305	3.1156105	1338	3.1264561	1371	3.1370374
1306	3.1159432	1339	3.1267806	1372	3.1373541
1307	3.1162756	1340	3.1271048	1373	3.1376705
1308	3.1166077	1341	3.1274288	1374	3.1379867
1309	3.1169396	1342	3.1277525	1375	3.1383027
1310	3.1172713	1343	3.1280760	1376	3.1386184
1311	3.1176027	1344	3.1283993	1377	3.1389339
1312	3.1179338	1345	3.1287223	1378	3.1392492
1313	3.1182647	1346	3.1290450	1379	3.1395643
1314	3.1185954	1347	3.1293676	1380	3.1398791
1315	3.1189257	1348	3.1296899	1381	3.1401937
1316	3.1192559	1349	3.1300119	1382	3.1405080
1317	3.1195858	1350	3.1303338	1383	3.1408222
1318	3.1199154	1351	3.1306553	1384	3.1411361
1319	3.1202448	1352	3.1309767	1385	3.1414498
1320	3.1205739	1353	3.1312978	1386	3.1417632
1321	3.1209028	1354	3.1316187	1387	3.1420765
1322	3.1212314	1355	3.1319393	1388	3.1423895
1323	3.1215598	1356	3.1322597	1389	3.1427022
1324	3.1218880	1357	3.1325798	1390	3.1430148
1325	3.1222159	1358	3.1328998	1391	3.1433271
1326	3.1225435	1359	3.1332195	1392	3.1436392
1327	3.1228709	1360	3.1335389	1393	3.1439511
1328	3.1231981	1361	3.1338581	1394	3.1442624
1329	3.1235250	1362	3.1341771	1395	3.1445742
1330	3.1238516	1363	3.1344958	1396	3.1448854
1331	3.1241780	1364	3.1348144	1397	3.1451964
1332	3.1245042	1365	3.1351326	1398	3.1455072
1333	3.1248301	1366	3.1354507	1399	3.1458177
1334	3.1251558	1367	3.1357685	1400	3.1461280

N.	Logarith.	N.	Logarith.	N.	Logarith.
1401	3.1464381	1434	3.1565491	1467	3.1664301
1402	3.1467480	1435	3.1568519	1468	3.1667260
1403	3.1470577	1436	3.1571544	1469	3.1670258
1404	3.1473671	1437	3.1574568	1470	3.1673173
1405	3.1476763	1438	3.1577589	1471	3.1676127
1406	3.1479853	1439	3.1580608	1472	3.1679078
1407	3.1482941	1440	3.1583625	1473	3.1682027
1408	3.1486026	1441	3.1586640	1474	3.1684975
1409	3.1489110	1442	3.1589653	1475	3.1687920
1410	3.1492191	1443	3.1592663	1476	3.1690863
1411	3.1495270	1444	3.1595672	1477	3.1693805
1412	3.1498347	1445	3.1598678	1478	3.1696744
1413	3.1501422	1446	3.1601683	1479	3.1699682
1414	3.1504494	1447	3.1604685	1480	3.1702617
1415	3.1507564	1448	3.1607686	1481	3.1705550
1416	3.1510632	1449	3.1610684	1482	3.1708482
1417	3.1513698	1450	3.1613680	1483	3.1711411
1418	3.1516762	1451	3.1616674	1484	3.1714339
1419	3.1519824	1452	3.1619666	1485	3.1717264
1420	3.1522883	1453	3.1622656	1486	3.1720188
1421	3.1525941	1454	3.1625644	1487	3.1723110
1422	3.1528996	1455	3.1628630	1488	3.1726029
1423	3.1532049	1456	3.1631614	1489	3.1728947
1424	3.1535100	1457	3.1634595	1490	3.1731863
1425	3.1538149	1458	3.1637575	1491	3.1734776
1426	3.1541195	1459	3.1640553	1492	3.1737688
1427	3.1544240	1460	3.1643528	1493	3.1740598
1428	3.1547282	1461	3.1646502	1494	3.1743506
1429	3.1550322	1462	3.1649474	1495	3.1746412
1430	3.1553360	1463	3.1652443	1496	3.1749316
1431	3.1556396	1464	3.1655411	1497	3.1752218
1432	3.1559430	1465	3.1658376	1498	3.1755118
1433	3.1562462	1466	3.1661340	1499	3.1758016
1434	3.1565491	1467	3.1664301	1500	3.1760913

N.	Logarith.	N.	Logarith.	N.	Logarith.
1501	3.1763807	1534	3.1858253	1567	3.1950690
1502	3.1766699	1535	3.1861084	1568	3.1953460
1503	3.1769590	1536	3.1863912	1569	3.1956229
1504	3.1772478	1537	3.1866739	1570	3.1958996
1505	3.1775365	1538	3.1869563	1571	3.1961762
1506	3.1778250	1539	3.1872386	1572	3.1964525
1507	3.1781132	1540	3.1875207	1573	3.1967287
1508	3.1784013	1541	3.1878026	1574	3.1970047
1509	3.1787892	1542	3.1880844	1575	3.1972806
1510	3.1789769	1543	3.1883659	1576	3.1975562
1511	3.1792645	1544	3.1886473	1577	3.1978317
1512	3.1795518	1545	3.1889285	1578	3.1981070
1513	3.1798389	1546	3.1892095	1579	3.1983821
1514	3.1801259	1547	3.1894903	1580	3.1986571
1515	3.1804126	1548	3.1897709	1581	3.1989319
1516	3.1806992	1549	3.1900514	1582	3.1992065
1517	3.1809856	1550	3.1903317	1583	3.1994809
1518	3.1812718	1551	3.1906118	1584	3.1997552
1519	3.1815578	1552	3.1908917	1585	3.2000293
1520	3.1818436	1553	3.1911714	1586	3.2003032
1521	3.1821292	1554	3.1914510	1587	3.2005769
1522	3.1824146	1555	3.1917304	1588	3.2008505
1523	3.1826999	1556	3.1920096	1589	3.2011239
1524	3.1829850	1557	3.1922886	1590	3.2013971
1525	3.1832698	1558	3.1925674	1591	3.2016702
1526	3.1835545	1559	3.1928461	1592	3.2019431
1527	3.1838390	1560	3.1931246	1593	3.2022158
1528	3.1841233	1561	3.1934029	1594	3.2024883
1529	3.1844075	1562	3.1936810	1595	3.2027607
1530	3.1846914	1563	3.1939590	1596	3.2030329
1531	3.1849752	1564	3.1942367	1597	3.2033049
1532	3.1852588	1565	3.1945143	1598	3.2035768
1533	3.1855421	1566	3.1947917	1599	3.2038485
1534	3.1858253	1567	3.1950690	1600	3.2041200

N.	Logarith.	N.	Logarith.	N.	Logarith.
1601	3.2043913	1634	3.2132521	1667	3.2219356
1602	3.2046625	1635	3.2135178	1668	3.2221960
1603	3.2049335	1636	3.2137833	1669	3.2224563
1604	3.2052044	1637	3.2140487	1670	3.2227165
1605	3.2054750	1638	3.2143139	1671	3.2229764
1606	3.2057455	1639	3.2145789	1672	3.2232363
1607	3.2060159	1640	3.2148438	1673	3.2234959
1608	3.2062860	1641	3.2151086	1674	3.2237555
1609	3.2065560	1642	3.2153732	1675	3.2240148
1610	3.2068259	1643	3.2156376	1676	3.2242740
1611	3.2070955	1644	3.2159018	1677	3.2245331
1612	3.2073650	1645	3.2161659	1678	3.2247920
1613	3.2076344	1646	3.2164298	1679	3.2250507
1614	3.2079035	1647	3.2166936	1680	3.2253093
1615	3.2081725	1648	3.2169572	1681	3.2255677
1616	3.2084414	1649	3.2172206	1682	3.2258260
1617	3.2087100	1650	3.2174839	1683	3.2260841
1618	3.2089785	1651	3.2177471	1684	3.2263421
1619	3.2092468	1652	3.2180100	1685	3.2265999
1620	3.2095150	1653	3.2142728	1686	3.2268576
1621	3.2097830	1654	3.2185355	1687	3.2271151
1622	3.2100508	1655	3.2187980	1688	3.2273724
1623	3.2103185	1656	3.2190603	1689	3.2276296
1624	3.2105860	1657	3.2193225	1690	3.2278867
1625	3.2108534	1658	3.2195845	1691	3.2281436
1626	3.2111205	1659	3.2108464	1692	3.2284004
1627	3.2113876	1660	3.2201081	1693	3.2286570
1628	3.2116544	1661	3.2203696	1694	3.2289134
1629	3.2119211	1662	3.2206310	1695	3.2291697
1630	3.2121876	1663	3.2208922	1696	3.2294258
1631	3.2124540	1664	3.2211533	1697	3.2296818
1632	3.2127201	1665	3.2215142	1698	3.2299377
1633	3.2129862	1666	3.2216750	1699	3.2301934
1634	3.2132521	1667	3.2219356	1700	3.2304489

N.	Logarith.	N.	Logarith.	N.	Logarith.
1701	3.2307043	1734	3.2380491	1767	3.2472365
1702	3.2309596	1735	3.2392995	1768	3.2474[illegible]
1703	3.2312146	1736	3.2395497	1769	3.2477278
1704	3.2314696	1737	3.2397998	1770	3.2479733
1705	3.2317244	1738	3.2400498	1771	3.2482186
1706	3.2319790	1739	3.2402996	1772	3.2484637
1707	3.2322335	1740	3.2405492	1773	3.2487087
1708	3.2324879	1741	3.2407988	1774	3.2489536
1709	3.2327421	1742	3.2410481	1775	3.2491984
1710	3.2329961	1743	3.2412974	1776	3.2494430
1711	3.2332500	1744	3.2415465	1777	3.2496874
1712	3.2335038	1745	3.2417954	1778	3.2499318
1713	3.2337574	1746	3.2420442	1779	3.2501759
1714	3.2340108	1747	3.2422929	1780	3.2504200
1715	3.2342641	1748	3.2425414	1781	3.2506639
1716	3.2345173	1749	3.2427898	1782	3.2509077
1717	3.2347703	1750	3.2430380	1783	3.2511513
1718	3.2350232	1751	3.2432861	1784	3.2513948
1719	3.2352760	1752	3.2435341	1785	3.2516382
1720	3.2355284	1753	3.2437819	1786	3.2518815
1721	3.2357809	1754	3.2440296	1787	3.2521246
1722	3.2360331	1755	3.2442771	1788	3.2523675
1723	3.2362853	1756	3.2445245	1789	3.2526103
1724	3.2365373	1757	3.2447718	1790	3.2528530
1725	3.2367891	1758	3.2450189	1791	3.2530956
1726	3.2370408	1759	3.2452658	1792	3.2533380
1727	3.2372923	1760	3.2455127	1793	3.2535803
1728	3.2375437	1761	3.2457594	1794	3.2538224
1729	3.2377950	1762	3.2460059	1795	3.2540645
1730	3.2380461	1763	3.2462523	1796	3.2543063
1731	3.2382971	1764	3.2464986	1797	3.2545481
1732	3.2385479	1765	3.2467447	1798	3.2547897
1733	3.2387986	1766	3.2469907	1799	3.2550312
1734	3.2380491	1767	3.2472365	1800	3.2552725

N.	Logarith.	N.	Logarith.	N.	Logarith.
1801	3.2555137	1834	3.2633993	1867	3.2711443
1802	3.2557548	1835	3.2636361	1868	3.2713769
1803	3.2559957	1836	3.2638727	1869	3.2716093
1804	3.2562365	1837	3.2641092	1870	3.2718416
1805	3.2564772	1838	3.2643455	1871	3.2720738
1806	3.2567177	1839	3.2645817	1872	3.2723058
1807	3.2569582	1840	3.2648178	1873	3.2725378
1808	3.2571984	1841	3.2650538	1874	3.2727696
1809	3.2574386	1842	3.2652896	1875	3.2730013
1810	3.2576786	1843	3.2655253	1876	3.2732328
1811	3.2579184	1844	3.2657609	1877	3.2734643
1812	3.2581582	1845	3.2659964	1878	3.2736956
1813	3.2583978	1846	3.2662317	1879	3.2739268
1814	3.2586373	1847	3.2664669	1880	3.2741578
1815	3.2588766	1848	3.2667020	1881	3.2743888
1816	3.2591158	1849	3.2669369	1882	3.2746196
1817	3.2593549	1850	3.2671717	1883	3.2748503
1818	3.2595939	1851	3.2674064	1884	3.2750809
1819	3.2598327	1852	3.2676410	1885	3.2753113
1820	3.2600714	1853	3.2678754	1886	3.2755417
1821	3.2603099	1854	3.2681097	1887	3.2757719
1822	3.2605484	1855	3.2683439	1888	3.2760020
1823	3.2607867	1856	3.2685780	1899	3.2762320
1824	3.2610248	1857	3.2688119	1890	3.2764618
1825	3.2612629	1858	3.2690457	1891	3.2766915
1826	3.2615008	1859	3.2692794	1892	3.2769211
1827	3.2617385	1860	3.2695129	1893	3.2771506
1828	3.2619762	1861	3.2697464	1894	3.2773800
1829	3.2622137	1862	3.2699797	1895	3.2776092
1830	3.2624511	1863	3.2702129	1896	3.2778383
1831	3.2626883	1864	3.2704459	1897	3.2780673
1832	3.2629255	1865	3.2706788	1898	3.2782962
1833	3.2631625	1866	3.2709116	1899	3.2785250
1834	3.2633993	1867	3.2711443	1900	3.2787536

N.	Logarith.	N.	Logarith.	N.	Logarith.
1901	3.2789821	1934	3.2864565	1967	3.2938044
1902	3.2792105	1935	3.2866810	1968	3.2940251
1903	3.2794388	1936	3.2869054	1969	3.2942457
1904	3.2796669	1937	3.2871296	1970	3.2944662
1905	3.2798950	1938	3.2873538	1971	3.2946866
1906	3.2801229	1939	3.2875778	1972	3.2939069
1907	3.2803507	1940	3.2878017	1973	3.2951271
1908	3.2805784	1941	3.2880255	1974	3.2953471
1909	3.2808059	1942	3.2882492	1975	3.2955671
1910	3.2810334	1943	3.2884728	1976	3.2957869
1911	3.2812607	1944	3.2886963	1977	3.2960067
1912	3.2814879	1945	3.2889196	1978	3.2962263
1913	3.2817150	1946	3.2891428	1979	3.2964458
1914	3.2819419	1947	3.2893659	1980	3.2966652
1915	3.2821688	1948	3.2895889	1981	3.2968845
1916	3.2823955	1949	3.2898118	1982	3.2971036
1917	3.2826221	1950	3.2900346	1983	3.2973227
1918	3.2828486	1951	3.2902573	1984	3.2975417
1919	3.2830750	1952	3.2904798	1985	3.2977605
1920	3.2833012	1953	3.2907022	1986	3.2979792
1921	3.2835274	1954	3.2909246	1987	3.2981979
1922	3.2837534	1955	3.2911468	1988	3.2984164
1923	3.2839793	1956	3.2913688	1989	3.2986348
1924	3.2842051	1957	3.2915908	1990	3.2988531
1925	3.2844307	1958	3.2918127	1991	3.2990713
1926	3.2846563	1959	3.2920344	1992	3.2992893
1927	3.2848817	1960	3.2922561	1993	3.2995073
1928	3.2851070	1961	3.2924776	1994	3.2997251
1929	3.2853322	1962	3.2926990	1995	3.2999429
1930	3.2855573	1963	3.2920203	1996	3.3001605
1931	3.2857823	1964	3.2931415	1997	3.3003781
1932	3.2860071	1965	3.2933626	1998	3.3005955
1933	3.2862318	1966	3.2935835	1999	3.3008128
1934	3.2864565	1967	3.2938044	2000	3.3010300

N	Logarith.	N.	Logarith.	N.	Logarith.
2001	3.3012471	2034	3.3083509	2067	3.3153405
2002	3.3014641	2035	3.3085644	2068	3.3155505
2003	3.3016809	2036	3.3087778	2069	3.3157605
2004	3.3018977	2037	3.3089910	2070	3.3159703
2005	3.3021144	2038	3.3092042	2071	3.3161801
2006	3.3023309	2039	3.3094172	2072	3.3163897
2007	3.3025474	2040	3.3096302	2073	3.3165993
2008	3.3027637	2041	3.3098430	2074	3.3168087
2009	3.3029799	2042	3.3100557	2075	3.3170181
2010	3.3031961	2043	3.3102684	2076	3.3172273
2011	3.3034121	2044	3.3104809	2077	3.3174365
2012	3.3036280	2045	3.3106933	2078	3.3176455
2013	3.3038438	2046	3.3109056	2079	3.3178545
2014	3.3040595	2047	3.3111178	2080	3.3180633
2015	3.3042751	2048	3.3113299	2081	3.3182721
2016	3.3044905	2049	3.3115420	2082	3.3184807
2017	3.3047059	2050	3.3117539	2083	3.3186893
2018	3.3049212	2051	3.3119657	2084	3.3188977
2019	3.3051363	2052	3.3121774	2085	3.3191061
2020	3.3053514	2053	3.3123889	2086	3.3193143
2021	3.3055663	2054	3.3126004	2087	3.3195224
2022	3.3057812	2055	3.3128118	2088	3.3197305
2023	3.3059959	2056	3.3130231	2089	3.3199384
2024	3.3062105	2057	3.3132343	2090	3.3201463
2025	3.3064250	2058	3.3134454	2091	3.3203540
2026	3.3066394	2059	3.3136563	2092	3.3205617
2027	3.3068537	2060	3.3138672	2093	3.3207692
2028	3.3070679	2061	3.3140780	2094	3.3209767
2029	3.3072820	2062	3.3142887	2095	3.3211840
2030	3.3074960	2063	3.3144992	2096	3.3213913
2031	3.3077099	2064	3.3147097	2097	3.3215984
2032	3.3079237	2065	3.3149200	2098	3.3218055
2033	3.3081374	2066	3.3151303	2099	3.3220124
2034	3.3083509	2067	3.3153405	2100	3.3222193

N.	Logarith.	N.	Logarith.	N.	Logarith.
2101	3.3224260	2134	3.3291944	2167	3.3358589
2102	3.3226327	2135	3.3293979	2168	3.3360593
2103	3.3228393	2136	3.3296012	2169	3.3362596
2104	3.3230457	2137	3.3298045	2170	3.3364597
2105	3.3232521	2138	3.3300077	2171	3.3366598
2106	3.3234584	2139	3.3302108	2172	3.3368598
2107	3.3236645	2140	3.3304138	2173	3.3370597
2108	3.3238706	2141	3.3306167	2174	3.3372595
2109	3.3240766	2142	3.3308195	2175	3.3374593
2110	3.3242825	2143	3.3310222	2176	3.3376589
2111	3.3244882	2144	3.3312248	2177	3.3378584
2112	3.3246939	2145	3.3314273	2178	3.3380579
2113	3.3248995	2146	3.3316297	2179	3.3382572
2114	3.3251050	2147	3.3318320	2180	3.3384565
2115	3.3253104	2148	3.3320343	2181	3.3386557
2116	3.3255157	2149	3.3322364	2182	3.3388547
2117	3.3257209	2150	3.3324385	2183	3.3390537
2118	3.3259260	2151	3.3326404	2184	3.339252
2119	3.3261350	2152	3.3328423	2185	3.339451
2120	3.3263359	2153	3.3330440	2186	3.339650
2121	3.3265407	2154	3.3332457	2187	3.339848
2122	3.3267454	2155	3.3334473	2188	3.340047
2123	3.3269500	2156	3.3336488	2189	3.340245
2124	3.3271545	2157	3.3338501	2190	3.340444
2125	3.3273589	2158	3.3340514	2191	3.340642
2126	3.3275633	2159	3.3342526	2192	3.34084
2127	3.3277675	2160	3.3344537	2193	3.34103
2128	3.3279716	2161	3.3346548	2194	3.34123
2129	3.3281757	2162	3.3348557	2195	3.341434
2130	3.3283796	2163	3.3350565	2196	3.34163
2131	3.3285834	2164	3.3352572	2197	3.341830
2132	3.3287872	2165	3.3354579	2198	3.342027
2133	3.3289909	2166	3.3356585	2199	3.342225
2134	3.3291944	2167	3.3358589	2200	3.342422

N.	Logarith.	N.	Logarith.	N.	Logarith.
2201	3.3426200	2234	3.3490832	2267	3.3554515
2202	3.3428173	2235	3.3492775	2268	3.3556430
2203	3.3430145	2236	3.3494718	2269	3.3558345
2204	3.3432116	2237	3.3496660	2270	3.3560259
2205	3.3434086	2238	3.3498601	2271	3.3562171
2206	3.3436055	2239	3.3500541	2272	3.3564083
2207	3.3438023	2240	3.3502480	2273	3.3565994
2208	3.3439991	2241	3.3504419	2274	3.3567905
2209	3.3441957	2242	3.3506356	2275	3.3569814
2210	3.3443923	2243	3.3508293	2276	3.3571723
2211	3.3445887	2244	3.3510228	2277	3.3573630
2212	3.3447851	2245	3.3512163	2278	3.3575537
2213	3.3449814	2246	3.3514098	2279	3.3577443
2214	3.3451776	2247	3.3516031	2280	3.3579348
2215	3.3453737	2248	3.3517963	2281	3.3581253
2216	3.3455698	2249	3.3519895	2282	3.3583156
2217	3.3457657	2250	3.3521825	2283	3.3585059
2218	3.3459615	2251	3.3523755	2284	3.3586961
2219	3.3461573	2252	3.3525684	2285	3.3588862
2220	3.3463530	2253	3.3527612	2286	3.3590762
2221	3.3465486	2254	3.3529539	2287	3.3592662
2222	3.3467441	2255	3.3531465	2288	3.3594560
2223	3.3469395	2256	3.3533391	2289	3.3596458
2224	3.3471348	2257	3.3535316	2290	3.3598355
2225	3.3473300	2258	3.3537239	2291	3.3600251
2226	3.3475252	2259	3.3539162	2292	3.3602146
2227	3.3477202	2260	3.3541084	2293	3.3604041
2228	3.3479152	2261	3.3543006	2294	3.3605934
2229	3.3481101	2262	3.3544926	2295	3.3607827
2230	3.3483049	2263	3.3546846	2296	3.3609719
2231	3.3484996	2264	3.3548764	2297	3.3611610
2232	3.3486942	2265	3.3550682	2298	3.3613500
2233	3.3488887	2266	3.3552599	2299	3.3615390
2234	3.3490832	2267	3.3554515	2300	3.3617278

N.	Logarith.	N.	Logarith.	N.	Logarith.
2301	3.3619166	2334	3.3681008	2367	3.3741983
2302	3.3621053	2335	3.3682869	2368	3.3743817
2303	3.3622939	2336	3.3684728	2369	3.3745651
2304	3.3624825	2337	3.3686587	2370	3.3747483
2305	3.3626709	2338	3.3688445	2371	3.3749316
2306	3.3628593	2339	3.3690302	2372	3.3751147
2307	3.3630476	2340	3.3692159	2373	3.3752977
2308	3.3632358	2341	3.3694014	2374	3.3754807
2309	3.3634239	2342	3.3695869	2375	3.3756636
2310	3.3636120	2343	3.3697723	2376	3.3758464
2311	3.3637999	2344	3.3699576	2377	3.3760292
2312	3.3639878	2345	3.3701428	2378	3.3762118
2313	3.3641756	2346	3.3703280	2379	3.3763944
2314	3.3643633	2347	3.3705131	2380	3.3765769
2315	3.3645510	2348	3.3706981	2381	3.3767594
2316	3.3647386	2349	3.3708830	2382	3.3769418
2317	3.3649260	2350	3.3710679	2383	3.3771240
2318	3.3651134	2351	3.3712526	2384	3.3773062
2319	3.3653007	2352	3.3714373	2385	3.3774884
2320	3.3654880	2353	3.3716219	2386	3.3776704
2321	3.3656751	2354	3.3718065	2387	3.3778524
2322	3.3658622	2355	3.3719909	2388	3.3780343
2323	3.3660492	2356	3.3721753	2389	3.3782161
2324	3.3662361	2357	3.3723596	2390	3.3783979
2325	3.3664230	2358	3.3725438	2391	3.3785796
2326	3.3666097	2359	3.3727279	2392	3.3787612
2327	3.3667964	2360	3.3729120	2393	3.3789427
2328	3.3669830	2361	3.3730960	2394	3.3791241
2329	3.3671695	2362	3.3732799	2395	3.3793055
2330	3.3673559	2363	3.3734637	2396	3.3794868
2331	3.3675423	2364	3.3736475	2397	3.3796680
2332	3.3677285	2365	3.3738311	2398	3.3798492
2333	3.3679147	2366	3.3740147	2399	3.3800302
2334	3.3681008	2367	3.3741983	2400	3.3802112

N.	Logarith.	N.	Logarith.	N.	Logarith.
2401	3.3803922	2434	3.3863206	2467	3.3921691
2402	3.3805730	2435	3.3864990	2468	3.3923452
2403	3.3807538	2436	3.3866773	2469	3.3925211
2404	3.3809345	2437	3.3868555	2470	3.3926969
2405	3.3811151	2438	3.3870337	2471	3.3928727
2406	3.3812956	2439	3.3872118	2472	3.3930485
2407	3.3814761	2440	3.3873898	2473	3.3932241
2408	3.3816565	2441	3.3875678	2474	3.3933997
2409	3.3818368	2442	3.3877457	2475	3.3935752
2410	3.3820170	2443	3.3879235	2476	3.3937506
2411	3.3821972	2444	3.3881012	2477	3.3939260
2412	3.3823773	2445	3.3882789	2478	3.3941013
2413	3.3825573	2446	3.3884565	2479	3.3942765
2414	3.3827373	2447	3.3886340	2480	3.3944517
2415	3.3829171	2448	3.3888114	2481	3.3946268
2416	3.3830969	2449	3.3889888	2482	3.3948018
2417	3.3832766	2450	3.3891661	2483	3.3949767
2418	3.3834563	2451	3.3893433	2484	3.3951516
2419	3.3836359	2452	3.3895205	2485	3.3953264
2420	3.3838154	2453	3.3896975	2486	3.3955011
2421	3.3839948	2454	3.3898746	2487	3.3956758
2422	3.3841741	2455	3.3900515	2488	3.3958504
2423	3.3843534	2456	3.3902284	2489	3.3960249
2424	3.3845326	2457	3.3904052	2490	3.3961993
2425	3.3847117	2458	3.3905819	2491	3.3963737
2426	3.3848908	2459	3.3907585	2492	3.3965480
2427	3.3850698	2460	3.3909351	2493	3.3967223
2428	3.3852487	2461	3.3911116	2494	3.3968964
2429	3.3854275	2462	3.3912880	2495	3.3970705
2430	3.3856063	2463	3 3914644	2496	3.3972446
2431	3.3857850	2464	3.3916407	2497	3.3974185
2432	3.3859636	2465	3.3918169	2498	3.3975924
2433	3.3861421	2466	3.3919931	2499	3.3977662
2434	3.3863206	2467	3.3921691	2500	3.3979400

N.	Logarith.	N.	Logarith.	N.	Logarith.
2501	3.3981137	2534	3.4038066	2567	3.4094259
2502	3.3982873	2535	3.4039780	2568	3.4055950
2503	3.3984608	2536	3.4041492	2569	3.4097641
2504	3.3986343	2537	3.4043205	2570	3.4099331
2505	3.3988077	2538	3.4044916	2571	3.4101021
2506	3.3989811	2539	3.4046627	2572	3.4102710
2507	3.3991543	2540	3.4048337	2573	3.4104398
2508	3.3993275	2541	3.4050047	2574	3.4106085
2509	3.3995007	2542	3.4051755	2575	3.4107772
2510	3.3996737	2543	3.4053464	2576	3.4109459
2511	3.3998467	2544	3.4055171	2577	3.4111144
2512	3.4000196	2545	3.4056878	2578	3.4112829
2513	3.4001925	2546	3.4058584	2579	3.4114513
2514	3.4003653	2547	3.4060289	2580	3.4116197
2515	3.4005380	2548	3.4061994	2581	3.4117880
2516	3.4007106	2549	3.4063698	2582	3.4119562
2517	3.4008832	2550	3.4065402	2583	3.4121244
2518	3.4010557	2551	3.4067105	2584	3.4122925
2519	3.4012282	2552	3.4068807	2585	3.4124605
2520	3.4014005	2553	3.4070508	2586	3.4126285
2521	3.4015728	2554	3.4072209	2587	3.4127964
2522	3.4017451	2555	3.4073909	2588	3.4129643
2523	3.4019173	2556	3.4075608	2589	3.4131320
2524	3.4020893	2557	3.4077307	2590	3.4132998
2525	3.4022614	2558	3.4079005	2591	3.4134674
2526	3.4024333	2559	3.4080703	2592	3.4136350
2527	3.4026052	2560	3.4082400	2593	3.4138025
2528	3.4027771	2561	3.4084096	2594	3.4139700
2529	3.4029488	2562	3.4085791	2595	3.4141374
2530	3.4031205	2563	3.4087486	2596	3.4143047
2531	3.4032921	2564	3.4089180	2597	3.4144719
2532	3.4034637	2565	3.4090874	2598	3.4146391
2533	3.4036352	2566	3.4092567	2599	3.4148063
2534	3.4038066	2567	3.4094259	2600	3.4149733

N.	Logarith.	N.	Logarith.	N.	Logarith.
2601	3.4151404	2634	3.4206158	2667	3.4260230
2602	3.4153073	2635	3.4207806	2668	3.4261858
2603	3.4154742	2636	3.4209454	2669	3.4263486
2604	3.4156410	2637	3.4211101	2670	3.4265113
2605	3.4158077	2638	3.4212748	2671	3.4266739
2606	3.4159744	2639	3.4214394	2672	3.4268365
2607	3.4161410	2640	3.4216039	2673	3.4269990
2608	3.4163076	2641	3.4217684	2674	3.4271614
2609	3.4164741	2642	3.4219328	2675	3.4273238
2610	3.4166405	2643	3.4220972	2676	3.4274861
2611	3.4168069	2644	3.4222614	2677	3.4276484
2612	3.4169732	2645	3.4224257	2678	3.4278106
2613	3.4171394	2646	3.4225898	2679	3.4279727
2614	3.4173056	2647	3.4227539	2680	3.4281348
2615	3.4174717	2648	3.4229180	2681	3.4282968
2616	3.4176377	2649	3.4230820	2682	3.4284588
2617	3.4178037	2650	3.4232459	2683	3.4286207
2618	3.4179696	2651	3.4234097	2684	3.4287825
2619	3.4181355	2652	3.4235735	2685	3.4289443
2620	3.4183013	2653	3.4237372	2686	3.4291060
2621	3.4184670	2654	3.4239009	2687	3.4292677
2622	3.4186327	2655	3.4240645	2688	3.4294293
2623	3.4187983	2656	3.4242281	2689	3.4295908
2624	3.4189638	2657	3.4243916	2690	3.4297523
2625	3.4191293	2658	3.4245550	2691	3.4299137
2626	3.4192947	2659	3.4247183	2692	3.4300751
2627	3.4194601	2660	3.4248816	2693	3.2302364
2628	3.4196254	2661	3.4250449	2694	3.4303976
2629	3.4197906	2662	3.4252080	2695	3.4305588
2630	3.4199557	2663	3.4253712	2696	3.4307199
2631	3.4201208	2664	3.4255342	2697	3.4308809
2632	3.4202859	2665	3.4256972	2698	3.4310419
2633	3.4204509	2666	3.4258601	2699	3.4312029
2634	3.4206158	2667	3.4260230	2700	3.4313638

N.	Logarith.	N.	Logarith.	N.	Logarith.
2701	3.4315246	2734	3.4367985	2767	3.4420092
2702	3.4316853	2735	3.4369573	2768	3.4421661
2703	3.4318460	2736	3.4371161	2769	3.4423229
2704	3.4320067	2737	3.4372748	2770	3.4424798
2705	3.4321673	2738	3.4374334	2771	3.4426365
2706	3.4323278	2739	3.4375920	2772	3.4427932
2707	3.4324882	2740	3.4377506	2773	3.4429499
2708	3.4326487	2741	3.4379090	2774	3.4431065
2709	3.4328090	2742	3.4380674	2775	3.4432630
2710	3.4329693	2743	3.4382258	2776	3.4434195
2711	3.4331295	2744	3.4383841	2777	3.4435759
2712	3.4332897	2745	3.4385423	2778	3.4437322
2713	3.4334498	2746	3.4387005	2779	3.4438885
2714	3.4336098	2747	3.4388587	2780	3.4440448
2715	3.4337698	2748	3.4390167	2781	3.4442010
2716	3.4339298	2749	3.4391747	2782	3.4443571
2717	3.4340896	2750	3.4393327	2783	3.4445132
2718	3.4342494	2751	3.4394906	2784	3.4446692
2719	3.4344092	2752	3.4396484	2785	3.4448252
2720	3.4345689	2753	3.4398062	2786	3.4449811
2721	3.4347285	2754	3.4399639	2787	3.4451370
2722	3.4348881	2755	3.4401216	2788	3.4452928
2723	3.4350476	2756	3.4402792	2789	3.4454485
2724	3.4352071	2757	3.4404368	2790	3.4456042
2725	3.4353665	2758	3.4405943	2791	3.4457598
2726	3.4355258	2759	3.4407517	2792	3.4459154
2727	3.4356851	2760	3.4409091	2793	3.4460709
2728	3.4358444	2761	3.4410664	2794	3.4462264
2729	3.4360035	2762	3.4412237	2795	3.4463818
2730	3.4361626	2763	3.4413809	2796	3.4465372
2731	3.4363217	2764	3.4415380	2797	3.4466925
2732	3.4364807	2765	3.4416951	2798	3.4468477
2733	3.4366396	2766	3.4418522	2799	3.4470029
2734	3.4367985	2767	3.4420092	2800	3.447158[illegible]

N.	Logarith.	N.	Logarith.	N.	Logarith.
2801	3.4473131	2834	3.4523998	2867	3.4574277
2802	3.4474681	2835	3.4525531	2868	3.4575791
2803	3.4476231	2836	3.4527062	2869	3.4577305
2804	3.4477780	2837	3.4528593	2870	3.4578819
2805	3.4479329	2838	3.4530124	2871	3.4580332
2806	3.4480877	2839	3.4531654	2872	3.4581844
2807	3.4482424	2840	3.4533183	2873	3.4583356
2808	3.4483971	2841	3.4534712	2874	3.4584867
2809	3.4485517	2842	3.4536241	2875	3.4586378
2810	3.4487063	2843	3.4537769	2876	3.4587889
2811	3.4488608	2844	3.4539296	2877	3.4589399
2812	3.4490153	2845	3.4540823	2878	3.4590908
2813	3.4491697	2846	3.4542349	2879	3.4592417
2814	3.4493241	2847	3.4543875	2880	3.4593925
2815	3.4494784	2848	3.4545400	2881	3.4595433
2816	3.4496326	2849	3.4546924	2882	3.4596940
2817	3.4497868	2850	3.4548449	2883	3.4598446
2818	3.4499410	2851	3.4549972	2884	3.4599953
2819	3.4500951	2852	3.4551495	2885	3.4601458
2820	3.4502491	2853	3.4553018	2886	3.4602963
2821	3.4504031	2854	3.4554540	2887	3.4604468
2822	3.4505570	2855	3.4556061	2888	3.4605972
2823	3.4507109	2856	3.4557582	2889	3.4607475
2824	3.4508647	2857	3.4559102	2890	3.4608978
2825	3.4510184	2858	3.4560622	2891	3.4610481
2826	3.4511721	2859	3.4562142	2892	3.4611983
2827	3.4513258	2860	3.4563660	2893	3.4613484
2828	3.4514794	2861	3.4565179	2894	3.4614985
2829	3.4516329	2862	3.4566696	2895	3.4616486
2830	3.4517864	2863	3.4568213	2896	3.4617986
2831	3.4519399	2864	3.4569730	2897	3.4619485
2832	3.4520932	2865	3.4571246	2898	3.4620984
2833	3.4522466	2866	3.4572762	2899	3.4622482
2834	3.4523998	2867	3.4574277	2900	3.4623980

N.	Logarith.	N.	Logarith.	N.	Logarith.
2901	3.4625477	2934	3.467466[illegible]	2967	3.47231[illegible]
2902	3.4626974	2935	3.4676081	2968	3.47246[illegible]
2903	3.4628470	2936	3.4677560	2969	3.47261[illegible]
2904	3.4629966	2937	3.4679039	2970	3.47275[illegible]
2905	3.4631461	2938	3.4680518	2971	3.472902[illegible]
2906	3.4632956	2939	3.4681996	2972	3.47304[illegible]
2907	3.4634450	2940	3.4683478	2973	3.47319[illegible]
2908	3.4635944	2941	3.4684950	2974	3.47334[illegible]
2909	3.4637437	2942	3.4686427	2975	3.47348[illegible]
2910	3.4638930	2943	3.4687903	2976	3.47363[illegible]
2911	3.4640422	2944	3.4689378	2977	3.47377[illegible]
2912	3.4641914	2945	3.4690858	2978	3.47392[illegible]
2913	3.4643405	2946	3.4692327	2979	3.47407[illegible]
2914	3.4644895	2947	3.4693801	2980	3.47421[illegible]
2915	3.4646386	2948	3.4695275	2981	3.47436[illegible]
2916	3.4647875	2949	3.4696748	2982	3.47450[illegible]
2917	3.4649364	2950	3.4698220	2983	3.47465[illegible]
2918	3.4650853	2951	3.4699692	2984	3.47479[illegible]
2919	3.4652341	2952	3.4701163	2985	3.4749443
2920	3.4653828	2953	3.4702634	2986	3.4750898
2921	3.4655316	2954	3.4704105	2987	3.4752352
2922	3.4656802	2955	3.4705575	2988	3.4753806
2923	3.4658288	2956	3.4707044	2989	3.4755259
2924	3.4659774	2957	3.4708513	2990	3.4756712
2925	3.4661259	2958	3.4709982	2991	3.4758164
2926	3.4662743	2959	3.4711450	2992	3.4759616
2927	3.4664227	2960	3.4712917	2993	3.4761067
2928	3.4665711	2961	3.4714384	2994	3.4762518
2929	3.4667194	2962	3.4715851	2995	3.4763968
2930	3.4668676	2963	3.4717317	2996	3.4765418
2931	3.4670158	2964	3.4718782	2997	3.4766[illegible]
2932	3.4671640	2965	3.4720247	2998	3.4768[illegible]
2933	3.4673121	2966	3.4721711	2999	3.47697[illegible]
2934	3.4674601	2967	3.4723175	3000	3.47712[illegible]

N.	Logarith.	N.	Logarith.	N.	Logarith.
3001	3.4772660	3034	3.4820156	3067	3.4867138
3002	3.4774107	3035	3.4821587	3068	3.4868554
3003	3.4775553	3036	3.4823018	3069	3.4869969
3004	3.4776999	3037	3.4824448	3070	3.4871384
3005	3.4778445	3038	3.4825878	3071	3.4872798
3006	3.4779890	3039	3.4827307	3072	3.4874212
3007	3.4781334	3040	3.4828736	3073	3.4875626
3008	3.4782778	3041	3.4830164	3074	3.4877039
3009	3.4784222	3042	3.4831592	3075	3.4878451
3010	3.4785665	3043	3.4833019	3076	3.4879863
3011	3.4787108	3044	3.4834446	3077	3.4881275
3012	3.4788550	3045	3.4835873	3078	3.4882686
3013	3.4789991	3046	3.4837299	3079	3.4884097
3014	3.4791432	3047	3.4838725	3080	3.4885507
3015	3.4792873	3048	3.4840150	3081	3.4886917
3016	3.4794313	3049	3.4841574	3082	3.4888326
3017	3.4795753	3050	3.4842998	3083	3.4889735
3018	3.4797192	3051	3.4844422	3084	3.4891144
3019	3.4798631	3052	3.4845845	3085	3.4892552
3020	3.4800069	3053	3.4847268	3086	3.4893959
3021	3.4801507	3054	3.4848690	3087	3.4895366
3022	3.4802945	3055	3.4850112	3088	3.4896773
3023	3.4804381	3056	3.4851533	3089	3.4898179
3024	3.4805818	3057	3.4852954	3090	3.4899585
3025	3.4807254	3058	3.4854375	3091	3.4900990
3026	3.4808689	3059	3.4855795	3092	3.4902395
3027	3.4810124	3060	3.4857214	3093	3.4937999
3028	3.4811559	3061	3.4858633	3094	3.4905203
3029	3.4812993	3062	3.4860052	3095	3.4906607
3030	3.4814426	3063	3.4861470	3096	3.4908009
3031	3.4855859	3064	3.4862888	3097	3.4909412
3032	3.4817292	3065	3.4864305	3098	3.4910814
3033	3.4818724	3066	3.4865721	3099	3.4912216
3034	3.4820156	3067	3.4867138	3100	3.4913617

N.	Logarith.	N.	Logarith.	N.	Logarith.
3101	3.4915018	3134	3.4960990	3167	3.5006481
3102	3.4916418	3135	3.4962375	3168	3.5007851
3103	3.4917818	3136	3.4963761	3169	3.5009222
3104	3.4919217	3137	3.4965145	3170	3.5010593
3105	3.4920616	3138	3.4966529	3171	3.5011962
3106	3.4922014	3139	3.4967913	3172	3.5013332
3107	3.4923413	3140	3.4969296	3173	3.5014701
3108	3.4924810	3141	3.4970679	3174	3.5016069
3109	3.4926207	3142	3.4972062	3175	3.5017437
3110	3.4927604	3143	3.4973444	3176	3.5018805
3111	3.4929000	3144	3.4974825	3177	3.5020172
3112	3.4930396	3145	3.4976206	3178	3.5021539
3113	3.4931791	3146	3.4977587	3179	3.5022905
3114	3.4933186	3147	3.4978967	3180	3.5024271
3115	3.4934580	3148	3.4980347	3181	3.5025637
3116	3.4935974	3149	3.4981727	3182	3.5027002
3117	3.4937368	3150	3.4983106	3183	3.5028366
3118	3.4938761	3151	3.4984484	3184	3.5029731
3119	3.4940154	3152	3.4985862	3185	3.5031094
3120	3.4941546	3153	3.4987240	3186	3.5032458
3121	3.4942938	3154	3.4988617	3187	3.5033821
3122	3.4944329	3155	3.4989994	3188	3.5035183
3123	3.4945720	3156	3.4991370	3189	3.5036545
3124	3.4947110	3157	3.4992846	3190	3.5037907
3125	3.4948500	3158	3.4994121	3191	3.5039268
3126	3.4949890	3159	3.4995496	3192	3.5040629
3127	3.4951279	3160	3.4996871	3193	3.5041989
3128	3.4952667	3161	3.4998245	3194	3.5043349
3129	3.4954056	3162	3.4999619	3195	3.5044709
3130	3.4955443	3163	3.5000992	3196	3.5046068
3131	3.4956831	3164	3.5002365	3197	3.5047426
3132	3.4958218	3165	3.5003737	3198	3.5048785
3133	3.4959604	3166	3.5005109	3199	3.5050142
3134	3.4960990	3167	3.5006481	3200	3.5051500

N.	Logarith.	N.	Logarith.	N.	Logarith
3201	3.5052857	3234	3.5097400	3267	3.5141491
3202	3.5054213	3235	3.5098743	3268	3.5142820
3203	3.5055569	3236	3.5100085	3269	3.5144149
3204	3.5056925	3237	3.5101427	3270	3.5145478
3205	3.5058280	3238	3.5102768	3271	3.5146805
3206	3.5059635	3239	3.5104109	3272	3.5148133
3207	3.5060990	3240	3.5105450	3273	3.5149460
3208	3.5062344	3241	3.5106790	3274	3.5150787
3209	3.5063697	3242	3.5108130	3275	3.5152113
3210	3.5065050	3243	3.5109469	3276	3.5153439
3211	3.5066403	3244	3.5110808	3277	3.5154764
3212	3.5067755	3245	3.5112147	3278	3.5156089
3213	3.5069107	3246	3.5113485	3279	3.5157414
3214	3.5070459	3247	3.5114823	3280	3.5158738
3215	3.5071810	3248	3.5116160	3281	3.5160062
3216	3.5073160	3249	3.5117497	3282	3.5161386
3217	3.5074511	3250	3.5118834	3283	3.5162709
3218	3.5075860	3251	3.5120170	3284	3.5164031
3219	3.5077210	3252	3.5121505	3285	3.5165354
3220	3.5078559	3253	3.5122841	3286	3.5166676
3221	3.5079907	3254	3.5124175	3287	3.5167997
3222	3.5081255	3255	3.5125510	3288	3.5169318
3223	3.5082603	3256	3.5126844	3289	3.5170639
3224	3.5083950	3257	3.5128178	3290	3.5171959
3225	3.5085297	3258	3.5129511	3291	3.5173279
3226	3.5086644	3259	3.5120844	3292	3.5174598
3227	3.5087990	3260	3.5132176	3293	3.5175917
3228	3.5089335	3261	3.5133508	3294	3.5177236
3229	3.5090680	3262	3.5134840	3295	3.5178554
3230	3.5092025	3263	3.5136171	3296	3.5179872
3231	3.5093370	3264	3.5137501	3297	3.5181189
3232	3.5094713	3265	3.5138832	3298	3.5182506
3233	3.5096057	3266	3.5140162	3299	3.5183823
3234	3.5097400	3267	3.5141491	3300	3.5185139

N.	Logarith.	N.	Logarith.	N.	Logarith.
3301	3.5186455	3334	3.5229656	3367	3.5272431
3302	3.5187771	3335	3.5230958	3368	3.5273721
3303	3.5189086	3336	3.5232260	3369	3.5275010
3304	3.5190400	3337	3.5233562	3370	3.5276299
3305	3.5191715	3338	3.5234863	3371	3.5277588
3306	3.5193028	3339	3.5236164	3372	3.5278876
3307	3.5194342	3340	3.5237465	3373	3.5280163
3308	3.5195655	3341	3.5238765	3374	3.5281451
3309	3.5196968	3342	3.5240064	3375	3.5282738
3310	3.5198280	3343	3.5241364	3376	3.5284024
3311	3.5199592	3344	3.5242663	3377	3.5285311
3312	3.5200903	3345	3.5243961	3378	3.5286596
3313	3.5202214	3346	3.5245259	3379	3.5287882
3314	3.5203525	3347	3.5246557	3380	3.5289167
3315	3.5204835	3348	3.5247854	3381	3.5290452
3316	3.5206145	3349	3.5249151	3382	3.5291736
3317	3.5207455	3350	3.5250448	3383	3.5293020
3318	3.5208764	3351	3.5251744	3384	3.5294303
3319	3.5210073	3352	3.5253040	3385	3.5295587
3320	3.5211381	3353	3.5254335	3386	3.5296869
3321	3.5212689	3354	3.5255631	3387	3.5298152
3322	3.5213996	3355	3.5256925	3388	3.5299434
3323	3.5215303	3356	3.5258219	3389	3.5300716
3324	3.5216610	3357	3.5259513	3390	3.5301997
3325	3.5217916	3358	3.5260807	3391	3.5303278
3326	3.5219222	3359	3.5262100	3392	3.5304558
3327	3.5220528	3360	3.5263393	3393	3.5305839
3328	3.5221833	3361	3.5264685	3394	3.5307118
3329	3.5223138	3362	3.5265977	3395	3.5308398
3330	3.5224442	3363	3.5267269	3396	3.5309677
3331	3.5225746	3364	3.5268560	3397	3.5310955
3332	3.5227050	3365	3.5269851	3398	3.5312234
3333	3.5228353	3366	3.5271141	3399	3.5313512
3334	3.5229656	3367	3.5272431	3400	3.5314789

N.	Logarith.	N.	Logarith.	N.	Logarith
3401	3.5316066	3434	3.5358003	3467	3.5399538
3402	3.5317343	3435	3.5359267	3468	3.5400791
3403	3.5318619	3436	3.5360532	3469	3.5402043
3404	3.5319895	3437	3.5361795	3470	3.5403295
3405	3.5321171	3438	3.5363059	3471	3.5404546
3406	3.5322446	3439	3.5364322	3472	3.5405797
3407	3.5323721	3440	3.5365584	3473	3.5407048
3408	3.5324996	3441	3.5366847	3474	3.5408298
3409	3.5326270	3442	3.5368109	3475	3.5409548
3410	3.5327544	3443	3.5369370	3476	3.5410798
3411	3.5328817	3444	3.5370631	3477	3.5412047
3412	3.5330090	3445	3.5371892	3478	3.5413296
3413	3.5331363	3446	3.5373153	3479	3.5414544
3414	3.5332635	3447	3.5374413	3480	3.5415792
3415	3.5333907	3448	3.5375672	3481	3.5417040
3416	3.5335179	3449	3.5376932	3482	3.5418288
3417	3.5336450	3450	3.5378191	3483	3.5419535
3418	3.5337721	3451	3.5379450	3484	3.5420781
3419	3.5338991	3452	3.5380708	3485	3.5422028
3420	3.5340261	3453	3.5381966	3486	3.5423274
3421	3.5341531	3454	3.5383223	3487	3.5424519
3422	3.5342800	3455	3.5384481	3488	3.5425765
3423	3.5344069	3456	3.5385737	3489	3.5427010
3424	3.5345338	3457	3.5386994	3490	3.5428254
3425	3.5346606	3458	3.5388250	3491	3.5429498
3426	3.5347874	3459	3.5389506	3492	3.5430742
3427	3.5349141	3460	3.5390761	3493	3.5431986
3428	3.5350408	3461	3.5392016	3494	3.5433229
3429	3.5351675	3462	3.5393271	3495	3.5434472
3430	3.5352941	3463	3.5394525	3496	3.5435714
3431	3.5354207	3464	3.5395779	3497	3.5436956
3432	3.5355473	3465	3.5397032	3498	3.5438198
3433	3.5356738	3466	3.5398286	3499	3.5439439
3434	3.5358003	3467	3.5399538	3500	3.5440680

N.	Logarith.	N.	Logarith.	N.	Logarith.
3501	3.5441921	3534	3.5442665	3567	3.5523031
3502	3.5443161	3535	3.5483894	3568	3.5524248
3503	3.5444401	3536	3.5485123	3569	3.5525465
3504	3.5445641	3537	3.5486351	3570	3.5526682
3505	3.5446880	3538	3.5487578	3571	3.5527898
3506	3.5448119	3539	3.5488806	3572	3.5529114
3507	3.5449358	3540	3.5490033	3573	3.5530330
3508	3.5450596	3541	3.5491259	3574	3.5531545
3509	3.5451834	3542	3.5492486	3575	3.5532760
3510	3.5453071	3543	3.5493712	3576	3.5533975
3511	3.5454308	3544	3.5494937	3577	3.5535189
3512	3.5455545	3545	3.5496162	3578	3.5536403
3513	3.5456781	3546	3.5497387	3579	3.5537617
3514	3.5458018	3547	3.5498612	3580	3.5538830
3515	3.5459253	3548	3.5499836	3581	3.5540043
3516	3.5460489	3549	3.5501060	3582	3.5541256
3517	3.5461724	3550	3.5502283	3583	3.5542468
3518	3.5462958	3551	3.5503507	3584	3.5543680
3519	3.5464193	3552	3.5504730	3585	3.5544892
3520	3.5465427	3553	3.5505952	3586	3.5546103
3521	3.5466660	3554	3.5507174	3587	3.5547314
3522	3.5467894	3555	3.5508396	3588	3.5548524
3523	3.5469126	3556	3.5509618	3589	3.5549735
3524	3.5470359	3557	3.5510839	3590	3.5550944
3525	3.5471591	3558	3.5512059	3591	3.5552154
3526	3.5472823	3559	3.5513280	3592	3.5553363
3527	3.5474055	3560	3.5514500	3593	3.5554572
3528	3.5475286	3561	3.5515720	3594	3.5555781
3529	3.5476517	3562	3.5516939	3595	3.5556989
3530	3.5477747	3563	3.5518158	3596	3.5558197
3531	3.5478977	3564	3.5519377	3597	3.5559404
3532	3.5480207	3565	3.5520595	3598	3.5560612
3533	3.5481436	3566	3.5521813	3599	3.5561818
3534	3.5482665	3567	3.5523031	3600	3.5563025

N.	Logarith.	N.	Logarith.	N.	Logarith.
3601	3.5564231	3634	3.5603849	3667	3.5643109
3602	3.5565437	3635	3.5605044	3668	3.5644293
3603	3.5566643	3636	3.5606239	3669	3.5645477
3604	3.5567848	3637	3.5607433	3670	3.5646661
3605	3.5569053	3638	3.5608627	3671	3.5647844
3606	3.5570257	3639	3.5609820	3672	3.5649027
3607	3.5571461	3640	3.5611014	3673	3.5650209
3608	3.5572665	3641	3.5612207	3674	3.5651392
3609	3.5573869	3642	3.5613399	3675	3.5652573
3610	3.5575072	3643	3.5614592	3676	3.5653755
3611	3.5576275	3644	3.5615784	3677	3.5654936
3612	3.5577477	3645	3.5616975	3678	3.5656117
3613	3.5578680	3646	3.5618167	3679	3.5657298
3614	3.5579881	3647	3.5619358	3680	3.5658478
3615	3.5581083	3648	3.5620548	3681	3.5659658
3616	3.5582284	3649	3.5621739	3682	3.5660838
3617	3.5583485	3650	3.5622929	3683	3.5662017
3618	3.5584686	3651	3.5624118	3684	3.5663196
3619	3.5585886	3652	3.5625308	3685	3.5664375
3620	3.5587086	3653	3.5626497	3686	3.5665553
3621	3.5588285	3654	3.5627685	3687	3.5666731
3622	3.5589484	3655	3.5628874	3688	3.5667909
3623	3.5590683	3656	3.5630062	3689	3.5669087
3624	3.5591882	3657	3.5631250	3690	3.5670264
3625	3.5593080	3658	3.5632437	3691	3.5671440
3626	3.5594278	3659	3.5633624	3692	3.5672617
3627	3.5595476	3660	3.5634811	3693	3.5673793
3628	3.5596673	3661	3.5635997	3694	3.5674969
3629	3.5597870	3662	3.5637183	3695	3.5676144
3630	3.5599066	3663	3.5638369	3696	3.5677320
3631	3.5600262	3664	3.5639555	3697	3.5678494
3632	3.5601458	3665	3.5640740	3698	3.5679669
3633	3.5602654	3666	3.5641925	3699	3.5680843
3634	3.5603849	3667	3.5643109	3700	3.5682017

N.	Logarith.	N.	Logarith.	N.	Logarith.
3701	3.5683191	3734	3.5721743	3767	3.5759956
3702	3.5684364	3735	3.5722906	3768	3.5761109
3703	3.5685537	3736	3.5724069	3769	3.5762261
3704	3.5686710	3737	3.5725231	3770	3.5763413
3705	3.5687882	3738	3.5726393	3771	3.5764565
3706	3.5689054	3739	3.5727555	3772	3.5765717
3707	3.5690226	3740	3.5728716	3773	3.5766868
3708	3.5691397	3741	3.5729877	3774	3.5768019
3709	3.5692568	3742	3.5731038	3775	3.5769169
3710	3.5693739	3743	3.5732198	3776	3.5770320
3711	3.5694910	3744	3.5733358	3777	3.5771470
3712	3.5696080	3745	3.5734518	3778	3.5772620
3713	3.5697249	3746	3.5735678	3779	3.5773769
3714	3.5698419	3747	3.5736837	3780	3.5774918
3715	3.5699588	3748	3.5737996	3781	3.5776067
3716	3.5700757	3749	3.5739154	3782	3.5777215
3717	3.5701926	3750	3.5740313	3783	3.5778363
3718	3.5703094	3751	3.5741471	3784	3.5779511
3719	3.5704262	3752	3.5742628	3785	3.5780659
3720	3.5705429	3753	3.5743786	3786	3.5781806
3721	3.5706597	3754	3.5744943	3787	3.5782953
3722	3.5707764	3755	3.5746099	3788	3.5784100
3723	3.5708930	3756	3.5747256	3789	3.5785246
3724	3.5710097	3757	3.5748412	3790	3.5786392
3725	3.5711263	3758	3.5749568	3791	3.5787538
3726	3.5712428	3759	3.5750723	3792	3.5788683
3727	3.5713594	3760	3.5751878	3793	3.5789828
3728	3.5714759	3761	3.5753033	3794	3.5790973
3729	3.5715924	3762	3.5754188	3795	3.5792118
3730	3.5717088	3763	3.5755342	3796	3.5793262
3731	3.5718252	3764	3.5756496	3797	3.5794406
3732	3.5719416	3765	3.5757650	3798	3.5795550
3733	3.5720580	3766	3.5758809	3799	3.5796693
3734	3.5721743	3767	3.5759956	3800	3.5797836

N.	Logarith.	N.	Logarith.	N.	Logarith.
3801	3.5798979	3834	3.5836521	3867	3.5873742
3802	3.5800121	3835	3.5837654	3868	3.5874865
3803	3.5801263	3836	3.5838786	3869	3.5875987
3804	3.5802405	3837	3.5839918	3870	3.5877110
3805	3.5803547	3838	3.5841050	3871	3.5878232
3806	3.5804688	3839	3.5842181	3872	3.5879353
3807	3.5805829	3840	3.5843312	3873	3.5880475
3808	3.5806969	3841	3.5844443	3874	3.5881596
3809	3.5808110	3842	3.5845574	3875	3.5882717
3810	3.5809250	3843	3.5846704	3876	3.5883838
3811	3.5810389	3844	3.5847834	3877	3.5884958
3812	3.5811529	3845	3.5848963	3878	3.5886078
3813	3.5812668	3846	3.5850093	3879	3.5887198
3814	3.5813807	3847	3.5851222	3880	3.5888317
3815	3.5814945	3848	3.5852351	3881	3.5889436
3816	3.5816084	3849	3.5853479	3882	3.5890555
3817	3.5817222	3850	3.5854607	3883	3.5891674
3818	3.5818359	3851	3.5855735	3884	3.5892792
3819	3.5819497	3852	3.5856863	3885	3.5893910
3820	3.5820634	3853	3.5857990	3886	3.5895028
3821	3.5821770	3854	3.5859117	3887	3.5896145
3822	3.5822907	3855	3.5860244	3888	3.5897262
3823	3.5824043	3856	3.5861370	3889	3.5898379
3824	3.5825179	3857	3.5862496	3890	3.5899496
3825	3.5826314	3858	3.5863622	3891	3.5900612
3826	3.5827450	3859	3.5864748	3892	3.5901728
3827	3.5828585	3860	3.5865873	3893	3.5902844
3828	3.5829719	3861	3.5866998	3894	3.5903959
3829	3.5830854	3862	3.5868123	3895	3.5905075
3830	3.5831988	3863	3.5869247	3896	3.5906189
3831	3.5833122	3864	3.5870371	3897	3.5907304
3832	3.5834255	3865	3.5871495	3898	3.5908418
3833	3.5835388	3866	3.5872618	3899	3.5909532
3834	3.5836521	3867	3.5873742	3900	3.5910646

N.	Logarith.	N.	Logarith.	N.	Logarith.
3901	3.5911759	3934	3.5948344	3967	3.5984622
3902	3.5912873	3935	3.5949447	3968	3.5985717
3903	3.5913985	3936	3.5950551	3969	3.5986811
3924	3.5915098	3937	3.5951654	3970	3.5987905
3905	3.5916210	3938	3.5952757	3971	3.5988999
3906	3.5917322	3939	3.5953860	3972	3.5990092
3907	3.5918434	3940	3.5954962	3973	3.5991186
3908	3.5919546	3941	3.5956064	3974	3.5992279
3909	3.5920657	3942	3.5957166	3975	3.5993371
3910	3.5921768	3943	3.5958268	3976	3.5994464
3911	3.5922878	3944	3.5969369	3977	3.5995556
3912	3.5923988	3945	3.5960470	3978	3.5996648
3913	3.5925098	3946	3.5961571	3979	3.5997739
3914	3.5926208	3947	3.5962671	3980	3.5998831
3915	3.5927318	3948	3.5963771	3981	3.5999922
3916	3.5928427	3949	3.5964871	3982	3.6001013
3917	3.5929536	3950	3.5965971	3983	3.6002103
3918	3.5930644	3951	3.5967070	3984	3.6003193
3919	3.5931753	3952	3.5968169	3985	3.6004283
3920	3.5932861	3953	3.5969268	3986	3.6015373
3921	3.5933968	3954	3.5970367	3987	3.6006462
3922	3.5935076	3955	3.5971465	3988	3.6007551
3923	3.5936183	3956	3.5972563	3989	3.6008640
3924	3.5937290	3957	3.5973660	3990	3.6009729
3925	3.5938397	3958	3.5974758	3991	3.6010817
3926	3.5939503	3959	3.5975855	3992	3.6011905
3927	3.5940609	3960	3.5976952	3993	3.6012993
3928	3.5941715	3961	3.5978048	3994	3.6014080
3929	3.5942820	3962	3.5979145	3995	3.6015168
3930	3.5943925	3963	3.5980241	3996	3.6016255
3931	3.5945030	3964	3.5981336	3997	3.6017341
3932	3.5946135	3965	3.5982432	3998	3.6018428
3933	3.5947239	3966	3.5983527	3999	3.6019514
3934	3.5948344	3967	3.5984622	4000	3.6020600

N.	Logarith.	N.	Logarith.	N.	Logarith.
4001	3.6021685	4034	3.6057359	4067	3.6092742
4002	3.6022771	4035	3.6058435	4068	3.6093809
4003	3.6023856	4036	3.6059512	4069	3.6094879
4004	3.6024941	4037	3.6060587	4070	3.6095944
4005	3.6026025	4038	3.6061663	4071	3.6097011
4006	3.6027109	4039	3.6062738	4072	3.6098078
4007	3.6028193	4040	3.6063814	4073	3.6099144
4008	3.6029277	4041	3.6064888	4074	3.6100210
4009	3.6030361	4042	3.6065963	4075	3.6101276
4010	3.6031444	4043	3.6067037	4076	3.6102342
4011	3.6032527	4044	3.6068111	4077	3.6103407
4012	3.6033609	4045	3.6069185	4078	3.6104472
4013	3.6034692	4046	3.6070259	4079	3.6105537
4014	3.6035774	4047	3.6071332	4080	3.6106602
4015	3.6036855	4048	3.6072405	4081	3.6107666
4016	3.6037937	4049	3.6073478	4082	3.6108730
4017	3.6039018	4050	3.6074550	4083	3.6109794
4018	3.6040099	4051	3.6075622	4084	3.6110857
4019	3.6041180	4052	3.6076694	4085	3.6111921
4020	3.6042261	4053	3.6077766	4086	3.6112984
4021	3.6043341	4054	3.6078837	4087	3.6114046
4022	3.6044421	4055	3.6079909	4088	3.6115199
4023	3.6045500	4056	3.6080979	4089	3.6116171
4024	3.6046580	4057	3.6082050	4090	3.6117233
4025	3.6047659	4058	3.6083120	4091	3.6118295
4026	3.6048738	4059	3.6084190	4092	3.6119356
4027	3.6049816	4060	3.6085260	4093	3.6120417
4028	3.6050895	4061	3.6086330	4094	3.6121478
4029	3.6051973	4062	3.6087399	4095	3.6122539
4030	3.6053050	4063	3.6088468	4096	3.6123599
4031	3.6054128	4064	3.6089537	4097	3.6124660
4032	3.6055205	4065	3.6090605	4098	3.6125720
4033	3.6056282	4066	3.6091674	4099	3.6126779
4034	3.6057359	4067	3.6092742	4100	3.6127839

N.	Logarith.	N.	Logarith.	N.	Logarith.
4101	3.6128898	4134	3.6163705	4167	3.6198235
4102	3.6129957	4135	3.6164755	4168	3.6199277
4103	3.6131015	4136	3.6165805	4169	3.6200319
4104	3.6132073	4137	3.6166855	4170	3.6201360
4105	3.6133132	4138	3.6167905	4171	3.6202402
4106	3.6134189	4139	3.6168954	4172	3.6203443
4107	3.6135247	4140	3.6170003	4173	3.6204484
4108	3.6136304	4141	3.6171052	4174	3.6205524
4109	3.6137361	4142	3.6172101	4175	3.6206565
4110	3.6138418	4143	3.6173149	4176	3.6207605
4111	3.6139475	4144	3.6174197	4177	3.6208645
4112	3.6140531	4145	3.6175245	4178	3.6209685
4113	3.6141587	4146	3.6176293	4179	3.6210724
4114	3.6142643	4147	3.6177340	4180	3.6211763
4115	3.6143698	4148	3.6178387	4181	3.6212802
4116	3.6144754	4149	3.6179434	4182	3.6213840
4117	3.6145809	4150	3.6180481	4183	3.6214879
4118	3.6146863	4151	3.6181527	4184	3.6215917
4119	3.6147918	4152	3.6182573	4185	3.6216955
4120	3.6148972	4153	3.6183619	4186	3.6217992
4121	3.6150026	4154	3.6184665	4187	3.6219030
4122	3.6151080	4155	3.6185710	4188	3.6220067
4123	3.6152133	4156	3.6186755	4189	3.6221104
4124	3.6153187	4157	3.6187800	4190	3.6222140
4125	3.6154240	4158	3.6188845	4191	3.6223177
4126	3.6155292	4159	3.6189889	4192	3.6224213
4127	3.6156345	4160	3.6190933	4193	3.6225249
4128	3.6157397	4161	3.6191977	4194	3.6226284
4129	3.6158449	4162	3.6193021	4195	3.6227320
4130	3.6159501	4163	3.6194064	4196	3.6228355
4131	3.6160552	4164	3.6195107	4197	3.6229390
4132	3.6161603	4165	3.6196150	4198	3.6230424
4133	3.6162654	4166	3.6197193	4199	3.6231459
4134	3.6163705	4167	3.6198235	4200	3.6232493

N.	Logarith.	N.	Logarith.	N.	Logarith.
4201	3.6233527	4234	3.6267509	4267	3.6301226
4202	3.6234560	4235	3.6268534	4268	3.6302244
4203	3.6234594	4236	3.6269559	4269	3.6303262
4204	3.6236627	4237	3.6270585	4270	3.6304279
4205	3.6237660	4238	3.6271610	4271	3.6305296
4206	3.6238693	4239	3.6272634	4272	3.6306312
4207	3.6239725	4240	3.6273659	4273	3.6307329
4208	3.6240757	4241	3.6274683	4274	3.6308345
4209	3.6241789	4242	3.6275707	4275	3.6309361
4210	3.6242821	4243	3.6276730	4276	3.6310377
4211	3.6243852	4244	3.6277754	4277	3.6311392
4212	3.6244884	4245	3.6278777	4278	3.6312408
4213	3.6245915	4246	3.6279800	4279	3.6313423
4214	3.6246945	4247	3.6280823	4280	3.6314438
4215	3.6247976	4248	3.6281845	4281	3.6315452
4216	3.6249006	4249	3.6282867	4282	3.6316467
4217	3.6250036	4250	3.6283889	4283	3.6317481
4218	3.6251066	4251	3.6284911	4284	3.6318495
4219	3.6252095	4252	3.6285933	4285	3.6319508
4220	3.6253124	4253	3.6286954	4286	3.6320522
4221	3.6254153	4254	3.6287975	4287	3.6321535
4222	3.6255182	4255	3.6288996	4288	3.6322548
4223	3.6256211	4256	3.6290016	4289	3.6323560
4224	3.6257239	4257	3.6291036	4290	3.6324573
4225	3.6258267	4258	3.6292057	4291	3.6325585
4226	3.6259295	4259	3.6293076	4292	3.6326597
4227	3.6260322	4260	3.6294096	4293	3.6327609
4228	3.6261350	4261	3.6205115	4294	3.6328620
4229	3.6262377	4262	3.6206134	4295	3.6329632
4230	3.6263404	4263	3.6207153	4296	3.6330643
4231	3.6264430	4264	3.6298172	4297	3.6331653
4232	3.6265457	4265	3.6299190	4298	3.6332664
4233	3.6266483	4266	3.6300208	4299	3.6333674
4234	3.6267509	4267	3.6301226	4300	3.6334685

N.	Logarith.	N.	Logarith.	N.	Logarith.
4301	3.6335694	4334	3.6368889	4367	3.6401832
4302	3.6336704	4335	3.6369891	4368	3.6402826
4303	3.6337713	4336	3.6370893	4369	3.6403820
4304	3.6338723	4337	3.6371894	4370	3.6404814
4305	3.6339732	4338	3.6372895	4371	3.6405808
4306	3.6340740	4339	3.6373896	4372	3.6406802
4307	3.6341749	4340	3.6374897	4373	3.6407795
4308	3.6342757	4341	3.6375898	4374	3.6408788
4309	3.6343765	4342	3.6376898	4375	3.6409781
4310	3.6344773	4343	3.6377898	4376	3.6410773
4311	3.6345780	4344	3.6378898	4377	3.6411765
4312	3.6346788	4345	3.6379898	4378	3.6412758
4313	3.6347795	4346	3.6380897	4379	3.6413749
4314	3.6348801	4347	3.6381896	4380	3.6414741
4315	3.6349808	4348	3.6382895	4381	3.6415733
4316	3.6350814	4349	3.6383894	4382	3.6416724
4317	3.6351820	4350	3.6384893	4383	3.6417715
4318	3.6352826	4351	3.6385891	4384	3.6418705
4319	3.6353832	4352	3.6386889	4385	3.6419696
4320	3.6354837	4353	3.6387887	4386	3.6420686
4321	3.6355843	4354	3.6388884	4387	3.6421676
4322	3.6356848	4355	3.6389882	4388	3.6422666
4323	3.6357852	4356	3.6390879	4389	3.6423656
4324	3.6358857	4357	3.6391876	4390	3.6424645
4325	3.6359861	4358	3.6392872	4391	3.6425634
4326	3.6360865	4359	3.6393869	4392	3.6426623
4327	3.6368769	4360	3.6394865	4393	3.6427612
4328	3.6362872	4361	3.6395861	4394	3.6428601
4329	3.6363876	4362	3.6396857	4395	3.6429589
4330	3.6364879	4363	3.6397852	4396	3.6430577
4331	3.6365882	4364	3.6398847	4397	3.6431565
4332	3.6366884	4365	3.6399842	4398	3.6432552
4333	3.6367887	4366	3.6400837	4399	3.6433540
4334	3.6368889	4367	3.6401832	4400	3.6434527

N.	Logarith.	N.	Logarith.	N.	Logarith.
4401	3.6435514	4434	3.6467957	4467	3.6500160
4402	3.6436500	4435	3.6468936	4468	3.6501132
4403	3.6437487	4436	3.6469915	4469	3.6502104
4404	3.6438473	4437	3.6470894	4470	3.6503075
4405	3.6439459	4438	3.6471873	4471	3.6504047
4406	3.6440445	4439	3.6472851	4472	3.6505018
4407	3.6441430	4440	3.6473830	4473	3.6505989
4408	3.6442416	4441	3.6474808	4474	3.6506960
4409	3.6443401	4442	3.6475785	4475	3.6507930
4410	3.6444386	4443	3.6476763	4476	3.6508901
4411	3.6445371	4444	3.6477740	4477	3.6509871
4412	3.6446355	4445	3.6478718	4478	3.6510841
4413	3.6447339	4446	3.6479695	4479	3.6511811
4414	3.6448323	4447	3.6480671	4480	3.6512780
4415	3.6449307	4448	3.6481648	4481	3.6513749
4416	3.6450291	4449	3.6482624	4482	3.6514719
4417	3.6451274	4450	3.6483200	4483	3 6515687
4418	3.6452257	4451	3.6484576	4484	3.6516656
4419	3.6453240	4452	3.6485552	4485	3.6517624
4420	3.6454223	4453	3.6486572	4486	3.6518593
4421	3.6455205	4454	3.6487502	4487	3.6519561
4422	3.6456187	4455	3.6488477	4488	3.6520528
4423	3.6457169	4456	3.6489452	4489	3.6521496
4424	3.6458151	4457	3.6490426	4490	3.6522463
4425	3.6459133	4458	3.6491401	4491	3.6523430
4426	3.6460114	4459	3.6492375	4492	3.6524397
4427	3.6461095	4460	3.6493349	4493	3.6525364
4428	3.6462076	4461	3.6494322	4494	3.6526331
4429	3.6463057	4462	3.6495296	4495	3.6527297
4430	3.6464037	4463	3.6496269	4496	3.6528263
4431	3.6465017	4464	3.6497242	4497	3.6529229
4432	3.6465997	4465	3.6498215	4498	3.6 30195
4433	3.6466977	4466	3.6499187	4499	3.6531160
4434	3.6467957	4467	3.6500160	4500	3.6532125

N.	Logarith.	N.	Logarith.	N.	Logarith.
4501	3.6533090	4534	3.6564815	4567	3.6596310
4502	3.6534055	4535	3.6565773	4568	3.6597261
4503	3.6535019	4536	3.6566730	4569	3.6598212
4504	3.6535984	4537	3.6567688	4570	3.6599162
4505	3.6536948	4538	3.6568645	4571	3.6600112
4506	3.6537912	4539	3.6569602	4572	3.6601062
4507	3.6538876	4540	3.6570559	4573	3.6602012
4508	3.6539839	4541	3.6571515	4574	3.6602962
4509	3.6540802	4542	3.6572471	4575	3.6603911
4510	3.6541765	4543	3.6573427	4576	3.6604860
4511	3.6542728	4544	3.6574383	4577	3.6605809
4512	3.6543691	4545	3.6575339	4578	3.6606758
4513	3.6544653	4546	3.6576294	4579	3.6607706
4514	3.6545616	4547	3.6577250	4580	3.6608655
4515	3.6546578	4548	3.6578205	4581	3.6609603
4516	3.6547539	4549	3.6579159	4582	3.6610551
4517	3.6548501	4550	3.6580114	4583	3.6611499
4518	3.6549462	4551	3.6581068	4584	3.6612446
4519	3.6550423	4552	3.6582023	4585	3.6613393
4520	3.6551384	4553	3.6582976	4586	3.6614340
4521	3.6552345	4554	3.6583930	4587	3.6615287
4522	3.6553306	4555	3.6584884	4588	3.6616234
4523	3.6554266	4556	3.6585837	4589	3.6617181
4524	3.6555226	4557	3.6586790	4590	3.6618127
4525	3.6556186	4558	3.6587743	4591	3.6619073
4526	3.6557145	4559	3.6588696	4592	3.6620019
4527	3.6558105	4560	3.6589648	4593	3.66209[illegible]
4528	3.6559064	4561	3.6590610	4594	3.66219[illegible]
4529	3.6560023	4562	3.6591553	4595	3.66228[illegible]
4530	3.6560982	4563	3.6592505	4596	3.66238[illegible]
4531	3.6561941	4564	3.6593456	4597	3.6624745
4532	3.6562899	4565	3.6594408	4598	3.6625690
4533	3.6563857	4566	3.6595359	4599	3.6626634
4534	3.6564815	4567	3.6596310	4600	3.6627578

N.	Logarith.	N.	Logarith.	N.	Logarith.
4601	3.6628522	4634	3.6659560	4667	3.6690378
4602	3.6629466	4635	3.6660497	4668	3.6691308
4603	3.6630410	4636	3.6661434	4669	3.6692239
4604	3.6631353	4637	3.6662371	4670	3.6693169
4605	3.6632296	4638	3.6663307	4671	3.6694099
4606	3.6633239	4639	3.6664244	4672	3.6695028
4607	3.6634182	4640	3.6665180	4673	3.6695958
4608	3.6635125	4641	3.6666116	4674	3.6696887
4609	3.6636067	4642	3.6667051	4675	3.6697816
4610	3.6637009	4643	3.6667987	4676	3.6698745
4611	3.6637951	4644	3.6668922	4677	3.6699674
4612	3.6638893	4645	3.6669857	4678	3.6700602
4613	3.6629835	4646	3.6670792	4679	3.6701530
4614	3.6640776	4647	3.6671727	4680	3.6702459
4615	3.6641717	4648	3.6672661	4681	3.6703386
4616	3.6642658	4649	3.6673595	4682	3.6704314
4617	3.6643599	4650	3.6674530	4683	3.6705242
4618	3.6644539	4651	3.6675463	4684	3.6706169
4619	3.6645480	4652	3.6676397	4685	3.6707096
4620	3.6646420	4653	3.6677331	4686	3.6708023
4621	3.6647360	4654	3.6678264	4687	3.6708950
4622	3.6648299	4655	3.6679197	4688	3.6709876
4623	3.6649239	4656	3.6680130	4689	3.6710802
4624	3.6650178	4657	3.6681062	4690	3.6711728
4625	3.6651117	4658	3.6681995	4691	3.6712654
4626	3.6652056	4659	3.6682927	4692	3.6713580
4627	3.6652995	4660	3.6683859	4693	3.6714506
4628	3.6653933	4661	3.6684791	4694	3.6715431
4629	3.6654872	4662	3.6685723	4695	3.6716356
4630	3.6655810	4663	3.6686654	4696	3.6717281
4631	3.6656748	4664	3.6687585	4697	3.6718206
4632	3.6657685	4665	3.6688516	4698	3.6719130
4633	3.6658623	4666	3.6689447	4699	3.6720054
4634	3.6659560	4667	3.6690378	4700	3.6720979

N.	Logarith.	N.	Logarith.	N.	Logarith.
4701	3.6721903	4734	3.6752283	4767	3.6782452
4702	3.6722826	4735	3.6753200	4768	3.6783362
4703	3.6723750	4736	3.6754117	4769	3.6784273
4704	3.6724673	4737	3.6755034	4770	3.6785184
4705	3.6725596	4738	3.6755951	4771	3.6786094
4706	3.6726519	4739	3.6756867	4772	3.6787004
4707	3.6727442	4740	3.6757783	4773	3.6787914
4708	3.6728365	4741	3.6758700	4774	3.6788824
4709	3.6729287	4742	3.6759615	4775	3.6789734
4710	3.6730209	4743	3.6760531	4776	3.6790643
4711	3.6731131	4744	3.6761447	4777	3.6791552
4712	3.6732053	4745	3.6762362	4778	3.6792461
4713	3.6732974	4746	3.6763277	4779	3.6793370
4714	3.6733896	4747	3.6764192	4780	3.6794279
4715	3.6734817	4748	3.6765107	4781	3.6795187
4716	3.6735738	4749	3.6766022	4782	3.6796096
4717	3.6736659	4750	3.6766936	4783	3.6797004
4718	3.6737579	4751	3.6767850	4784	3.6797912
4719	3.6738500	4752	3.6768764	4785	3.6798819
4720	3.6739420	4753	3.6769678	4786	3.6799727
4721	3.6740340	4754	3.6770592	4787	3.6800634
4722	3.6741260	4755	3.6771505	4788	3.6801541
4723	3.6742179	4756	3.6772418	4789	3.6802448
4724	3.6743099	4757	3.6773332	4790	3.6803355
4725	3.6744018	4758	3.6774244	4791	3.6804262
4726	3.6744937	4759	3.6775157	4792	3.6805168
4727	3.6745856	4760	3.6776069	4793	3.680607[illegible]
4728	3.6746775	4761	3.6776982	4794	3.6806980
4729	3.6747693	4762	3.6777894	4795	3.6807886
4730	3.6748611	4763	3.6778806	4796	3.6808792
4731	3.6749529	4764	3.6779718	4797	3.6809697
4732	3.6750447	4765	3.6780629	4798	3.6810602
4733	3.6751365	4766	3.6781540	4799	3.6811507
4734	3.6752283	4767	3.6782452	4800	3.6812412

N.	Logarith.	N.	Logarith	N.	Logarith.
4801	3.6813317	4834	3.6843066	4867	3.6872613
4802	3.6814222	4835	3.6843965	4868	3.6873506
4803	3.6815126	4836	3.6844863	4869	3.6874398
4804	3.6816030	4837	3.6845761	4870	3.6875290
4805	3.6816934	4838	3.6846659	4871	3.6876181
4806	3.6817838	4839	3.6847556	4872	3.6877073
4807	3.6818741	4840	3.6848454	4873	3.6877964
4808	3.6819645	4841	3.6849351	4874	3.6878855
4809	3.6820548	4842	3.6850248	4875	3.6879746
4810	3.6821451	4843	3.6851145	4876	3.6880637
4811	3.6822354	4844	3.6852041	4877	3.6881528
4812	3.6823256	4845	3.6852938	4878	3.6882418
4813	3.6824159	4846	3.6853834	4879	3.6883308
4814	3.6825061	4847	3.6854730	4880	3.6884198
4815	3.6825963	4848	3.6855626	4881	3.6885088
4816	3.6826865	4849	3.6856522	4882	3.6885978
4817	3.6827766	4850	3.6857417	4883	3.6886867
4818	3.6828668	4851	3.6858313	4884	3.6887756
4819	3.6829569	4852	3.6859208	4885	3.6888646
4820	3.6830470	4853	3.6860103	4886	3.6889535
4821	3.6831371	4854	3.6860998	4887	3.6890423
4822	3.6832272	4855	3.6861892	4888	3.6891312
4823	3.6833173	4856	3.6862787	4889	3.6892200
4824	3.6834073	4857	3.6863681	4890	3.6893089
4825	3.6834973	4858	3.6864575	4891	3.6893977
4826	3.6835873	4859	3.6865469	4892	3.6894864
4827	3.6836773	4860	3.6866363	4893	3.6895752
4828	3.6837673	4861	3.6867256	4894	3.6896640
4829	3.6838572	4862	3.6868149	4895	3.6897527
4830	3.6839471	4863	3.6869043	4896	3.6898414
4831	3.6840370	4864	3.6869936	4897	3.6899301
4832	3.6841269	4865	3.6870828	4898	3.6900188
4833	3.6842168	4866	3.6871721	4899	3.6901074
4834	3.6843066	4867	3.6872613	4900	3.6901961

N.	Logarith.	N.	Logarith.	N.	Logarith.
4901	3.6902847	4934	3.6931991	4967	3.6960942
4902	3.6903733	4935	3.6932872	4968	3.6961816
4903	3.6904619	4936	3.6933752	4969	3.6962690
4904	3.6905505	4937	3.6934631	4970	3.6963564
4905	3.6906390	4938	3.6935511	4971	3.6964438
4906	3.6907275	4939	3.6936390	4972	3.6965311
4907	3.6908161	4940	3.6937269	4973	3.6966185
4908	3.6909046	4941	3.6938148	4974	3.6967058
4909	3.6909930	4942	3.6939027	4975	3.6967931
4910	3.6910815	4943	3.6939906	4976	3.6968804
4911	3.6911699	4944	3.6940785	4977	3.6969676
4912	3.6912584	4945	3.6941663	4978	3.6970549
4913	3.6913468	4946	3.6942541	4979	3.6971421
4914	3.6914352	4947	3.6943419	4980	3.6972293
4915	3.6915235	4948	3.6944297	4981	3.6973165
4916	3.6916119	4949	3.6945174	4982	3.6974037
4917	3.6917002	4950	3.6946052	4983	3.6974909
4918	3.6917885	4951	3.6946929	4984	3.6975780
4919	3.6918768	4952	3.6947806	4985	3.6976652
4920	3.6919651	4953	3.6948683	4986	3.6977523
4921	3.6920534	4954	3.6949560	4987	3.6978394
4922	3.6921416	4955	3.6950437	4988	3.6979264
4923	3.6922298	4956	3.6951313	4989	3.6980135
4924	3.6923180	4957	3.6952189	4990	3.6981005
4925	3.6924062	4958	3.6953065	4991	3.6981876
4926	3.6924944	4959	3.6953941	4992	3.6982746
4927	3.6925826	4960	3.6954817	4993	3.6983616
4928	3.6926707	4961	3.6955692	4994	3.6984485
4929	3.6927588	4962	3.6956568	4995	3.6985355
4930	3.6928469	4963	3.6957443	4996	3.6986224
4931	3.6929350	4964	3.6958318	4997	3.6987093
4932	3.6930231	4965	3.6959193	4998	3.6987963
4933	3.6931111	4966	3.6960067	4999	3.6988831
4934	3.6931991	4967	3.6960942	5000	3.6989700

N.	Logarith.	N.	Logarith.	N.	Logarith.
5001	3.6990569	5034	3.7019132	5067	3.7047509
5002	3.6991437	5035	3.7019995	5068	3.7048366
5003	3.6992305	5036	3.7020857	5069	3.7049223
5004	3.6993173	5037	3.7021719	5070	3.7050080
5005	3.6994041	5038	3.7022582	5071	3.7050936
5006	3.6994908	5039	3.7023444	5072	3.7051792
5007	3.6995776	5040	3.7024305	5073	3.7052649
5008	3.6996643	5041	3.7025167	5074	3.7053505
5009	3.6997510	5042	3.7026028	5075	3.7054360
5010	3.6998377	5043	3.7026890	5076	3.7055216
5011	3.6999244	5044	3.7027751	5077	3.7056072
5012	3.7000111	5045	3.7028612	5078	3.7056927
5013	3.7000977	5046	3 7029472	5079	3.7057782
5014	3.7001843	5047	3.7030333	5080	3.7058637
5015	3.7002709	5048	3.7031193	5081	3.7059492
5016	3.7003575	5049	3.7032054	5082	3.7060347
5017	3.7004441	5050	3.7032914	5083	3.7061201
5018	3.7005307	5051	3.7033774	5084	3.7062055
5019	3.7006172	5052	3.7034633	5085	3.7062910
5020	3.7007037	5053	3.7035493	5086	3.7063764
5021	3.7007902	5054	3.7036352	5087	3.7064617
5022	3.7008767	5055	3.7037212	5088	3.7065471
5023	3.7009632	5056	3.7038071	5089	3.7066324
5024	3.7010496	5057	3.7038929	5090	3.7067178
5025	3.7011361	5058	3.7039788	5091	3.7068031
5026	3.7012225	5059	3.7040647	5092	3.7068884
5027	3.7013089	5060	3.7041505	5093	3.7069737
5028	3.7013953	5061	3.7042363	5094	3.7070589
5029	3.7014816	5062	3.7043221	5095	3.7071442
5030	3.7015680	5063	3.7044079	5096	3.7072294
5031	3.7016543	5064	3.7044937	5097	3.7073146
5032	3.7017406	5065	3.7045794	5098	3.7073998
5033	3.7018269	5066	3.7046652	5099	3.7074850
5034	3.7019132	5067	3.7047509	5100	3.7075702

N	Logarith.	N.	Logarith.	N.	Logarith.
5101	3.7076553	5134	3.7104559	5167	3.7132385
5102	3.7077405	5135	3.7105404	5168	3.7133225
5103	3.7078256	5136	3.7106250	5169	3.7134065
5104	3.7079107	5137	3.7107096	5170	3.7134905
5105	3.7079957	5138	3.7107941	5171	3.7135745
5106	3.7080808	5139	3.7108786	5172	3.7136585
5107	3.7081659	5140	3.7109631	5173	3.7137425
5108	3.7082509	5141	3.7110476	5174	3.7138264
5109	3.7083359	5142	3.7111321	5175	3.7139104
5110	3.7084209	5143	3.7112165	5176	3.7139943
5111	3.7085059	5144	3.7113010	5177	3.7140782
5112	3.7085908	5145	3.7113854	5178	3.7141620
5113	3.7086758	5146	3.7114698	5179	3.7142459
5114	3.7087607	5147	3.7115542	5180	3.7143298
5115	3.7088456	5148	3.7116385	5181	3.7144136
5116	3.7089305	5149	3.7117229	5182	3.7144974
5117	3.7090154	5150	3.7118072	5183	3.7145812
5118	3.7091003	5151	3.7118915	5184	3.7146650
5119	3.7091851	5152	3.7119759	5185	3.7147488
5120	3.7092700	5153	3.7120601	5186	3.7148325
5121	3.7093548	5154	3.7121444	5187	3.7149162
5122	3.7094396	5155	3.7122287	5188	3.7150000
5123	3.7095244	5156	3.7123129	5189	3.7150837
5124	3.7096091	5157	3.7123971	5190	3.7151674
5125	3.7096939	5158	3.7124813	5191	3.7152510
5126	3.7097786	5159	3.7125655	5192	3.7153347
5127	3.7098633	5160	3.7126497	5193	3.7154183
5128	3.7099480	5161	3.7127339	5194	3.7155019
5129	3.7100327	5162	3.7128180	5195	3.7155856
5130	3.7101174	5163	3.7129021	5196	3.7156691
5131	3.7102020	5164	3.7129862	5197	3.7157527
5132	3.7102866	5165	3.7130703	5198	3.7158363
5133	3.7103713	5166	3.7131544	5199	3.7159198
5134	3.7104559	5167	3.7132385	5200	3.7160033

N.	Logarith.	N.	Logarith.	N.	Logarith.
5201	3.7160869	5234	3.7188337	5267	3.7215633
5202	3.7161703	5235	3.7189167	5268	3.7216458
5203	3.7162538	5236	3.7189996	5269	3.7217282
5204	3.7163373	5237	3.7190826	5270	3.7218106
5205	3.7164207	5238	3.7191655	5271	3.7218930
5206	3.7165042	5239	3.7192484	5272	3.7219754
5207	3.7165876	5240	3.7193313	5273	3.7220578
5208	3.7166710	5241	3.7194142	5274	3.7221401
5209	3.7167544	5242	3.7194970	5275	3.7222225
5210	3.7168377	5243	3.7195799	5276	3.7223048
5211	3.7169211	5244	3.7196627	5277	3.7223871
5212	3.7170044	5245	3.7197455	5278	3.7224694
5213	3.7170877	5246	3.7198283	5279	3.7225517
5214	3.7171710	5247	3.7199111	5280	3.7226339
5215	3.7172543	5248	3.7199938	5281	3.7227162
5216	3.7173376	5249	3.7200766	5282	3.7227984
5217	3.7174208	5250	3.7201593	5283	3.7228806
5218	3.7175041	5251	3.7202420	5284	3.7229628
5219	3.7175873	5252	3.7203247	5285	3.7230450
5220	3.7176705	5253	3.7204074	5286	3.7231272
5221	3.7177537	5254	3.7204901	5287	3.7232093
5222	3.7178369	5255	3.7205727	5288	3.7232914
5223	3.7179200	5256	3.7206554	5289	3.7233736
5224	3.7180032	5257	3.7207380	5290	3.7234557
5225	3.7180863	5258	3.7208206	5291	3.7235378
5226	3.7181694	5259	3.7209032	5292	3.7236198
5227	3.7182525	5260	3.7209857	5293	3.7237019
5228	3.7183356	5261	3.7210683	5294	3.7237839
5229	3.7184186	5262	3.7211508	5295	3.7238660
5230	3.7185017	5263	3.7212334	5296	3.7239480
5231	3.7185847	5264	3.7213159	5297	3.7240300
5232	3.7186677	5265	3.7213984	5298	3.7241120
5233	3.7187507	5266	3.7214809	5299	3.7241939
5234	3.7188337	5267	3.7215633	5300	3.7242759

N.	Logarith.	N.	Logarith.	N.	Logarith.
5301	3.7243578	5334	3.7270530	5367	3.7297316
5302	3.7244397	5335	3.7271344	5368	3.7298125
5303	3.7245216	5336	3.7272158	5369	3.7298934
5304	3.7246035	5337	3.7272972	5370	3.7299743
5305	3.7246854	5338	3.7273786	5371	3.7300551
5306	3.7247672	5339	3.7274599	5372	3.7301360
5307	3.7248491	5340	3.7275413	5373	3.7302168
5308	3.7249309	5341	3.7276226	5374	3.7302977
5309	3.7250127	5342	3.7277039	5375	3.7303785
5310	3.7250945	5343	3.7277852	5376	3.7304593
5311	3.7251763	5344	3.7278664	5377	3.7305400
5312	3.7252581	5345	3.7279477	5378	3.7306208
5313	3.7253398	5346	3.7280290	5379	3.7307015
5314	3.7254215	5347	3.7281102	5380	3.7307823
5315	3.7255033	5348	3.7281914	5381	3.7308630
5316	3.7255850	5349	3.7282726	5382	3.7309437
5317	3.7256667	5350	3.7283538	5383	3.7310244
5318	3.7257483	5351	3.7284349	5384	3.7311051
5319	3.7258300	5352	3.7285161	5385	3.7311857
5320	3.7259116	5353	3.7285972	5386	3.7312663
5321	3.7259933	5354	3.7286784	5387	3.7313470
5322	3.7260749	5355	3.7287595	5388	3.7314276
5323	3.7261565	5356	3.7288406	5389	3.7315082
5324	3.7262380	5357	3.7289216	5390	3.7315888
5325	3.7263196	5358	3.7290027	5391	3.7316692
5326	3.7264012	5359	3.7290838	5392	3.7317499
5327	3.7264827	5360	3.7291648	5393	3.7318305
5328	3.7265642	5361	3.7292458	5394	3.7319109
5329	3.7266457	5362	3.7293268	5395	3.7319914
5330	3.7267272	5363	3.7294078	5396	3.7320719
5331	3.7268087	5364	3.7294888	5397	3.7321524
5332	3.7268901	5365	3.7295697	5398	3.7322329
5333	3.7269716	5366	3.7296507	5399	3.7323133
5334	3.7270530	5367	3.7297316	5400	3.7323938

N.	Logarith.	N.	Logarith.	N.	Logarith.
5401	3.7324742	5434	3.7351196	5467	3.7377491
5402	3.7325546	5435	3.7351995	5468	3.7378285
5403	3.7326350	5436	3.7352794	5469	3.7379079
5404	3.7327153	5437	3.7353593	5470	3.7379873
5405	3.7327957	5438	3.7354392	5471	3.7380667
5406	3.7328760	5439	3.7355191	5472	3.7381461
5407	3.7329564	5440	3.7355989	5473	3.7382254
5408	3.7330367	5441	3.7356787	5474	3.7383048
5409	3.7331170	5442	3.7357585	5475	3.7383841
5410	3.7331973	5443	3.7358383	5476	3.7384634
5411	3.7332775	5444	3.7359181	5477	3.7385427
5412	3.7333578	5445	3.7359979	5478	3.7386220
5413	3.7334380	5446	3.7360776	5479	3.7387013
5414	3.7335142	5447	3.7361574	5480	3.7387806
5415	3.7335985	5448	3.7362371	5481	3.7388598
5416	3.7336787	5449	3.7363168	5482	3.7389390
5417	3.7337588	5450	3.7363965	5483	3.7390182
5418	3.7338390	5451	3.7364762	5484	3.7390974
5419	3.7339191	5452	3.7365558	5485	3.7391766
5420	3.7339993	5453	3.7366355	5486	3.7392558
5421	3.7340794	5454	3.7367151	5487	3.7393350
5422	3.7341595	5455	3.7367948	5488	3.7394141
5423	3.7342396	5456	3.7368744	5489	3.7394932
5424	3.7343197	5457	3.7369540	5490	3.7395723
5425	3.7343997	5458	3.7370335	5491	3.7396514
5426	3.7344798	5459	3.7371131	5492	3.7397305
5427	3.7345598	5460	3.7371926	5493	3.7398096
5428	3.7346398	5461	3.7372722	5494	3.7398886
5429	3.7347198	5462	3.7373517	5495	3.7399677
5430	3.7347998	5463	3.7374312	5496	3.7400467
5431	3.7348798	5464	3.7375107	5497	3.7401257
5432	3.7349598	5465	3.7375902	5498	3.7402047
5433	3.7350397	5466	3.7376696	5499	3.7402837
5434	3.7351196	5467	3.7377491	5500	3.7403627

N.	Logarith.	N.	Logarith.	N.	Logarith.
5501	3.7404416	5534	3.7430392	5567	3.7456212
502	3.7405206	5535	3.7431176	5568	3.7456992
5503	3.7405995	5536	3.7431961	5569	3.7457772
5504	3.7406784	5537	3.7432745	5570	3.7458552
5505	3 7407573	5538	3.7433530	5571	3.7459332
5506	3.7408362	5539	3.7434314	5572	3.7460111
5507	3.7409151	5540	3.7435098	5573	3.7460890
5508	3.7409939	5541	3.7435891	5574	3.7461670
5509	3.7410728	5542	3.7436665	5575	3.7462449
5510	3.7411516	5543	3.7437449	5576	3.7463228
5511	3.7412304	5544	3.7438232	5577	3.7464006
5512	3.7413092	5545	3.7439015	5578	3.7464785
5513	3.7413880	5546	3.7439799	5579	3.7465564
5514	3.7414668	5547	3.7440582	5580	3.7466342
5515	3 7415455	5548	3.7441365	5581	3.7467120
5516	3.7416243	5549	3.7442147	5582	3.7467898
5517	3.7417030	5550	3.7442930	5583	3.7468676
5518	3.7417817	5551	3.7443712	5584	3.7469454
5519	3.7418604	5552	3.7444495	5585	3.7470232
5520	3.7419391	5553	3.7445277	5586	3.7471009
5521	3.7420177	5554	3.7446059	5587	3.7471787
5522	3.7420964	5555	3.7446841	5588	3.7472564
5523	3.7421750	5556	3.7447622	5589	3.7473341
5524	3.7422537	5557	3.7448414	5590	3.7474118
5525	3.7423323	5558	3.7449185	5591	3.7474895
5526	3.7424109	5559	3.7449967	5592	3.7475672
5527	3.7424895	5560	3.7450748	5593	3.7476448
5528	3.7425680	5561	3.7451529	5594	3.7477225
5529	3.7426466	5562	3.7452310	5595	3.7478001
5530	3.7427251	5563	3.7453091	5596	3.7478777
5531	3.7428037	5564	3.7453871	5597	3.7479553
5532	3.7428822	5565	3.7454652	5598	3.7480329
5533	3.7429607	5566	3.7455432	5599	3.7481105
5534	3.7430392	5567	3.7456212	5600	3.7481880

N.	Logarith.	N.	Logarith.	N.	Logarith.
5601	3.7482656	5634	3.7508168	5667	3.7533532
5602	3.7483431	5635	3.7598939	5668	3.7534298
5603	3.7484206	5636	3.7509710	5669	3.7535065
5604	3.7484981	5637	3.7510480	5670	3.7535831
5605	3.7485756	5638	3.7511251	5671	3.7536596
5606	3.7486531	5639	3.7512021	5672	3.7537362
5607	3.7487306	5640	3.7512791	5673	3.7538128
5608	3.7488080	5641	3.7513561	5674	3.7538893
5609	3.7488854	5642	3.7514331	5675	3.7539659
5610	3.7489629	5643	3.7515100	5676	3.7540424
5611	3.7490403	5644	3.7515870	5677	3.7541189
5612	3.7491177	5645	3.7516639	5678	3.7541954
5613	3.7491950	5646	3.7517409	5679	3.7542719
5614	3.7492724	5647	3.7518178	5680	3.7543483
5615	3.7493498	5648	3.7518947	5681	3.7544248
5616	3.7494271	5649	3.7519716	5682	3.7545012
5617	3.7495044	5650	3.7520484	5683	3.7545777
5618	3.7495817	5651	3.7521253	5684	3.7546541
5619	3.7496590	5652	3.7522022	5685	3.7547305
5620	3.7497363	5653	3.7522790	5686	3.7548069
5621	3.7498136	5654	3.7523558	5687	3.7548832
5622	3.7498908	5655	3.7524326	5688	3.7549596
5623	3.7499681	5656	3.7525094	5689	3.7550359
5624	3.7500453	5657	3.7525862	5690	3.7551123
5625	3.7501225	5658	3.7526629	5691	3.7551886
5626	3.7501997	5659	3.7527397	5692	3.7556249
5627	3.7502769	5660	3.7528164	5693	3.7553412
5628	3.7503541	5661	3.7528932	5694	3.7554175
5629	3.7504312	5662	3.7529699	5695	3.7554937
5630	3.7505084	5663	3.7530466	5696	3.7555700
5631	3.7505855	5664	3.7531232	5697	3.7556462
5632	3.7506626	5665	3.7531999	5698	3.7557224
5633	3.7507398	5666	3.7532766	5699	3.7557987
5634	3.7508168	5667	3.7533532	5700	3.7558749

N.	Logarith.	N.	Logarith.	N.	Logarith.
5701	3.7559510	5734	3.7584577	5767	3.7609500
5702	3.7560272	5735	3.7585334	5768	3.7610253
5703	3.7561034	5736	3.7586091	5769	3.7611005
5704	3.7561795	5737	3.7586848	5770	3.7611758
5705	3.7562556	5738	3.7587605	5771	3.7612511
5706	3.7563318	5739	3.7588362	5772	3.7613263
5707	3.7564079	5740	3.7589119	5773	3.7614016
5708	3.7564840	5741	3.7589875	5774	3.7614768
5709	3.7565600	5742	3.7590632	5775	3.7615520
5710	3.7566361	5743	3.7591388	5776	3.7616272
5711	3.7567122	5744	3.7592144	5777	3.7617024
5712	3.7567882	5745	3.7592900	5778	3.7617775
5713	3.7568642	5746	3.7593656	5779	3.7618527
5714	3.7569402	5747	3.7594412	5780	3.7619278
5715	3.7570162	5748	3.7595168	5781	3.7620030
5716	3.7570922	5749	3.7595923	5782	3.7620781
5717	3.7571682	5750	3.7596678	5783	3.7621532
5718	3.7572441	5751	3.7597434	5784	3.7622283
5719	3.7573201	5752	3.7558189	5785	3.7623034
5720	3.7573960	5753	3.7598944	5786	3.7623784
5721	3.7574719	5754	3.7599699	5787	3.7624535
5722	3.7575479	5755	3.7600453	5788	3.7625285
5723	3.7576237	5756	3.7601208	5789	3.7626035
5724	3.7576996	5757	3.7601962	5790	3.7626786
5725	3.7577755	5758	3.7602717	5791	3.7627536
5726	3.7578513	5759	3.7603471	5792	3.7628286
5727	3.7579272	5760	3.7604225	5793	3.7629036
5728	3.7580030	5761	3.7604979	5794	3.7629785
5729	3.7580788	5762	3.7605733	5795	3.7630534
5730	3.7581546	5763	3.7606486	5796	3.7631284
5731	3.7582304	5764	3.7607240	5797	3.7632033
5732	3.7583062	5765	3.7607993	5798	3.7632782
5733	3.7583819	5766	3.7608746	5799	3.7633531
5734	3.7584577	5767	3.7609500	5800	3.7634280

N.	Logarith.	N.	Logarith.	N.	Logarith.
5801	3.7635029	5834	3.7659664	5867	3.7684161
5802	3.7635777	5835	3.7660409	5868	3.7684901
5803	3.7636526	5836	3.7661153	5869	3.7685641
5804	3.7637274	5837	3.7661897	5870	3.7686381
5805	3.7638022	5838	3.7660641	5871	3.7687121
5806	3.7638770	5839	3.7663385	5872	3.7687860
5807	3.7639518	5840	3.7664128	5873	3.7688600
5808	3.7640266	5841	3.7664872	5874	3.7689339
5809	3.7641014	5842	3.7665616	5875	3.7690079
5810	3.7641761	5843	3.7666359	5876	3.7690818
5811	3.7642509	5844	3.7697102	5877	3.7691557
5812	3.7643256	5845	3.7667845	5878	3.7692296
5813	3.7644003	5846	3.7668588	5879	3.7693035
5814	3.7644750	5847	3.7669331	5880	3.7693773
5815	3.7645497	5848	3.7670074	5881	3.7694512
5816	3.7646244	5849	3.7670816	5882	3.7695250
5817	3.7646991	5850	3.7671559	5883	3.7695988
5818	3.7647737	5851	3.7672301	5884	3.7696727
5819	3.7648484	5852	3.7673043	5885	3.7697465
5820	3.7649230	5853	3.7673785	5886	3.7698203
5821	3.7649976	5854	3.7674527	5887	3.7698940
5822	3.7650722	5855	3.7675269	5888	3.7699678
5823	3.7651468	5856	3.7676011	5889	3.7700416
5824	3.7652214	5857	3.7676752	5890	3.7701153
5825	3.7652959	5858	3.7677494	5891	3.7701890
5826	3.7653705	5859	3.7678235	5892	3.7702627
5827	3.7654450	5860	3.7678976	5893	3.7703364
5828	3.7655195	5861	3.7679717	5894	3.7704101
5829	3.7655941	5862	3.7680458	5895	3.7704838
5830	3.7656686	5863	3.7681199	5896	3.7705575
5831	3.7657430	5864	3.7681940	5897	3.7706311
5832	3.7658175	5865	3.7682680	5898	3.7707048
5833	3.7658920	5866	3.7683421	5899	3.7707784
5834	3.7659664	5867	3.7684161	5900	3.7708520

N.	Logarith.	N.	Logarith.	N.	Logarith.
5901	3.7709256	5934	3.7733475	5967	3.7757560
5902	3.7709998	5935	3.7734207	5968	3.7758288
5903	3.7710728	5936	3.7734939	5969	3.7759016
5904	3.7711463	5937	3.7735670	5970	3.7759743
5905	3.7712199	5938	3.7736402	5971	3.7760471
5906	3.7712934	5939	3.7737133	5972	3.7761198
5907	3.7713670	5940	3.7737864	5973	3.7761925
5908	3.7714405	5941	3.7738596	5974	3.7762652
5909	3.7715140	5942	3.7739326	5975	3.7763379
5910	3.7715875	5943	3.7740057	5976	3.7764106
5911	3.7716610	5944	3.7740788	5977	3.7764833
5912	3.7717344	5945	3.7741519	5978	3.7765559
5913	3.7718079	5946	3.7742249	5979	3.7766286
5914	3.7718813	5947	3.7742979	5980	3.7767012
5915	3.7719547	5948	3.7743710	5981	3.7767738
5916	3.7720282	5949	3.7744440	5982	3.7768464
5917	3.7721016	5950	3.7745170	5983	3.7769190
5918	3.7721750	5951	3.7745899	5984	3.7769916
5919	3.7722483	5952	3.7746629	5985	3.7770642
5920	3.7723217	5953	3.7747359	5986	3.7771367
5921	3.7723951	5954	3.7748088	5987	3.7772093
5922	3.7724684	5955	3.7748818	5988	3.7772818
5923	3.7725417	5956	3.7749547	5989	3.7773543
5924	3.7726150	5957	3.7750276	5990	3.7774268
5925	3.7726884	5958	3.7751005	5991	3.7774993
5926	3.7727616	5959	3.7751734	5992	3.7775718
5927	3.7728349	5960	3.7752463	5993	3.7776443
5928	3.7729082	5961	3.7753191	5994	3.7777167
5929	3.7729814	5962	3.7753920	5995	3.7777892
5930	3.7730547	5963	3.7754648	5996	3.7778616
5931	3.7731279	5964	3.7755376	5997	3.7779340
5932	3.7732011	5965	3.7756104	5998	3.7780065
5933	3.7732743	5966	3.7756832	5999	3.7780789
5934	3.7733475	5967	3.7757560	6000	3.7781512

N.	Logarith.	N.	Logarith.	N.	Logarith.
6001	3.7782236	6034	3.7806053	6067	3.7829740
6002	3.7782960	6035	3.7806773	6068	3.7830456
6003	3.7783683	6036	3.7807492	6069	3.7831171
6004	3.7784407	6037	3.7808212	6070	3.7831887
6005	3.7785130	6038	3.7808931	6071	3.7832602
6006	3.7785853	6039	3.7809650	6072	3.7833318
6007	3.7786576	6040	3.7810369	6073	3.7834033
6008	3.7787299	6041	3.7811088	6074	3.7834748
6009	3.7788022	6042	3.7811807	6075	3.7835463
6010	3.7788745	6043	3.7812526	6076	3.7836178
6011	3.7789467	6044	3.7813245	6077	3.7836892
6012	3.7790190	6045	3.7813963	6078	3.7837607
6013	3.7790912	6046	3.7814681	6079	3.7838321
6014	3.7791634	6047	3.7815400	6080	3.7839036
6015	3.7792356	6048	3.7816118	6081	3.7839750
6016	3.7793078	6049	3.7816836	6082	3.7840464
6017	3.7793800	6050	3.7817554	6083	3.7841178
6018	3.7794522	6051	3.7818272	6084	3.7841892
6019	3.7795243	6052	3.7818989	6085	3.7842606
6020	3.7795965	6053	3.7819707	6086	3.7843319
6021	3.7796686	6054	3.7820424	6087	3.7844033
6022	3.7797408	6055	3.7821141	6088	3.7844746
6023	3.7798129	6056	3.7821859	6089	3.7845460
6024	3.7798850	6057	3.7822576	6090	3.7846173
6025	3.7799571	6058	3.7823293	6091	3.7846886
6026	3.7800291	6059	3.7824010	6092	3.7847599
6027	3.7801012	6060	3.7824726	6093	3.7848312
6028	3.7801732	6061	3.7825443	6094	3.7849024
6029	3.7802453	6062	3.7826159	6095	3.7849737
6030	3.7803173	6063	3.7826876	6096	3.7850450
6031	3.7803893	6064	3.7827592	6097	3.7851162
6032	3.7804613	6065	3.7828308	6098	3.7851874
6033	3.7805333	6066	3.7829024	6099	3.7852586
6034	3.7806053	6067	3.7829740	6100	3.7853298

N.	Logarith.	N.	Logarith.	N	Logarith.
6101	3.7854010	6134	3.7877438	6167	3.7900739
6102	3.7854722	6135	3.7878146	6168	3.7901444
6103	3.7855434	6136	3.7878853	6169	3.7902148
6104	3.7856145	6137	3.7879561	6170	3.7902852
6105	3.7856857	6138	3.7880269	6171	3.7903555
6106	3.7857568	6139	3.7880976	6172	3.7904259
6107	3.7858279	6140	3.7881684	6173	3.7904963
6108	3.7858990	6141	3.7882391	6174	3.7905666
6109	3.7859701	6142	3.7883098	6175	3.7906370
6110	3.7860412	6143	3.7883805	6176	3.7907073
6111	3.7861123	6144	3.7884512	6177	3.790777
6112	3.7861833	6145	3.7885219	6178	3.790847
6113	3.7862544	6146	3.7885926	6179	3.790918
6114	3.7863254	6147	3.7886632	6180	3.790988
6115	3.7863965	6148	3.7887339	6181	3.791058
6116	3.7864675	6149	3.7888045	6182	3.791129
6117	3.7865385	6150	3.7888751	6183	3.791199
6118	3.7866095	6151	3.7889457	6184	3.791269
6119	3.7866805	6152	3.7890163	6185	3.791339
6120	3.7867514	6153	3.7890869	6186	3.791409
6121	3.7868224	6154	3.7891575	6187	3.791480
6122	3.7868933	6155	3.7892281	6188	3.791550
6123	3.7869643	6156	3.7892986	6189	3.791620
6124	3.7870352	6157	3.7893691	6190	3.791690
6125	3.7871061	6158	3.7894397	6191	3.791760
6126	3.7871770	6159	3.7895102	6192	3.791830
6127	3.7872479	6160	3.7895807	6193	3.791901
6128	3.7873188	6161	3.7896512	6194	3.791971
6129	3.7873896	6162	3.7897217	6195	3.792041
6130	3.7874605	6163	3.7897922	6196	3.792111
6131	3.7875313	6164	3.7898626	6197	3.792181
6132	3.7876021	6165	3.7899331	6198	3.792251
6133	3.7876730	6166	3.7900035	6199	3.792321
6134	3.7877438	6167	3.7900739	6200	3.792391

N.	Logarith.	N.	Logarith.	N.	Logarith.
6201	3.7924617	6234	3.7947668	6267	3.7970597
6202	3.7925318	6235	3.7948365	6268	3.7971290
6203	3.7926018	6236	3.7949061	6269	3.7971983
6204	3.7926718	6237	3.7949757	6270	3.7972675
6205	3.7927418	6238	3.7950454	6271	3.7973368
6206	3.7928118	6239	3.7951150	6272	3.7974060
6207	3.7928817	6240	3.7951846	6273	3.7974753
6208	3.7929517	6241	3.7952542	6274	3.7975445
6209	3.7930217	6242	3.7953238	6275	3.7976137
6210	3.7930916	6243	3.7953933	6276	3.7976829
6211	3.7931615	6244	3.7954629	6277	3.7977521
6212	3.7932314	6245	3.7955324	6278	3.7978213
6213	3.7933014	6246	3.7956020	6279	3.7978905
6214	3.7933712	6247	3.7956715	6280	3.7979596
6215	3.7934411	6248	3.7957410	6281	3.7980288
6216	3.7935110	6249	3.7958105	6282	3.7980979
6217	3.7935809	6250	3.7958800	6283	3.7981671
6218	3.7936507	6251	3.7959495	6284	3.7982362
6219	3.7937206	6252	3.7960190	6285	3.7983053
6220	3.7937904	6253	3.7960884	6286	3.7983744
6221	3.7938602	6254	3.7961579	6287	3.7984435
6222	3.7939300	6255	3.7962273	6288	3.7985125
6223	3.7939998	6256	3.7962967	6289	3.7985816
6224	3.7940696	6257	3.7963662	6290	3.7986506
6225	3.7941394	6258	3.7964356	6291	3.7987197
6226	3.7942091	6259	3.7965050	6292	3.7987887
6227	3.7942789	6260	3.7965743	6293	3.7988577
6228	3.7943486	6261	3.7966437	6294	3.7989267
6229	3.7944183	6262	3.7967131	6295	3.7989957
6230	3.7944880	6263	3.7967824	6296	3.7990647
6231	3.7945578	6264	3.7968517	6297	3.7991337
6232	3.7946274	6265	3.7969211	6298	3.7992027
6233	3.7946971	6266	3.7969904	6299	3.7992716
6234	3.7947668	6267	3.7970597	6300	3.7993405

N.	Logarith.	N.	Logarith.	N.	Logarith.
6301	3.7994095	6334	3.8016781	6367	3.8039348
6302	3.7994784	6335	3.8017466	6368	3.8040031
6303	3.7995473	6336	3.8018152	6369	3.8040712
6304	3.7996162	6337	3.8018837	6370	3.8041394
6305	3.7996851	6338	3.8019522	6371	3.8042076
6306	3.7997540	6339	3.8020208	6372	3.8042758
6307	3.7998228	6340	3.8020893	6373	3.8043439
6308	3.7998917	6341	3.8021578	6374	3.8044121
6309	3.7999605	6342	3.8022262	6375	3.8044802
6310	3.8000294	6343	3.8022947	6376	3.8045483
6311	3.8000982	6344	3.8023632	6377	3.8046164
6312	3.8001670	6345	3.8024316	6378	3.8046845
6313	3.8002358	6346	3.8025001	6379	3.8047526
6314	3.8003046	6347	3.8025685	6380	3.8048207
6315	3.8003734	6348	3.8026369	6381	3.8048887
6316	3.8004421	6349	3.8027053	6382	3.8049568
6317	3.8005109	6350	3.8027737	6383	3.8050248
6318	3.8005796	6351	3.8028421	6384	3.8050929
6319	3.8006484	6352	3.8029105	6385	3.8051609
6320	3.8007171	6353	3.8029789	6386	3.8052289
6321	3.8007858	6354	3.8030472	6387	3.8052969
6322	3.8008545	6355	3.8031156	6388	3.8053649
6323	3.8009232	6356	3.8031839	6389	3.8054329
6324	3.8009919	6357	3.8032522	6390	3.8055009
6325	3.8010605	6358	3.8033205	6391	3.8055688
6326	3.8011292	6359	3.8033888	6392	3.8056368
6327	3.8011978	6360	3.8034571	6393	3.8057047
6328	3.8012665	6361	3.8035254	6394	3.8057726
6329	3.8013351	6362	3.8035937	6395	3.8058405
6330	3.8014037	6363	3:8036619	6396	3.8059085
6331	3.8014723	6364	3.8037302	6397	3.8059763
6332	3.8015409	6365	3.8037984	6398	3.8060442
6333	3.8016095	6366	3.8038666	6399	3.8061121
6334	3.8016781	6367	3.8039348	6400	3.8061800

N.	Logarith.	N.	Logarith.	N.	Logarith.
6401	3.8062478	6434	3.8084811	6467	3.8107029
6402	3.8063157	6435	3.8085485	6468	3.8107700
6403	3.8063835	6436	3.8086160	6469	3.8108371
6404	3.8064513	6437	3.8086835	6470	3.8109043
6405	3.8065191	6438	3.8087510	6471	3.8109714
6406	3.8065869	6439	3.8088184	6472	3.8110385
6407	3.8066547	6440	3.8088859	6473	3.8111056
6408	3.8067225	6441	3.8089533	6474	3.8111727
6409	3.8067903	6442	3.8090207	6475	3.8112398
6410	3.8068580	6443	3.8090881	6476	3.8113068
6411	3.8069258	6444	3.8091555	6477	3.8113739
6412	3.8069935	6445	3.8092229	6478	3.8114409
6413	3.8070612	6446	3.8092903	6479	3.8115080
6414	3.8071290	6447	3.8093577	6480	3.8115750
6415	3.8071967	6448	3.8094250	6481	3.8116420
6416	3.8072644	6449	3.8094924	6482	3.8117090
6417	3.8073320	6450	3.8095597	6483	3.8117760
6418	3.8073997	6451	3.8096270	6484	3.8118430
6419	3.8074674	6452	3.8096944	6485	3.8119100
6420	3.8075350	6453	3.8097617	6486	3.8119769
6421	3.8076027	6454	3.8098290	6487	3.8120439
6422	3.8076703	6455	3.8098962	6488	3.8121108
6423	3.8077379	6456	3.8099635	6489	3.8121778
6424	3.8078055	6457	3.8100308	6490	3.8122447
6425	3.8078731	6458	3.8100980	6491	3.8123116
6426	3.8079407	6459	3.8101653	6492	3.8123785
6427	3.8080083	6460	3.8102325	6493	3.8124454
6428	3.8080759	6461	3.8102997	6494	3.8125123
6429	3.8081434	6462	3.8103670	6495	3.8125792
6430	3.8082110	6463	3.8104342	6496	3.8126460
6431	3.8082785	6464	3.8105013	6497	3.8127129
6432	3.8083460	6465	3.8105685	6498	3.8127797
6433	3.8084136	6466	3.8106357	6499	3.8128465
6434	3.8084811	6467	3.8107029	6500	3.8129134

N.	Logarith.	N.	Logarith.	N.	Logarith.
6501	3.8129802	6534	3.8151791	6567	3.8173670
6502	3.8130470	6535	3.8152456	6568	3.8174331
6503	3.8131138	6536	3.8153120	6569	3.8174993
6504	3.8131805	6537	3.8153785	6570	3.8175654
6505	3.8132473	6538	3.8154449	6571	3.8176315
6506	3.8133141	6539	3.8155113	6572	3.8176976
6507	3.8133808	6540	3.8155777	6573	3.8177636
6508	3.8134475	6541	3.8156441	6574	3.8178297
6509	3.8135143	6542	3.8157105	6575	3.8178958
6510	3.8135810	6543	3.8157769	6576	3.8179618
6511	3.8136477	6544	3.8158433	6577	3.8180278
6512	3.8137144	6545	3.8159096	6578	3.8180939
6513	3.8137811	6546	3.8159760	6579	3.8181599
6514	3.8138478	6547	3.8160423	6580	3.8182259
6515	3.8139144	6548	3.8161087	6581	3.8182919
6516	3.8139811	6549	3.8161750	6582	3.8183579
6517	3.8140477	6550	3.8162413	6583	3.8184239
6518	3.8141144	6551	3.8163076	6584	3.8184898
6519	3.8141810	6552	3.8163739	6585	3.8185558
6520	3.8142476	6553	3.8164402	6586	3.8186217
6521	3.8143142	6554	3.8165064	6587	3.8186877
6522	3.8143808	6555	3.8165727	6588	3.8187536
6523	3.8144474	6556	3.8166389	6589	3.8188195
6524	3.8145140	6557	3.8167052	6590	3.8188854
6525	3.8145805	6558	3.8167714	6591	3.8189513
6526	3.8146471	6559	3.8168376	6592	3.8190172
6527	3.8147136	6560	3.8169038	6593	3.8190831
6528	3.8147801	6561	3.8169700	6594	3.8191489
6529	3.8148467	6562	3.8170362	6595	3.8192148
6530	3.8149132	6563	3.8171024	6596	3.8192806
6531	3.8149797	6564	3.8171686	6597	3.8193465
6532	3.8150462	6565	3.8172347	6598	3.8194123
6533	3.8151127	6566	3.8173009	6599	3.8194781
6534	3.8151791	6567	3.8173670	6600	3.8195439

N.	Logarith.	N.	Logarith.	N.	Logarith
6601	3.8196097	6634	3.8217755	6667	3.8239305
6602	3.8196755	6635	3.8218409	6668	3.8239956
6603	3.8197413	6636	3.8219064	6669	3.8240607
6604	3.8198071	6637	3.8219718	6670	3.8241258
6605	3.8198728	6638	3.8220372	6671	3.8241909
6606	3.8199386	6639	3.8221027	6672	3.8242560
6607	3.8200043	6640	3.8221681	6673	3.8243211
6608	3.8200700	6641	3.8222335	6674	3.8243862
6609	3.8201358	6642	3.8222989	6675	3.8244513
6610	3.8202015	6643	3.8223643	6676	3.8245163
6611	3.8202672	6644	3.8224296	6677	3.8245814
6612	3.8203328	6645	3.8224950	6678	3.8246464
6613	3.8203985	6646	3.8225603	6679	3.8247114
6614	3.8204642	6647	3.8226257	6680	3.8247765
6615	3.8205298	6648	3.8226910	6681	3.8248415
6616	3.8205955	6649	3.8227563	6682	3.8249065
6617	3.8206611	6650	3.8228216	6683	3.8249715
6618	3.8207268	6651	3.8228869	6684	3.8250364
6619	3.8207924	6652	3.8229522	6685	3.8251014
6620	3.8208580	6653	3.8230175	6686	3.8251664
6621	3.8209236	6654	3.8230828	6687	3.8252313
6622	3.8209892	6655	3.8231481	6688	3.8252963
6623	3.8210548	6656	3.8232133	6689	3.8253612
6624	3.8211203	6657	3.8332786	6690	3.8254261
6625	3.8211859	6658	3.8233438	6691	3.8254910
6626	3.8212514	6659	3.8234090	6692	3.8255559
6627	3.8213170	6660	3.8234742	6693	3.8256208
6628	3.8213825	6661	3.8235394	6694	3.8256857
6629	3.8214480	6662	3.8236046	6695	3.8257506
6630	3.8215135	6663	3.8236698	6696	3.8258154
6631	3.8215790	6664	3.8237350	6697	3.8258803
6632	3.8216445	6665	3.8238002	6698	3.8259451
6633	3.8217100	6666	3.8238653	6699	3.8260100
6634	3.8217755	6667	3.8239305	6700	3.8260748

N.	Logarith.	N.	Logarith.	N.	Logarith.
6701	3.8261396	6734	3.8282731	6767	3.8303962
6702	3.8262044	6735	3.8283376	6768	3.8304603
6703	3.8262692	6736	3.8284021	6769	3.8305245
6704	3.8263340	6737	3.8284665	6770	3.8305887
6705	3.8263988	6738	3.8285310	6771	3.8306528
6706	3.8264635	6739	3.8285955	6772	3.8307169
6707	3.8265283	6740	3.8286599	6773	3.8307811
6708	3.8265931	6741	3.8287243	6774	3.8308452
6709	3.8266578	6742	3.8287887	6775	3.8309093
6710	3.8267225	6743	3.8288532	6776	3.8309734
6711	3.8267872	6744	3.8289176	6777	3.8310375
6712	3.8268519	6745	3.8289820	6778	3.8311016
6713	3.8269166	6746	3.8290463	6779	3.8311656
6714	3.8269813	6747	3.8291107	6780	3.8312297
6715	3.8270460	6748	3.8291751	6781	3.8312937
6716	3.8271107	6749	3.8292394	6782	3.8313678
6717	3.8271753	6750	3.8293038	6783	3.8314218
6718	3.8272400	6751	3.8293681	6784	3.8314858
6719	3.8273046	6752	3.8294324	6785	3.8315499
6720	3.8273693	6753	3.8294967	6786	3.8316139
6721	3.8274339	6754	3.8295611	6787	3.8316778
6722	3.8274985	6755	3.8296254	6788	3.8317418
6723	3.8275631	6756	3.8296896	6789	3.8318058
6724	3.8276277	6757	3.8297539	6790	3.8318698
6725	3.8276923	6758	3.8298182	6791	3.8319337
6726	3.8277569	6759	3.8298824	6792	3.8319977
6727	3.8278214	6760	3.8299467	6793	3.8320616
6728	3.8278860	6761	3.8300109	6794	3.8321255
6729	3.8279505	6762	3.8300752	6795	3.8321895
6730	3.8280151	6763	3.8301394	6796	3.8322534
6731	3.8280796	6764	3.8302036	6797	3.8323173
6732	3.8281441	6765	3.8302678	6798	3.8323812
6733	3.8282086	6766	3.8303320	6799	3.8324450
6734	3.8282731	6767	3.8303962	6800	3.8325089

N.	Logarith.	N.	Logarith.	N.	Logarith.
6801	3.8325728	6834	3.8346750	6867	3.8367670
6802	3.8326366	6835	3.8347385	6868	3.8368303
6803	3.8327005	6836	3.8348021	6869	3.8368935
6804	3.8327643	6837	3.8348656	6870	3.8369567
6805	3.8328281	6838	3.8349291	6871	3.8370199
6806	3.8328919	6839	3.8349926	6872	3.8370832
6807	3.8329558	6840	3.8350561	6873	3.8371463
6808	3.8330195	6841	3.8351196	6874	3.8372095
6809	3.8330833	6842	3.8351831	6875	3.8372727
6810	3.8331471	6843	3.8352465	6876	3.8373359
6811	3.8332109	6844	3.8353100	6877	3.8373990
6812	3.8332746	6845	3.8353735	6878	3.8374622
6813	3.8333384	6846	3.8354369	6879	3.8375253
6814	3.8334021	6847	3.8355003	6880	3.8375884
6815	3.8334659	6848	3.8355638	6881	3.8376516
6816	3.8335296	6849	3.8356272	6882	3.8377147
6817	3.8335933	6850	3.8356906	6883	3.8377778
6818	3.8336570	6851	3.8357540	6884	3.8378409
6819	3.8337207	6852	3.8358174	6885	3.8379039
6820	3.8337844	6853	3.8358807	6886	3.8379670
6821	3.8338480	6854	3.8359441	6887	3.8380301
6822	3.8339117	6855	3.8360075	6888	3.8380931
6823	3.8339754	6856	3.8360708	6889	3.8381562
6824	3.8340390	6857	3.8361341	6890	3.8382192
6825	3.8341027	6858	3.8361975	6891	3.8382822
6826	3.8341663	6859	3.8362608	6892	3.8383453
6827	3.8342299	6860	3.8363241	6893	3.8384083
6828	3.8342935	6861	3.8363874	6894	3.8384713
6829	3.8343571	6862	3.8364507	6895	3.8385343
6830	3.8344207	6863	3.8365140	6896	3.8385973
6831	3.8344843	6864	3.8365773	6897	3.8386602
6832	3.8345479	6865	3.8366405	6898	3.8387232
6833	3.8346114	6866	3.8367038	6899	3.8387861
6834	3.8346750	6867	3.8367670	6900	3.8388491

N.	Logarith.	N.	Logarith.	N.	Logarith.
6901	3.8389120	6934	3.8409838	6967	3.8430458
6902	3.8389750	6935	3.8410465	6968	3.8431081
6903	3.8390379	6936	3.8411091	6969	3.8431705
6904	3.8391008	6937	3.8411717	6970	3.8432328
6905	3.8391637	6938	3.8412343	6971	3.8432951
6906	3.8392266	6939	3.8412969	6972	3.8433574
6907	3.8392895	6940	3.8413595	6973	3.8434197
6908	3.8393523	6941	3.8414220	6974	3.8434819
6909	3.8394152	6942	3.8414846	6975	3.8435442
6910	3.8394780	6943	3.8415472	6976	3.8436065
6911	3.8395409	6944	3.8416097	6977	3.8436687
6912	3.8396037	6945	3.8416722	6978	3.8437310
6913	3.8396666	6946	3.8417348	6979	3 8437932
6914	3.8397294	6947	3.8417973	6980	3.8438554
6915	3.8397922	6948	3.8418598	6981	3.8439176
6916	3.8398550	6949	3.8419223	6982	3.8439798
6917	3.8399178	6950	3.8419848	6983	3.8440420
6918	3.8399806	6951	3.8420473	6984	3.8441042
6919	3.8400433	6952	3.8421098	6985	3.8441664
6920	3.8401061	6953	3.8421722	6986	3.8442286
6921	3.8401688	6954	3.8422347	6987	3.8442907
6922	3.8402316	6955	3.8422971	6988	3.8443529
6923	3.8402943	6956	3.8423596	6989	3.8444150
6924	3.8403571	6957	3.8424220	6990	3.8444772
6925	3.8404198	6958	3.8424844	6991	3.8445393
6926	3.8404825	6959	3.8425468	6992	3.8446014
6927	3.8405452	6960	3.8426092	6993	3.8446635
6928	3.8406079	6961	3.8426716	6994	3.8447256
6929	3.8406706	6962	3.8427340	6995	3.8447877
6930	3.8407332	6963	3.8427964	6996	3.8448498
6931	3.8407959	6964	3.8428588	6997	3.8449119
6932	3.8408586	6965	3.8429211	6998	3.8449739
6933	3.8409212	6966	3.8429835	6999	3.8450360
6934	3.8409838	6967	3.8430458	7000	3.8450980

N.	Logarith.	N.	Logarith.	N.	Logarith.
7001	3.8451601	7034	3.847202[illegible]	7067	3.8492351
7002	3.8452221	7035	3.8472641	7068	3.8492965
7003	3.8452841	7036	3.8473258	7069	3.8493580
7004	3.8453461	7037	3.8473876	7070	3.8494194
7005	3.8454081	7038	3.8474493	7071	3.8494808
7006	3.8454701	7039	3.8475110	7072	3.8495423
7007	3.8455321	7040	3.8475727	7073	3.8496037
7008	3.8455941	7041	3.8476343	7074	3.8496651
7009	3.8456561	7042	3.8476960	7075	3.8497264
7010	3.8457180	7043	3.8477577	7076	3.8497878
7011	3.8457800	7044	3.8478193	7077	3.8498492
7012	3.8458419	7045	3.8478810	7078	3.8499106
7013	3.8459038	7046	3.8479426	7079	3.8499719
7014	3.8459658	7047	3.8480043	7080	3.8500333
7015	3.8460277	7048	3.8480659	7081	3.8500946
7016	3.8460896	7049	3.8481275	7082	3.8501559
7017	3.8461515	7050	3.8481891	7083	3.8502172
7018	3.8462134	7051	3.8482507	7084	3.8502786
7019	3.8462752	7052	3.8483123	7085	3.8503399
7020	3.8463371	7053	3.8483739	7086	3.8504011
7021	3.8463990	7054	3.8484355	7087	3.8504624
7022	3.8464608	7055	3.8484970	7088	3.8505237
7023	3.8465227	7056	3.8485586	7089	3.8505850
7024	3.8465845	7057	3.8486201	7090	3.8506462
7025	3.8466463	7058	3.8486817	7091	3.8507075
7026	3.8467081	7059	3.8487432	7092	3.8507687
7027	3.8467700	7060	3.8488047	7093	3.8508300
7028	3.8468318	7061	3.8488662	7094	3.8508912
7029	3.8468935	7062	3.8489277	7095	3.8509524
7030	3.8469553	7063	3.8489892	7096	3.8510136
7031	3.8470171	7064	3.8490507	7097	3.8510748
7032	3.8470789	7065	3.8491122	7098	3.8511360
7033	3.8471406	7066	3.8491736	7099	3.8511972
7034	3.8472024	7067	3.8492351	7100	3.8512583

N.	Logarith.	N.	Logarith.	N.	Logarith.
7101	3.8513195	7134	3.8533331	7167	3.8553374
7102	3.8513807	7135	3.8533940	7168	3.8553980
7103	3.8514418	7136	3.8534548	7169	3.8554586
7104	3.8515030	7137	3.8535157	7170	3.8555192
7105	3.8515641	7138	3.8535765	7171	3.8555797
7106	3.8516252	7139	3.8536374	7172	3.8556403
7107	3.8516863	7140	3.8536982	7173	3.8557008
7108	3.8517474	7141	3.8537590	7174	3.8557614
7109	3.8518085	7142	3.8538198	7175	3.8558219
7110	3.8518696	7143	3.8538806	7176	3.8558824
7111	3.8519307	7144	3.8539414	7177	3.8550429
7112	3.8519917	7145	3.8540022	7178	3.8560035
7113	3.8520528	7146	3.8540630	7179	3.8560640
7114	3.8521139	7147	3.8541238	7180	3.8561244
7115	3.8521749	7148	3.8541845	7181	3.8561849
7116	3.8522359	7149	3.8542453	7182	3.8562454
7117	3.8522970	7150	3.8543060	7183	3.8563059
7118	3.8523580	7151	3.8543668	7184	3.8563663
7119	3.8524190	7152	3.8544275	7185	3.8564268
7120	3.8524800	7153	3.8544882	7186	3.8564872
7121	3.8525410	7154	3.8545489	7187	3.8565476
7122	3.8526020	7155	3.8546096	7188	3.8566081
7123	3.8526629	7156	3.8546703	7189	3.8566685
7124	3.8527239	7157	3.8547310	7190	3.8567289
7125	3.8527849	7158	3.8547917	7191	3.8567893
7126	3.8528458	7159	3.8548524	7192	3.8568497
7127	3.8529068	7160	3.8549130	7193	3.8569101
7128	3.8529677	7161	3.8549737	7194	3.8569704
7129	3.8530286	7162	3.8550343	7195	3.8570308
7130	3.8530895	7163	3.8550949	7196	3.8570912
7131	3.8531504	7164	3.8551556	7197	3.8571515
7132	3.8532113	7165	3.8552162	7198	3.8572118
7133	3.8532722	7166	3.8552768	7199	3.8572722
7134	3.8533331	7167	3.8553374	7200	3.8573325

N.	Logarith.	N.	Logarith.	N.	Logarith.
7201	3.8573928	7234	3.8593785	7267	3.8613552
7202	3.8574531	7235	3.8594385	7268	3.8614149
7203	3.8575134	7236	3.8594986	7269	3.8614747
7204	3.8575737	7237	3.8595586	7270	3.8615344
7205	3.8576340	7238	3.8596186	7271	3.8615941
7206	3.8576943	7239	3.8596786	7272	3.8616539
7207	3.8577545	7240	3.8597386	7273	3.8617136
7208	3.8578148	7241	3.8597985	7274	3.8617733
7209	3.8578750	7242	3.8598585	7275	3.8618330
7210	3.8579353	7243	3.8599185	7276	3.8618927
7211	3.8570955	7244	3.8599784	7277	3.8619524
7212	3.8580557	7245	3.8600384	7278	3.8620120
7213	3.8581159	7246	3.8600983	7279	3.8620717
7214	3.8581761	7247	3.8601583	7280	3.8621314
7215	3.8582363	7248	3.8602182	7281	3.8621910
7216	3.8582965	7249	3.8602781	7282	3.8622507
7217	3.8583567	7250	3.8603380	7283	3.8623103
7218	3.8584169	7251	3.8603979	7284	3.8623699
7219	3.8584770	7252	3.8604578	7285	3.8624296
7220	3.8585372	7253	3.8605177	7286	3.8624892
7221	3.8585973	7254	3.8605776	7287	3.8625488
7222	3.8586575	7255	3.8606374	7288	3.8626084
7223	3.8587176	7256	3.8606973	7289	3.8626679
7224	3.8587777	7257	3.8607571	7290	3.8627275
7225	3.8588379	7258	3.8608170	7291	3.8627871
7226	3.8588980	7259	3.8608768	7292	3.8628467
7227	3.8589581	7260	3.8609366	7293	3.8629062
7228	3.8590181	7261	3.8609964	7294	3.8629658
7229	3.8590782	7262	3.8610562	7295	3.8630253
7230	3.8591383	7263	3.8611160	7296	3.8630848
7231	3.8591984	7264	3.8611758	7297	3.8631443
7232	3.8592584	7265	3.8612356	7298	3.8632039
7233	3.8593185	7266	3.8612954	7299	3.8632634
7234	3.8593785	7267	3.8613552	7300	3.8633229

N.	Logarith.	N.	Logarith.	N.	Logarith.
7301	3.8633823	7334	3.8653409	7367	3.8672907
7302	3.8634418	7335	3.8654001	7368	3.8673496
7303	3.8635013	7336	3.8654593	7369	3.8674086
7304	3.8635608	7337	3.8655185	7370	3.8674675
7305	3.8636202	7338	3.8655777	7371	3.8675264
7306	3.8636797	7339	3.8656369	7372	3.8675853
7307	3.8637391	7340	3.8656961	7373	3.8676442
7308	3.8637985	7341	3.8657552	7374	3.8677031
7309	3.8638580	7342	3.8658144	7375	3.8677620
7310	3.8639174	7343	3.86[illegible]8735	7376	3.8678209
7311	3.8639768	7344	3.8659327	7377	3.8678798
7312	3.8640362	7345	3.8659918	7378	3.8679387
7313	3.8640956	7346	3.8660509	7379	3.8679975
7314	3.8641550	7347	3.8661100	7380	3.8680564
7315	3.8642143	7348	3.8661691	7381	3.8681152
7316	3.8642737	7349	3.8662282	7382	3.8681740
7317	3.8643331	7350	3.8662873	7383	3.8682329
7318	3.8643924	7351	3.8663464	7384	3.8682917
7319	3.8644517	7352	3.8664055	7385	3.8683505
7320	3.8645111	7353	3.8664646	7386	3.8684093
7321	3.8645704	7354	3.8665236	7387	3.8684681
7322	3.8646297	7355	3.8665827	7388	3.8685269
7323	3.8646890	7356	3.8666417	7389	3.8685857
7324	3.8647483	7357	3.8667008	7390	3.8686444
7325	3.8648076	7358	3.8667598	7391	3.8687032
7326	3.8648669	7359	3.8668188	7392	3.8687620
7327	3.8649262	7360	3.8668778	7393	3.8688207
7328	3.8649855	7361	3.8669368	7394	3.8688794
7329	3.8650447	7362	3.8669958	7395	3.8689382
7330	3.8651040	7363	3.8670548	7396	3.8689969
7331	3.8651632	7364	3.8671138	7397	3.8690556
7332	3.8652225	7365	3.8671728	7398	3.86[illegible]143
7333	3.8652817	7366	3.8672317	7399	3.8691730
7334	3.8653409	7367	3.8672907	7400	3.8692317

N.	Logarith.	N.	Logarith.	N.	Logarith.
7401	3.8692904	7434	3.8712226	7467	3.8731461
7402	3.8693491	7435	3.8712810	7468	3.8732043
7403	3.8694077	7436	3.8713394	7469	3.8732625
7404	3.8694664	7437	3.8713978	7470	3.8733206
7405	3.8695251	7438	3.8714562	7471	3.8733787
7406	3.8695837	7439	3.8715146	7472	3.8734369
7407	3.8696423	7440	3.8715729	7473	3.8734950
7408	3.8697010	7441	3.8716313	7474	3.8735531
7409	3.8697596	7442	3.8716897	7475	3.8736112
7410	3.8698182	7443	3.8717480	7476	3.8736693
7411	3.8698768	7444	3.8718064	7477	3.8737274
7412	3.8699354	7445	3.8718647	7478	3.8737855
7413	3.8699940	7446	3.8719230	7479	3.8738435
7414	3.8700526	7447	3.8719814	7480	3.8739016
7415	3.8701112	7448	3.8720397	7481	3.8739597
7416	3.8701697	7449	3.8720980	7482	3.8740177
7417	3.8702283	7450	3.8721563	7483	3.8740757
7418	3.8702868	7451	3.8722146	7484	3.8741338
7419	3.8703454	7452	3.8722728	7485	3.8741918
7420	3.8704039	7453	3.8723311	7486	3.8742498
7421	3.8704624	7454	3.8723894	7487	3.8743078
7422	3.8705209	7455	3.8724476	7488	3.8743658
7423	3.8705795	7456	3.8725059	7489	3.8744238
7424	3.8706380	7457	3.8725641	7490	3.8744818
7425	3.8706965	7458	3.8726224	7491	3.8745398
7426	3.8707549	7459	3.8726806	7492	3.8745978
7427	3.8708134	7460	3.8727388	7493	3.8746557
7428	3.8708719	7461	3.8727970	7494	3.8747137
7429	3.8709304	7462	3.8728552	7495	3.8747716
7430	3.8709888	7463	3.8729134	7496	3.8748296
7431	3.8710473	7464	3.8739716	7497	3.8748875
7432	3.8711057	7465	3.8730298	7498	3.8749454
7433	3.8711641	7466	3.8730880	7499	3.8750034
7434	3.8712226	7467	3.8731461	7500	3.8750613

N.	Logarith.	N.	Logarith.	N.	Logarith.
7501	3.8751192	7534	3.8770256	7567	3.8789237
7502	3.8755771	7535	3.8770833	7568	3.8789811
7503	3.8752349	7536	3.8771409	7569	3.8790385
7504	3.8752928	7537	3.8771985	7570	3.8790959
7505	3.8753507	7538	3.8772561	7571	3.8791532
7506	3.8754086	7539	3.8773137	7572	3.8792106
7507	3.8754664	7540	3.8773713	7573	3.8792680
7508	3.8755243	7541	3.8774289	7574	3.8793253
7509	3.8755821	7542	3.8774865	7575	3.8793826
7510	3.8756399	7543	3.8775441	7576	3.8794400
7511	3.8756978	7544	3.8776017	7577	3.8794973
7512	3.8757556	7545	3.8776592	7578	3.8795546
7513	3.8758134	7546	3.8777168	7579	3.8796119
7514	3.8758712	7547	3.8777743	7580	3.8796692
7515	3.8759290	7548	3.8778319	7581	3.8797265
7516	3.8759868	7549	3.8778894	7582	3.8797838
7517	3.8760445	7550	3.8779469	7583	3.8798411
7518	3.8761023	7551	3.8780045	7584	3.8798983
7519	3.8761601	7552	3.8780620	7585	3.8799556
7520	3.8762178	7553	3.8781195	7586	3.8800128
7521	3.8762756	7554	3.8781770	7587	3.8800701
7522	3.8763333	7555	3.8782345	7588	3.8801273
7523	3.8763911	7556	3.8782919	7589	3.8801846
7524	3.8764488	7557	3.8783494	7590	3.8802418
7525	3.8765065	7558	3.8784069	7591	3.8802990
7526	3.8765642	7559	3.8784643	7592	3.8803562
7527	3.8766219	7560	3.8785218	7593	3.8804134
7528	3.8766796	7561	3.8785792	7594	3.8804706
7529	3.8767373	7562	3.8786367	7595	3.8805278
7530	3.8767950	7563	3.8786941	7596	3.8805850
7531	3.8768526	7564	3.8787515	7597	3.8806421
7532	3.8769103	7565	3.8788089	7598	3.8806993
7533	3.8769680	7566	3.8788663	7599	3.8807564
7534	3.8770256	7567	3.8789237	7600	3.8808136

N	Logarith.	N.	Logarith.	N.	Logarith.
7601	3.8808707	7634	3.8827522	7667	3.8846255
7602	3.8809279	7635	3.8828090	7668	3.8846821
7603	3.8809850	7636	3.8828659	7669	3.8847387
7604	3.8810421	7637	3.8829228	7670	3.8847954
7605	3.8810992	7638	3.8829797	7671	3.8848520
7606	3.8811563	7639	3.8830365	7672	3.8849086
7607	3.8812134	7640	3.8830934	7673	3.8849652
7608	3.8812705	7641	3.8831502	7674	3.8850218
7609	3.8813276	7642	3.8832070	7675	3.8850784
7610	3.8813847	7643	3.8832639	7676	3.8851350
7611	3.8814417	7644	3.8833207	7677	3.8851915
7612	3.8814988	7645	3.8833775	7678	3.8852481
7613	3.8815558	7646	3.8834343	7679	3.8853047
7614	3.8816129	7647	3.8834911	7680	3.8853612
7615	3.8816699	7648	3.8835479	7681	3.8854178
7616	3.8817269	7649	3.8836047	7682	3.8854743
7617	3.8817849	7650	3.8836614	7683	3.8855308
7618	3.8818410	7651	3.8837182	7684	3.8855874
7619	3.8818980	7652	3.8837750	7685	3.8856439
7620	3.8819550	7653	3.8838317	7686	3.8857004
7621	3.8820120	7654	3.8838885	7687	3.8857569
7622	3.8820689	7655	3.8839452	7688	3.8858134
7623	3.8821259	7656	3.8840019	7689	3.8858699
7624	3.8821829	7657	3.8840586	7690	3.8859263
7625	3.8822398	7658	3.8841154	7691	3.8859828
7626	3.8822968	7659	3.8841721	7692	3.8860393
7627	3.8823537	7660	3.8842288	7693	3.8860957
7628	3.8824107	7661	3.8842855	7694	3.8861522
7629	3.8824676	7662	3.8843421	7695	3.8862086
7630	3.8825245	7663	3.8843988	7696	3.8862651
7631	3.8825815	7664	3.8844555	7697	3.8863215
7632	3.8826384	7665	3.8845122	7698	3.8863779
7633	3.8826953	7666	3.8845688	7699	3.8864343
7634	3.8827522	7667	3.8846255	7700	3.8864907

N.	Logarith.	N.	Logarith.	N.	Logarith.
7701	3.8865471	7734	3.8884042	7767	3.8902533
7702	3.8866035	7735	3.8884603	7768	3.8903092
7703	3.8866599	7736	3.8885165	7769	3.8903651
7704	3.8867163	7737	3.8885726	7770	3.8904210
7705	3.8867726	7738	3.8886287	7771	3.8904769
7706	3.8868290	7739	3.8886848	7772	3.8905328
7707	3.8868854	7740	3.8887410	7773	3.8905887
7708	3.8869417	7741	3.8887971	7774	3.8906445
7709	3.8869980	7742	3.8888532	7775	3.8907004
7710	3.8870544	7743	3.8889093	7776	3.8907563
7711	3.8871107	7744	3.8889653	7777	3.8908121
7712	3.8871670	7745	3.8890214	7778	3.8908679
7713	3.8872233	7746	3.8890775	7779	3.8909238
7714	3.8872796	7747	3.8891336	7780	3.8909796
7715	3.8873359	7748	3.8891896	7781	3.8910354
7716	3.8873922	7749	3.8892457	7782	3.8910912
7717	3.8874485	7750	3.8893017	7783	3.8911470
7718	3.8875048	7751	3.8893577	7784	3.8912028
7719	3.8875610	7752	3.8894138	7785	3.8912586
7720	3.8876173	7753	3.8894698	7786	3.8913144
7721	3.8876736	7754	3.8895258	7787	3.8913702
7722	3.8877298	7755	3.8895818	7788	3.8914259
7723	3.8877860	7756	3.8896378	7789	3.8914817
7724	3.8878423	7757	3.8896938	7790	3.8915375
7725	3.8878985	7758	3.8897498	7791	3.8915932
7726	3.8879547	7759	3.8898058	7792	3.8916489
7727	3.8880109	7760	3.8898617	7793	3.8917047
7728	3.8880671	7761	3.8899177	7794	3.8917604
7729	3.8881233	7762	3.8899736	7795	3.8918161
7730	3.8881795	7763	3.8900296	7796	3.8918718
7731	3.8882357	7764	3.8900855	7797	3.8919275
7732	3.8882918	7765	3.8901415	7798	3.8919832
7733	3.8883480	7766	3.8901974	7799	3.8920389
7734	3.8884042	7767	3.8902533	7800	3.8920946

N.	Logarith.	N.	Logarith.	N.	Logarith.
7801	3.8921503	7834	3.8939836	7867	3.8958091
7802	3.8922059	7835	3.8940390	7868	3.8958644
7803	3.8922616	7836	3.8940944	7869	3.8959195
7804	3.8923173	7837	3.8941498	7870	3.8959747
7805	3.8923729	7838	3.8942053	7871	3.8960299
7806	3.8924285	7839	3.8942607	7872	3.8960851
7807	3.8924842	7840	3.8943161	7873	3.8961403
7808	3.8925398	7841	3.8943715	7874	3.8961954
7809	3.8925954	7842	3.8944268	7875	3.8962506
7810	3.8926510	7843	3.8944822	7876	3.8963057
7811	3.8927066	7844	3.8945376	7877	3.8963608
7812	3.8927622	7845	3.8945929	7878	3.8964160
7813	3.8928178	7846	3.8946483	7879	3.8964711
7814	3.8928734	7847	3.8947037	7880	3.8965262
7815	3.8929290	7848	3.8947590	7881	3.8965813
7816	3.8929846	7849	3.8948143	7882	3.8966364
7817	3.8930401	7850	3.8948697	7883	3.8966915
7818	3.8930957	7851	3.8949250	7884	3.8967466
7819	3.8931512	7852	3.8949803	7885	3.8968017
7820	3.8932068	7853	3.8950356	7886	3.8968568
7821	3.8932623	7854	3.8950909	7887	3.8969118
7822	3.8933178	7855	3.8951462	7888	3.8969669
7823	3.8933733	7856	3.8952015	7889	3.8970220
7824	3.8934288	7857	3.8952568	7890	3.8970770
7825	3.8934843	7858	3.8953120	7891	3.8971320
7826	3.8935398	7859	3.8953673	7892	3.8971871
7827	3.8935953	7860	3.8954225	7893	3.8972421
7828	3.8936508	7861	3.8954778	7894	3.8972971
7829	3.8937063	7862	3.8955330	7895	3.8973521
7830	3.8937618	7863	3.8955883	7896	3.8974071
7831	3.8938172	7864	3.8956435	7897	3.8974621
7832	3.8938727	7865	3.8956987	7898	3.8975171
7833	3.8939281	7866	3.8957539	7899	3.8975721
7834	3.8939836	7867	3.8958091	7900	3.8976271

N	Logarith.	N	Logarith.	N	Logarith.
7901	3.8976821	7934	3.8994922	7967	3.9012948
7902	3.8977370	7935	3.8995469	7968	3.9013493
7903	3.8977920	7936	3.8996017	7969	3.9014038
7904	3.8978469	7937	3.8996564	7970	3.9014583
7905	3.8979019	7938	3.8997111	7971	3.9015128
7906	3.8979568	7939	3.8997658	7972	3.9015673
7907	3.8980117	7940	3.8998205	7973	3.9016218
7908	3.8980667	7941	3.8998752	7974	3.9016762
7909	3.8981216	7942	3.8999229	7975	3.9017307
7910	3.8981765	7943	3.8999846	7976	3.9017851
7911	3.8982314	7944	3.9000392	7977	3.9018396
7912	3.8982863	7945	3.9000939	7978	3.9018940
7913	3.8983412	7946	3.9001486	7979	3.9019485
7914	3.8983960	7947	3.9002032	7980	3.9020029
7915	3.8984509	7948	3.9002579	7981	3.9020573
7916	3.8985058	7949	3.9003125	7982	3.9021117
7917	3.8985606	7950	3.9003671	7983	3.9021661
7918	3.8986155	7951	3.9004218	7984	3.9022205
7919	3.8986703	7952	3.9004764	7985	3.9022749
7920	3.8987252	7953	3.9005310	7986	3.9023293
7921	3.8987800	7954	3.9005856	7987	3.9023837
7922	3.8988348	7955	3.9006402	7988	3.9024381
7923	3.8988897	7956	3.9006948	7989	3.9024924
7924	3.8989445	7957	3.9007494	7990	3.9025468
7925	3.8989993	7958	3.9008039	7991	3.9026011
7926	3.8990541	7959	3.9008585	7992	3.9026555
7927	3.8991089	7960	3.9009131	7993	3.9027098
7928	3.8991636	7961	3.9009676	7994	3.9027641
7929	3.8992184	7962	3.9010222	7995	3.9028185
7930	3.8992732	7963	3.9010767	7996	3.9028728
7931	3.8993279	7964	3.9011313	7997	3.9029271
7932	3.8993827	7965	3.9011858	7998	3.9029814
7933	3.8994375	7966	3.9012403	7999	3.9030357
7934	3.8994922	7967	3.9012948	8000	3.9030900

N.	Logarith.	N.	Logarith.	N.	Logarith.
8001	3.9031443	8034	3.9049318	8067	3.9067121
8002	3.9031985	8035	3.9049859	8068	3.9067659
8003	3.9032528	8036	3.9050399	8069	3.9068197
8004	3.9033071	8037	3.9050940	8070	3.9068735
8005	3.9033613	8038	3.9051480	8071	3.9069273
8006	3.9034156	8039	3.9052020	8072	3.9069812
8007	3.9034698	8040	3.9052560	8073	3.9070350
8008	3.9035241	8041	3.9053101	8074	3.9070887
8009	3.9035783	8042	3.9053641	8075	3.9071425
8010	3.9036325	8043	3.9054181	8076	3.9071963
8011	3.9036867	8044	3.9054721	8077	3.9072501
8012	3.9037409	8045	3.9055260	8078	3.9073038
8013	3.9037951	8046	3.9055800	8079	3.9073576
8014	3.9038493	8047	3.9056340	8080	3.9074114
8015	3.9039035	8048	3.9056880	8081	3.9074651
8016	3.9039577	8049	3.9057419	8082	3.9075188
8017	3.9040119	8050	3.9057959	8083	3.9075726
8018	3.9040661	8051	3.9058498	8084	3.9076263
8019	3.9041202	8052	3.9059038	8085	3.9076800
8020	3.9041744	8053	3.9059577	8086	3.9077337
8021	3.9042285	8054	3.9060116	8087	3.9077874
8022	3.9042827	8055	3.9060655	8088	3.9078411
8023	3.9043368	8056	3.9061195	8089	3.9078948
8024	3.9043909	8057	3.9061734	8090	3.9079485
8025	3.9044450	8058	3.9062273	8091	3.9080022
8026	3.9044992	8059	3.9062812	8092	3.9080559
8027	3.9045533	8060	3.9063350	8093	3.9081095
8028	3.9046074	8061	3.9063889	8094	3.9081632
8029	3.9046615	8062	3.9064428	8095	3.9082169
8030	3.9047155	8063	3.9064967	8096	3.9082705
8031	3.9047696	8064	3.9065505	8097	3.9083241
8032	3.9048237	8065	3.9066044	8098	3.9083778
8033	3.9048778	8066	3.9066582	8099	3.9084314
8034	3.9049318	8067	3.9067121	8100	3.9084850

N.	Logarith.	N.	Logarith.	N.	Logarith.
8101	3.9085386	8134	3.9103042	8167	3.9120626
8102	3.9085922	8135	3.9103576	8168	3.9121157
8103	3.9086458	8136	3.9104109	8169	3.9121689
8104	3.9086994	8137	3.9104643	8170	3.9122221
8105	3.9087530	8138	3.9105177	8171	3.9122752
8106	3.9088066	8139	3.9105710	8172	3.9123284
8107	3.9088602	8140	3.9106244	8173	3.9123815
8108	3.9089137	8141	3.9106778	8174	3.9124346
8109	3.9089673	8142	3.9107311	8175	3.9124878
8110	3.9090209	8143	3.9107844	8176	3.9125409
8111	3.9090744	8144	3.9108378	8177	3.9125940
8112	3.9091279	8145	3.9108911	8178	3.9126471
8113	3.9091815	8146	3.9109444	8179	3.9127002
8114	3.9092350	8147	3.9109977	8180	3.9127533
8115	3.9092885	8148	3.9110510	8181	3.9128064
8116	3.9093420	8149	3.9111043	8182	3.9128595
8117	3.9093955	8150	3.9111576	8183	3.9129126
8118	3.9094490	8151	3.9112109	8184	3.9129656
8119	3.9095025	8152	3.9112642	8185	3.9130187
8120	3.9095560	8153	3.9113174	8186	3.9130717
8121	3.9096095	8154	3.9113707	8187	3.9131248
8122	3.9096630	8155	3.9114240	8188	3.9131778
8123	3.9097165	8156	3.9114772	8189	3.9132309
8124	3.9097699	8157	3.9115305	8190	3.9132839
8125	3.9098234	8158	3.9115838	8191	3.9133369
8126	3.9098768	8159	3.9116369	8192	3.9133899
8127	3.9099303	8160	3.9116902	8193	3.9134430
8128	3.9099837	8161	3.9117434	8194	3.9134960
8129	3.9100371	8162	3.9117966	8195	3.9135490
8130	3.9100905	8163	3.9118498	8196	3.9136019
8131	3.9101440	8164	3.9119030	8197	3.9136549
8132	3.9101974	8165	3.9119562	8198	3.9137079
8133	3.9102508	8166	3.9120094	8199	3.9137609
8134	3.9103042	8167	3.9120626	8200	3.9138139

N.	Logarith.	N.	Logarith.	N.	Logarith.
8201	3.9138668	8234	3.9156109	8267	3.9173479
8202	3.9139198	8235	3.9156636	8268	3.9174005
8203	3.9139727	8236	3.9157163	8269	3.9174530
8204	3.9140257	8237	3.9157691	8270	3.9175055
8205	3.9140786	8238	3.9158218	8271	3.9175580
8206	3.9141315	8239	3.9158745	8272	3.9176105
8207	3.9141844	8240	3.9159272	8273	3.9176630
8208	3.9142373	8241	3.9159799	8274	3.9177155
8209	3.9142903	8242	3.9160326	8275	3.9177680
8210	3.9143432	8243	3.9160853	8276	3.9178205
8211	3.9143961	8244	3.9161380	8277	3.9178730
8212	3.9144489	8245	3.9161907	8278	3.9179254
8213	3.9145018	8246	3.9162433	8279	3.9179779
8214	3.9145547	8247	3.9162960	8280	3.9180303
8215	3.9146076	8248	3.9163487	8281	3.9180828
8216	3.9146604	8249	3.9164013	8282	3.9181352
8217	3.9147133	8250	3.9164539	8283	3.9181877
8218	3.9147661	8251	3.9165066	8284	3.9182401
8219	3.9148190	8252	3.9165592	8285	3.9182925
8220	3.9148718	8253	3.9166118	8286	3.9183449
8221	3.9149246	8254	3.9166645	8287	3.9183973
8222	3.9149775	8255	3.9167171	8288	3.9184497
8223	3.9150303	8256	3.9167697	8289	3.9185021
8224	3.9150831	8257	3.9168223	8290	3.9185545
8225	3.9151359	8258	3.9168749	8291	3.9186069
8226	3.9151887	8259	3.9169275	8292	3.9186593
8227	3.9152415	8260	3.9169800	8293	3.9187117
8228	3.9152943	8261	3.9170326	8294	3.9187640
8229	3.9153471	8262	3.9170852	8295	3.9188164
8230	3.9153998	8263	3.9171378	8296	3.9188687
8231	3.9154526	8264	3.9171903	8297	3.9189211
8232	3.9155054	8265	3.9172429	8298	3.9189734
8233	3.9155581	8266	3.9172954	8299	3.9190258
8234	3.9156109	8267	3.9173479	8300	3.9190781

N.	Logarith.	N.	Logarith.	N.	Logarith.
8301	3.9191304	8334	3.9208535	8367	3.9225698
8302	3.9191827	8335	3.9209056	8368	3.9226217
8303	3.9192350	8336	3.9209577	8369	3.9226736
8304	3.9192873	8337	3.9210098	8370	3.9227255
8305	3.9193396	8338	3.9210619	8371	3.9227773
8306	3.9193919	8339	3.9211140	8372	3.9228292
8307	3.9194442	8340	3.9211661	8373	3.9228811
8308	3.9194965	8341	3.9212181	8374	3.9229330
8309	3.9195488	8342	3.9212702	8375	3.9229848
8310	3.9196010	8343	3.9213222	8376	3.9230367
8311	3.9196533	8344	3.9213743	8377	3.9230885
8312	3.9197055	8345	3.9214263	8378	3.9231404
8313	3.9197578	8346	3.9214784	8379	3.9231922
8314	3.9198100	8347	3.9215304	8380	3.9232440
8315	3.9198623	8348	3.9215824	8381	3.9232958
8316	3.9199145	8349	3.9216345	8382	3.9233477
8317	3.9199667	8350	3.9216865	8383	3.9233995
8318	3.9200189	8351	3.9217385	8384	3.9234513
8319	3.9200711	8352	3.9217905	8385	3.9235031
8320	3.9201233	8353	3.9218425	8386	3.9235549
8321	3.9201755	8354	3.9218945	8387	3.9236066
8322	3.9202277	8355	3.9219465	8388	3.9236584
8323	3.9202799	8356	3.9219984	8389	3.9237102
8324	3.9203321	8357	3.9220504	8390	3.9237620
8325	3.9203842	8358	3.9221024	8391	3.9238137
8326	3.9204364	8359	3.9221543	8392	3.9238655
8327	3.9204886	8360	3.9222063	8393	3.9239172
8328	3.9205407	8361	3.9222582	8394	3.9239690
8329	3.9205929	8362	3.9223102	8395	3.9240207
8330	3.9206450	8363	3.9223621	8396	3.9240724
8331	3.9206971	8364	3.9224140	8397	3.9241242
8332	3.9207493	8365	3.9224659	8398	3.9241759
8333	3.9208014	8366	3.9225179	8399	3.9242276
8334	3.9208535	8367	3.9225698	8400	3.9242793

N.	Logarith.	N.	Logarith.	N.	Logarith.
8401	3.9243310	8434	3.9260336	8467	3.9277296
8402	3.9243827	8435	3.9260851	8468	3.9277808
8403	3.9244344	8436	3.9261366	8469	3.9278321
8404	3.9244860	8437	3.9261880	8470	3.9278834
8405	3.9245377	8438	3.9262395	8471	3.9279347
8406	3.9245894	8439	3.9262910	8472	3.9279859
8407	3.9246410	8440	3.9263424	8473	3.9280372
8408	3.9246927	8441	3.9263939	8474	3.9280885
8409	3.9247444	8442	3.9264453	8475	3.9281397
8410	3.9247960	8443	3.9264968	8476	3.9281909
8411	3.9248476	8444	3.9265482	8477	3.9282422
8412	3.9248993	8445	3.9265997	8478	3.9282934
8413	3.9249509	8446	3.9266511	8479	3.9283446
8414	3.9250025	8447	3.9267025	8480	3.9283959
8415	3.9250541	8448	3.9267539	8481	3.9284471
8416	3.9251057	8449	3.9268053	8482	3.9284983
8417	3.9251573	8450	3.9268567	8483	3.9285495
8418	3.9252089	8451	3.9269081	8484	3.9286007
8419	3.9252605	8452	3.9269595	8485	3.9286518
8420	3.9253121	8453	3.9270109	8486	3.9287030
8421	3.9253637	8454	3.9270622	8487	3.9287542
8422	3.9254152	8455	3.9271136	8488	3.9288054
8423	3.9254668	8456	3.9271650	8489	3.9288565
8424	3.9255184	8457	3.9272163	8490	3.9289077
8425	3.9255699	8458	3.9272677	8491	3.9289588
8426	3.9256215	8459	3.9273190	8492	3.9290100
8427	3.9256730	8460	3.9273704	8493	3.9290611
8428	3.9257245	8461	3.9274217	8494	3.9291123
8429	3.9257761	8462	3.9274730	8495	3.9291634
8430	3.9258276	8463	3.9275243	8496	3.9292145
8431	3.9258791	8464	3.9275757	8497	3.9292656
8432	3.9259306	8465	3.9276270	8498	3.9293167
8433	3.9259821	8466	3.9276783	8499	3.9293678
8434	3.9260336	8467	3.9277296	8500	3.9294189

N.	Logarith.	N.	Logarith.	N.	Logarith.
8501	3.9294700	8534	3.9311526	8567	3.9328288
8502	3.9295211	8535	3.9312035	8568	3.9328795
8503	3.9295722	8536	3.9312544	8569	3.9329301
8504	3.9296233	8537	3.9313053	8570	3.9329808
8505	3.9296743	8538	3.9313561	8571	3.9330315
8506	3.9297254	8539	3.9314070	8572	3.9330822
8507	3.9297764	8540	3.9314579	8573	3.9331328
8508	3.9298275	8541	3.9315087	8574	3.9331835
8509	3.9298785	8542	3.9315596	8575	3.9332341
8510	3.9299296	8543	3.9316104	8576	3.9332848
8511	3.9299806	8544	3.9316612	8577	3.9333354
8512	3.9300316	8545	3.9317121	8578	3.9333860
8513	3.9300826	8546	3.9317629	8579	3.9334367
8514	3.9301336	8547	3.9318137	8580	3.9334873
8515	3.9301847	8548	3.9318645	8581	3.9335379
8516	3.9302357	8549	3.9319153	8582	3.9335885
8517	3.9302866	8550	3.9319661	8583	3.9336391
8518	3.9303376	8551	3.9320169	8584	3.9336897
8519	3.9303886	8552	3.9320677	8585	3.9337403
8520	3.9304396	8553	3.9321185	8586	3.9337909
8521	3.9304906	8554	3.9321692	8587	3.9338415
8522	3.9305415	8555	3.9322200	8588	3.9338920
8523	3.9305925	8556	3.9322708	8589	3.9339426
8524	3.9306434	8557	3.9323215	8590	3.9339932
8525	3.9306944	8558	3.9323733	8591	3.9340437
8526	3.9307453	8559	3.9324230	8592	3.9340943
8527	3.9307963	8560	3.9324738	8593	3.9341448
8528	3.9308472	8561	3.9325245	8594	3.9341953
8529	3.9308981	8562	3.9325752	8595	3.9342459
8530	3.9309490	8563	3.9326259	8596	3.9342964
8531	3.9309999	8564	3.9326767	8597	3.9343469
8532	3.9310508	8565	3.9327274	8598	3.9343974
8533	3.9311017	8566	3.9327781	8599	3.9344479
8534	3.9311526	8567	3.9328288	8600	3.9344984

N.	Logarith.	N.	Logarith.	N.	Logarith.
8601	3.9345489	8634	3.9362120	8667	3.9378688
8602	3.9345994	8635	3.9362623	8668	3.9379189
8603	3.9346499	8636	3.9363126	8669	3.9379690
8604	3.9347004	8637	3.9363629	8670	3.9380191
8605	3.9347509	8638	3.9364132	8671	3.9380692
8606	3.9348013	8639	3.9364635	8672	3.9381193
8607	3.9348518	8640	3.9365137	8673	3.9381693
8608	3.9349023	8641	3.9365640	8674	3.9382194
8609	3.9349527	8642	3.9366143	8675	3.9382695
8610	3.9350032	8643	3.9366645	8676	3.9383195
8611	3.9350536	8644	3.9367148	8677	3.9383696
8612	3.9351040	8645	3.9367650	8678	3.9384196
8613	3.9351544	8646	3.9368152	8679	3.9384697
8614	3.9352040	8647	3.9368655	8680	3.9385197
8615	3.9352553	8648	3.9369157	8681	3.9385698
8616	3.9353057	8649	3.9369659	8682	3.9386198
8617	3.9353561	8650	3.9370161	8683	3.9386698
8618	3.9354065	8651	3.9370663	8684	3.9387198
8619	3.9354569	8652	3.9371165	8685	3.9387698
8620	3.9355073	8653	3.9371667	8686	3.9388198
8621	3.9355576	8654	3.9372169	8687	3.9388698
8622	3.9356080	8655	3.9372671	8688	3.9389198
8623	3.9356584	8656	3.9373172	8689	3.9389698
8624	3.9357087	8657	3.9373674	8690	3.9390198
8625	3.9357591	8658	3.9374176	8691	3.9390697
8626	3.9358095	8659	3.9374677	8692	3.9391197
8627	3.9358598	8660	3.9375179	8693	3.9391697
8628	3.9359101	8661	3.9375680	8694	3.9392196
8629	3.9359605	8662	3.9376182	8695	3.9392696
8630	3.9360108	8663	3.9376683	8696	3.9393195
8631	3.9360611	8664	3.9377184	8697	3.9393695
8632	3.9361114	8665	3.9377686	8698	3.9394194
8633	3.9361617	8666	3.9378187	8699	3.9394693
8634	3.9362120	8667	3.9378688	8700	3.9395193

N.	Logarith.	N.	Logarith.	N.	Logarith.
8701	3.9395692	8734	3.9412132	8767	3.9428510
8702	3.9396191	8735	3.9412629	8768	3.9429005
8703	3.9396690	8736	3.9413126	8769	3.9429501
8704	3.9397189	8737	3.9413623	8770	3.9429996
8705	3.9397688	8738	3.9414120	8771	3.9430491
8706	3.9398187	8739	3.9414617	8772	3.9430986
8707	3.9398685	8740	3.9415114	8773	3.9431481
8708	3.9399184	8741	3.9415611	8774	3.9431976
8709	3.9399683	8742	3.9416108	8775	3.9432471
8710	3.9400182	8743	3.9416605	8776	3.9433966
8711	3.9400680	8744	3.9417101	8777	3.9433461
8712	3.9401179	8745	3.9417598	8778	3.9433956
8713	3.9401677	8746	3.9418095	8779	3.9434450
8714	3.9402176	8747	3.9418591	8780	3.9434945
8715	3.9402674	8748	3.9419088	8781	3.9435440
8716	3.9403172	8749	3.9419584	8782	3.9435934
8717	3.9403670	8750	3.9420081	8783	3.9436429
8718	3.9404619	8751	3.9420577	8784	3.9436923
8719	3.9404667	8752	3.9421073	8785	3.9437418
8720	3.9405165	8753	3.9421569	8786	3.9437912
8721	3.9405663	8754	3.9422065	8787	3.9438406
8722	3.9406161	8755	3.9422561	8788	3.9438900
8723	3.9406659	8756	3.9423058	8789	3.9439395
8724	3.9407157	8757	3.9423553	8790	3.9439889
8725	3.9407654	8758	3.9424049	8791	3.9440383
8726	3.9408152	8759	3.9424545	8792	3.9440877
8727	3.9408650	8760	3.9425041	8793	3.9441371
8728	3.9409147	8761	3.9425537	8794	3.9441865
8729	3.9409645	8762	3.9426032	8795	3.9442358
8730	3.9410142	8763	3.9426528	8796	3.9442852
8731	3.9410640	8764	3.9427024	8797	3.9443346
8732	3.9411137	8765	3.9427519	8798	3.9443840
8733	3.9411635	8766	3.9428015	8799	3.9444333
8734	3.9412132	8767	3.9428510	8800	3.9444827

N.	Logarith.	N.	Logarith.	N.	Logarith.
8801	3.9445320	8834	3.9461574	8867	3.9477767
8802	3.9445814	8835	3.9462066	8868	3.9478257
8803	3.9446307	8836	3.9462557	8869	3.9478747
8804	3.9446800	8837	3.9463048	8870	3.9479236
8805	3.9447294	8838	3.9463540	8871	3.9479726
8806	3.9447787	8839	3.9464031	8872	3.9480215
8807	3.9448280	8840	3.9464523	8873	3.9480705
8808	3.9448773	8841	3.9465014	8874	3.9481194
8809	3.9449266	8842	3.9465505	8875	3.9481684
8810	3.9449759	8843	3.9465996	8876	3.9482173
8811	3.9450252	8844	3.9466487	8877	3.9482662
8812	3.9450745	8845	3.9466678	8878	3.9483151
8813	3.9451238	8846	3.9467469	8879	3.9483641
8814	3.9451730	8847	3.9467960	8880	3.9484130
8815	3.9452223	8848	3.9468451	8881	3.9484619
8816	3.9452716	8849	3.9468942	8882	3.9485108
8817	3.9453208	8850	3.9469433	8883	3.9485597
8818	3.9453701	8851	3.9469923	8884	3.9486085
8819	3.9454193	8852	3.9470414	8885	3.9486574
8820	3.9454686	8853	3.9470905	8886	3.9487063
8821	3.9455178	8854	3.9471395	8887	3.9487552
8822	3.9455671	8855	3.9471886	8888	3.9488040
8823	3.9456163	8856	3.9472376	8889	3.9488529
8824	3.9456655	8857	3.9472866	8890	3.9489018
8825	3.9457147	8858	3.9473357	8891	3.9489506
8826	3.9457639	8859	3.9473847	8892	3.9489995
8827	3.9458131	8860	3.9474337	8893	3.9490483
8828	3.9458623	8861	3.9474827	8894	3.9490971
8829	3.9459115	8862	3.9475317	8895	3.9491460
8830	3.9459607	8863	3.9475807	8896	3.9491948
8831	3.9460099	8864	3.9476297	8897	3.9492436
8832	3.9460591	8865	3.9476787	8898	3.9492924
8833	3.9461082	8866	3.9477277	8899	3.9493412
8834	3.9461574	8867	3.9477767	8900	3.9493900

N.	Logarith.	N.	Logarith.	N.	Logarith.
8901	3.9494388	8934	3.9510459	8967	3.9526472
8902	3.9494876	8935	3.9510946	8968	3.9526956
8903	3.9495364	8936	3.9511432	8969	3.9527440
8904	3.9495852	8937	3.9511918	8970	3.9527924
8905	3.9496339	8938	3.9512404	8971	3.9528409
8906	3.9496827	8939	3.9512889	8972	3.9528893
8907	3.9497315	8940	3.9513375	8973	3.9529373
8908	3.9497802	8941	3.9513861	8974	3.9529861
8909	3.9498290	8942	3.9514347	8975	3.9530345
8910	3.9498777	8943	3.9514832	8976	3.9530828
8911	3.9499264	8944	3.9515318	8977	3.9531312
8912	3.9499752	8945	3.9515803	8978	3.9531796
8913	3.9500239	8946	3.9516289	8979	3.9532280
8914	3.9500726	8947	3.9516774	8980	3.9532767
8915	3.9501213	8948	3.9517260	8981	3.9533247
8916	3.9501701	8949	3.9517745	8982	3.9533730
8917	3.9502188	8950	3.9518230	8983	3.9534214
8918	3.9502675	8951	3.9518716	8984	3.9534697
8919	3.9503162	8952	3.9519201	8985	3.9535181
8920	3.9503649	8953	3.9519686	8986	3.9535664
8921	3.9504135	8954	3.9520171	8987	3.9536147
8922	3.9504622	8955	3.9520656	8988	3.9536631
8923	3.9505109	8956	3.9521141	8989	3.9577114
8924	3.9505596	8957	3.9521626	8990	3.9537597
8925	3.9506082	8958	3.9522111	8991	3 9538080
8926	3.9506569	8959	3.9522595	8992	3.9538563
8927	3.9507055	8960	3.9523080	8993	3.9539046
8928	3.9507542	8961	3.9523565	8994	3.9539529
8929	3.9508028	8962	3.9524049	8995	3.9540012
8930	3.9508515	8963	3.9524534	8996	3.9540494
8931	3.9509001	8964	3.9525018	8997	3.9540977
8932	3.9509487	8965	3.9525503	8998	3.9541460
8933	3.9509973	8966	3.9525987	8999	3.9541943
8934	3.9510459	8967	3.9526472	9000	3.9542425

N.	Logarith.	N.	Logarith.	N.	Logarith.
9001	3.9542908	9034	3.9558801	9067	3.9574636
9002	3.9543390	9035	3.9559282	9068	3.9575115
9003	3.9543872	9036	3.9559762	9069	3.9575594
9004	3.9544355	9037	3.9560243	9070	3.9576073
9005	3.9544837	9038	3.9560723	9071	3.9576552
9006	3.9545319	9039	3.9561204	9072	3.9577030
9007	3.9545802	9040	3.9561684	9073	3.9577509
9008	3.9546284	9041	3.9562165	9074	3.9577988
9009	3.9546766	9042	3.9562645	9075	3.9578466
9010	3.9547248	9043	3.9563125	9076	3.9578945
9011	3.9547730	9044	3.9563606	9077	3.9579423
9012	3.9548212	9045	3.9564086	9078	3.9579902
9013	3.9548694	9046	3.9564566	9079	3.9580380
9014	3.9549176	9047	3.9565046	9080	3.9580858
9015	3.9549657	9048	3.9565526	9081	3.9581337
9016	3.9550139	9049	3.9566006	9082	3.9581815
9017	3.9550621	9050	3.9566486	9083	3.9582293
9018	3.9551102	9051	3.9566966	9084	3.9582771
9019	3.9551584	9052	3.9567445	9085	3.9583249
9020	3.9552065	9053	3.9567925	9086	3.9583727
9021	3.9552547	9054	3.9568405	9087	3.9584205
9022	3.9553028	9055	3.9568885	9088	3.9584683
9023	3.9553510	9056	3.9569364	9089	3.9585161
9024	3.9553991	9057	3.9569844	9090	3.9585639
9025	3.9554472	9058	3.9570323	9091	3.9586117
9026	3.9554953	9059	3.9570803	9092	3.9586594
9027	3.9555434	9060	3.9571282	9093	3.9587072
9028	3.9555915	9061	3.9571761	9094	3.9587549
9029	3.9556397	9062	3.9572241	9095	3.9588027
9030	3.9556877	9063	3.9572720	9096	3.9588505
9031	3.9557358	9064	3.9573199	9097	3.9588982
9032	3.9557839	9065	3.9573678	9098	3.9589459
9033	3.9558320	9066	3.9574157	9099	3.9589937
9034	3.9558801	9067	3.9574636	9100	3.9590414

N.	Logarith.	N.	Logarith.	N.	Logarith.
9101	3.9590891	9134	3.9606610	9167	3.9622272
9102	3.9591368	9135	3.9607086	9168	3.9622746
9103	3.9591845	9136	3.9607561	9169	3.9623220
9104	3.9592322	9137	3.9608036	9170	3.9623693
9105	3.9592799	9138	3.9608512	9171	3.9624167
9106	3.9593276	9139	3.9608987	9172	3.9624640
9107	3.9593753	9140	3.9609462	9173	3.9625114
9108	3.9594230	9141	3.9609937	9174	3.9625587
9109	3.9594707	9142	3.9610412	9175	3.9626061
9110	3.9595184	9143	3.9610887	9176	3.9626534
9111	3.9595660	9144	3.9611362	9177	3.9627007
9112	3.9596137	9145	3.9611837	9178	3.9627481
9113	3.9596614	9146	3.9612312	9179	3.9627954
9114	3.9597090	9147	3.9612787	9180	3.9628427
9115	3.9597567	9148	3.9613262	9181	3.9628900
9116	3.9598043	9149	3.9613736	9182	3.9629373
9117	3.9598520	9150	3.9614211	9183	3.9629846
9118	3.9598996	9151	3.9614686	9184	3.9630319
9119	3.9599472	9152	3.9615160	9185	3.9630792
9120	3.9599948	9153	3.9615635	9186	3.9631264
9121	3.9600425	9154	3.9616109	9187	3.9631737
9122	3.9600901	9155	3.9616583	9188	3.9632210
9123	3.9601377	9156	3.9617058	9189	3.9632683
9124	3.9601853	9157	3.9617532	9190	3.9633155
9125	3.9602329	9158	3.9618006	9191	3.9633628
9126	3.9602805	9159	3.9618481	9192	3.9634100
9127	3.9603280	9160	3.9618955	9193	3.9634573
9128	3.9603756	9161	3.9619429	9194	3.9635045
9129	3.9604232	9162	3.9619903	9195	3.9635517
9130	3.9604708	9163	3.9620377	9196	3.9635990
9131	3.9605183	9164	3.9620851	9197	3.9636462
9132	3.9605659	9165	3.9621325	9198	3.9636934
9133	3.9606135	9166	3.9621799	9199	3.9637406
9134	3.9606610	9167	3.9622272	9200	3.9637878

N.	Logarith.	N.	Logarith.	N.	Logarith.
9201	3.9638350	9234	3.9653899	9267	3.9669392
9202	3.9638822	9235	3.9654369	9268	3.9669860
9203	3.9639294	9236	3.9654839	9269	3.9670329
9204	3.9639766	9237	3.9655309	9270	3.9670797
9205	3.9640238	9238	3.9655780	9271	3.9671266
9206	3.9640710	9239	3.9656250	9272	3.9671734
9207	3.9641181	9240	3.9656720	9273	3.9672203
9208	3.9641653	9241	3.9657190	9274	3.9672671
9209	3.9642125	9242	3.9657660	9275	3.9673139
9210	3.9642596	9243	3.9658130	9276	3.9673607
9211	3.9643068	9244	3.9658599	9277	3.9674076
9212	3.9643539	9245	3.9659069	9278	3.9674544
9213	3.9644011	9246	3.9669539	9279	3.9675012
9214	3.9644482	9247	3.9660009	9280	3.9675480
9215	3.9644953	9248	3.9660478	9281	3.9675948
9216	3.9645425	9249	3.9660948	9282	3.9676416
9217	3.9645896	9250	3.9661417	9283	3.9676883
9218	3.9646367	9251	3.9661887	9284	3.9677351
9219	3.9646838	9252	3.9662356	9285	3.9677819
9220	3.9647309	9253	3.9662826	9286	3.9678287
9221	3.9647780	9254	3.9663295	9287	3.9678754
9222	3.9648251	9255	3.9663764	9288	3.9679222
9223	3.9648722	9256	3.9664233	9289	3.9679690
9224	3.9649193	9257	3.9664703	9290	3.9680157
9225	3.9649664	9258	3.9665172	9291	3.9680625
9226	3.9650134	9259	3.9665641	9292	3.9681092
9227	3.9650605	9260	3.9666110	9293	3.9681559
9228	3.9651076	9261	3.9666579	9294	3.9682027
9229	3.9651546	9262	3.9667048	9295	3.9682494
9230	3.9652017	9263	3.9667517	9296	3.9682961
9231	3.9652488	9264	3.9667985	9297	3.9683428
9232	3.9652958	9265	3.9668454	9298	3.9683895
9233	3.9653424	9266	3.9668923	9299	3.9684362
9234	3.9653899	9267	3.9669392	9300	3.9684829

N.	Logarith.	N.	Logarith.	N.	Logarith.
9301	3.9685296	9334	3.9700678	9367	3.9716005
9302	3.9685763	9335	3.9701143	9368	3.9716469
9303	3.9686230	9336	3.9701608	9369	3.9716932
9304	3.9686697	9337	3.9702074	9370	3.9717396
9305	3.9687164	9338	3.9702539	9371	3.9717859
9306	3.9687630	9339	3.9703004	9372	3.9718323
9307	3.9688097	9340	3.9703469	9373	3.9718786
9308	3.9688564	9341	3.9703934	9374	3.9719249
9309	3.9689030	9342	3.9704399	9375	3.9719713
9310	3.9689497	9343	3.9704863	9376	3.9720176
9311	3.9689963	9344	3.9705328	9377	3.9720639
9312	3.9690430	9345	3.9705793	9378	3.9721102
9313	3.9690896	9346	3.9706258	9379	3.9721565
9314	3.9691363	9347	3.9706788	9380	3.9722028
9315	3.9691829	9348	3.9707187	9381	3.9722491
9316	3.9692295	9349	3.9707652	9382	3.9722954
9317	3.9692761	9350	3.9708116	9383	3.9723417
9318	3.9693227	9351	3.9708581	9384	3.9723880
9319	3.9693693	9352	3.9709045	9385	3.9724343
9320	3.9694159	9353	3.9709509	9386	3.9724805
9321	3.9694625	9354	3.9709974	9387	3.9725268
9322	3.9695091	9355	3.9710438	9388	3.9725731
9323	3.9695557	9356	3.9710902	9389	3.9726193
9324	3.9696023	9357	3.9711366	9390	3.9726656
9325	3.9696488	9358	3.9711830	9391	3.9727118
9326	3.9696954	9359	3.9712294	9392	3.9727581
9327	3.9697420	9360	3.9712758	9393	3.9728043
9328	3.9697885	9361	3.9713222	9394	3.9728506
9329	3.9698351	9362	3.9713686	9395	3.9728968
9330	3.9698816	9363	3.9714150	9396	3.9729430
9331	3.9699282	9364	3.9714614	9397	3.9729892
9332	3.9699747	9365	3.9715078	9398	3.9730354
9333	3.9700213	9366	3.9715542	9399	3.9730816
9334	3.9700678	9367	3.9716005	9400	3.9731279

N.	Logarith.	N.	Logarith.	N.	Logarith.
9401	3.9731741	9434	3.9746959	9467	3.9762124
9402	3.9732202	9435	3.9747419	9468	3.9762582
9403	3.9732664	9436	3.9747879	9469	3.9763041
9404	3.9733126	9437	3.9748340	9470	3.9763500
9405	3.9733588	9438	3.9748800	9471	3.9763958
9406	3.9734050	9439	3.9749260	9472	3.9764417
9407	3.9734511	9440	3.9749720	9473	3.9764875
9408	3.9734973	9441	3.9750180	9474	3.9765334
9409	3.9735435	9442	3.9750640	9475	3.9765792
9410	3.9735896	9443	3.9751100	9476	3.9766251
9411	3.9736358	9444	3.9751560	9477	3.9766709
9412	3.9736819	9445	3.9752020	9478	3.9767167
9413	3.9737281	9446	3.9752479	9479	3.9767625
9414	3.9737742	9447	3.9752939	9480	3.9768083
9415	3.9732203	9448	3.9753399	9481	3.9768541
9416	3.9738664	9449	3.9753858	9482	3.9768999
9417	3.9739126	9450	3.9754318	9483	3.9769457
9418	3.9739587	9451	3.9754778	9484	3.9769915
9419	3.9740048	9452	3.9755237	9485	3.9770373
9420	3.9740509	9453	3.9755697	9486	3.9770831
9421	3.9740970	9454	3.9756156	9487	3 9772289
9422	3.9741431	9455	3.9756615	9488	3.9771747
9423	3.9741892	9456	3.9757075	9489	3.9772204
9424	3.9742353	9457	3.9757534	9490	3.9772662
9425	3.9742814	9458	3.9757993	9491	3.9773120
9426	3.9743274	9459	3.9758452	9492	3.9773577
9427	3.9743735	9460	3.9758911	9493	3.9774035
9428	3.9744196	9461	3.9759370	9494	3.9774492
9429	3.9744656	9462	3.9759829	9495	3.9774950
9430	3.9745117	9463	3.9750288	9496	3.9775407
9431	3.9745577	9464	3.9760747	9497	3.9775864
9432	3.9746038	9465	3.9761206	9498	3.9776322
.433	3.9746498	9466	3.9761665	9499	3.9776770
9434	3.9746959	9467	3.9762124	9500	3.9777236

N	Logarith.	N.	Logarith.	N.	Logarith.
9501	3.9777693	9534	3.9792751	9567	3.9807758
9502	3.9778150	9535	3.9793207	9568	3.9808212
9503	3.9778607	9536	3.9793662	9569	3.9808666
9504	3.9779064	9537	3.9794118	9570	3.9809119
9505	3.9779521	9538	3.9794573	9571	3.9809573
9506	3.9779978	9539	3.9795028	9572	3.9810027
9507	3.9780435	9540	3.9795484	9573	3.9810481
9508	3.9780892	9541	3.9795939	9574	3.9810934
9509	3.9781348	9542	3.9796394	9575	3.9811388
9510	3.9781805	9543	3.9796849	9576	3.9811841
9511	3.9782262	9544	3.9797304	9577	3.9812295
9512	3.9782718	9545	3.9797759	9578	3.9812748
9513	3.9783175	9546	3.9798214	9579	3.9813202
9514	3.9783631	9547	3.9798669	9580	3.981365
9515	3.9784088	9548	3.9799124	9581	3.9814108
9516	3.9784544	9549	3.9799579	9582	3.9814562
9517	3.9785001	9550	3.9800034	9583	3.9815015
9518	3.9785457	9551	3.9800488	9584	3.9815468
9519	3.9785913	9552	3.9800943	9585	3.9815921
9520	3.9786369	9553	3.9801398	9586	3.9816374
9521	3.9786826	9554	3.9801852	9587	3.9816827
9522	3.9787282	9555	3.9802307	9588	3.9817280
9523	3.9787738	9556	3.9802761	9589	3.9817733
9524	3.9788194	9557	3.9803216	9590	3.9818186
9525	3.9788650	9558	3.9803670	9591	3.9818639
9526	3.9789106	9559	3.9804126	9592	3.9819092
9527	3.9789562	9560	3.9804579	9593	3.9819544
9528	3.9790017	9561	3.9805033	9594	3.9819997
9529	3.9790473	9562	3.9805487	9595	3.9820450
9530	3.9790929	9563	3.9805942	9596	3.9820902
9531	3.9791385	9564	3.9806396	9597	3.9821355
9532	3.9791840	9565	3.9806850	9598	3.9821807
9533	3.9792296	9566	3.9807304	9599	3.9822260
9534	3.9792751	9567	3.9807758	9600	3.9822712

N.	Logarith.	N.	Logarith.	N.	Logarith.
9601	3.9823165	9634	3.9838066	9667	3.9852917
9602	3.9823617	9635	3.9838517	9668	3.9853366
9603	3.9824069	9636	3.9838968	9669	3.9853816
9604	3.9824522	9637	3.9839419	9670	3.9854265
9605	3.9824974	9638	3.9839869	9671	3.9854714
9606	3.9825426	9639	3.9840320	9672	3.9855163
9607	3.9825878	9640	3.9840770	9673	3.9855612
9608	3.9826330	9641	3.9841221	9674	3.9856061
9609	3.9826782	9642	3.9841671	9675	3.9856510
9610	3.9827234	9643	3.9842122	9676	3.9856959
9611	3.9827686	9644	3.9842572	9677	3.9857407
9612	3.9828138	9645	3.9843022	9678	3.9857856
9613	3.9828589	9646	3.9843473	9679	3.9858305
9614	3.9829041	9647	3.9843923	9680	3.9858754
9615	3.9829493	9648	3.9844373	9581	3.9859202
9616	3.9829945	9649	3.9844823	9682	3.9859651
9617	3.9830396	9650	3.9845273	9683	3.9860099
9618	3.9830848	9651	3.9845723	9684	3.9860548
9619	3.9831299	9652	3.9846173	9685	3.9860996
9620	3.9831751	9653	3 9846623	9686	3.9861445
9621	3.9832202	9654	3.9847073	9687	3.9861893
9522	3.9832654	9655	3.9847523	9688	3.9862341
9623	3.9833105	9556	3.9847973	9689	3.9862790
9624	3.9833556	9657	3.9848422	9690	3.9863238
9625	3.9834007	9658	3.9848872	9691	3.9863686
9626	3.9834459	9659	3.9849322	9692	3.9864134
9627	3.9834910	9660	3.9849771	9693	3.9864582
9628	3.9835361	9661	3.9850221	9694	3.9865030
9629	3.9835812	9662	3.9850670	9695	3.9865478
9630	3.9836263	9663	3.9851120	9696	3.9865928
9631	3.9836714	9664	3.9851569	9697	3.9866374
9532	3.9837165	9665	3.9852019	9698	3.9866822
9633	3.9837616	9666	3.9852468	9699	3.9867270
9634	3.9838066	9667	3.9852917	9700	3.9867717

N.	*Logarith.*	N.	*Logarith.*	N.	*Logarith.*
9701	3.9868165	9734	3.9882913	9767	3.9897612
9702	3.9868613	9735	3.9883360	9768	3.9898056
9703	3.9869060	9736	3.9883806	9769	3.9898501
9704	3.9869508	9737	3.9884252	9770	3.9898946
9705	3.9869955	9738	3.9884698	9771	3.9899390
9706	3.9870403	9739	3.9885144	9772	3.9899835
9707	3.9870850	9740	3.9885590	9773	3.9900279
9708	3.9871298	9741	3.9886035	9774	3.9900723
9709	3.9871745	9742	3.9886481	9775	3.9901168
9710	3.9872192	9743	3.9886927	9776	3.9901612
9711	3.9872640	9744	3.9887373	9777	3.9902056
9712	3.9873087	9745	3.9887818	9778	3.9902500
9713	3.9873534	9746	3.9888264	9779	3.9902944
9714	3.9873981	9747	3.9888710	9780	3.9903389
9715	3.9874428	9748	3.9889155	9781	3.9903833
9716	3.9874875	9749	3.9889601	9782	3.9904277
9717	3.9875322	9750	3.9890046	9783	3.9904721
9718	3.9875769	9751	3.9890492	9784	3.9905164
9719	3.9876216	9752	3.9890937	9785	3.9905608
9720	3.9876663	9753	3.9891382	9786	3.9906052
9721	3.9877109	9754	3.9891828	9787	3.9906496
9722	3.9877556	9755	3.9892273	9788	3.9906940
9723	3.9878003	9756	3.9892718	9789	3.9907383
9724	3.9878449	9757	3.9893163	9790	3.9907827
9725	3.9878896	9758	3.9893608	9791	3.9908270
9726	3.9879343	9759	3.9894053	9792	3.9908714
9727	3.9879789	9760	3.9894498	9793	3.9909158
9728	3.9880236	9761	3.9894943	9794	3.9909601
9729	3.9880682	9762	3.9895388	9795	3.9910044
9730	3.9881128	9763	3.9895833	9796	3.9910488
9731	3.9881575	9764	3.9896278	9797	3.9910931
9732	3.9882021	9765	3.9896722	9798	3.9911374
9733	3.9882467	9766	3.9897167	9799	3.9911818
9734	3.9882913	9767	3.9897612	9800	3.9912261

N.	Logarith.	N.	Logarith.	N.	Logarith.
9801	3.9912704	9834	3.9927302	9867	3.9941851
9802	3.9913147	9835	3.9927744	9868	3.9942291
9803	3.9913590	9836	3.9928185	9869	3.9942731
9804	3.9914033	9837	3.9928627	9870	3.9943172
9805	3.9914476	9838	3.9929068	9871	3.9943612
9806	3.9914919	9839	3.9929510	9872	3.9944051
9807	3.9915362	9840	3.9929951	9873	3.9944491
9808	3.9915805	9841	3.9930392	9874	3.9944931
9809	3.9916247	9842	3.9930834	9875	3.9945371
9810	3.9916690	9843	3.9931275	9876	3.9945811
9811	3.9917133	9844	3.9931716	9877	3.9946251
9812	3.9917575	9845	3.9932157	9878	3.9946690
9813	3.9918018	9846	3.9932598	9879	3.9947130
9814	3.9918461	9847	3.9933039	9880	3.9947569
9815	3.9918903	9848	3.9933480	9881	3.9948009
9816	3.9919345	9849	3.9933921	9882	3.9948448
9817	3.9919788	9850	3.9934362	9883	3.9948888
9818	3.9920230	9851	3.9934803	9884	3.9949327
9819	3.9920673	9852	3.9935244	9885	3.9949767
9820	3.9921115	9853	3.9935685	9886	3.9950206
9821	3.9921557	9854	3.9936126	9887	3.9950645
9822	3.9921999	9855	3.9936566	9888	3.9951085
9823	3.9922441	9856	3.9937007	9889	3.9951524
9824	3.9922884	9857	3 9937448	9890	3.9951963
9825	3.9923326	9858	3.9937888	9891	3 9952402
9826	3.9923768	9859	3.9938329	9892	3.9952841
9827	3.9924210	9860	3.9938769	9893	3.9953280
9828	3.9924651	9861	3.9939210	9894	3.9953719
9829	3.9925093	9862	3.9939650	9895	3.9954158
9830	3.9925535	9863	3.9940090	9896	3.9954597
9831	3.9925977	9864	3.9940531	9897	3.9955036
9832	3.9926419	9865	3.9940971	9898	3.9955474
9833	3.9926860	9866	3.9941411	9899	3.9955913
9834	3.9927302	9867	3.9941851	9900	3.9956352

A Table of Logarithms.

N.	Logarith.	N.	Logarith.	N.	Logarith.
9901	3.9956791	9934	3.9971242	9967	3.9985645
9902	3.9957229	9935	3.9971679	9968	3.9986080
9903	3.9957668	9936	3.9972116	9969	3.9986516
9904	3.9958106	9937	3.9972553	9970	3.9986952
9905	3.9958545	9938	3.9972990	9971	3.9987387
9906	3.9958983	9939	3.9973427	9972	3.9987823
9907	3.9959422	9940	3.9973864	9973	3.9988258
9908	3.9959860	9941	3.9974301	9974	3.9988694
9909	3.9960298	9942	3.9974738	9975	3.9989129
9910	3.9960737	9943	3.9975174	9976	3.9989564
9911	3.9961175	9944	3.9975611	9977	3.9990000
9912	3.9961613	9945	3.9976048	9978	3.9990435
9913	3.9962051	9946	3.9976485	9979	3.9990870
9914	3.9962489	9947	3.9976921	9980	3.9991305
9915	3.9962927	9948	3.9977358	9981	3.9991741
9916	3.9963365	9949	3.9977794	9982	3.9992176
9917	3.9963803	9950	3.9978231	9983	3.9992611
9918	3.9964241	9951	3.9978667	9984	3.9993046
9919	3.9964679	9952	3.9979104	9985	3.9993481
9920	3.9965117	9953	3.9979540	9986	3.9993916
9921	3.9965554	9954	3.9979976	9987	3.9994350
9922	3.9965992	9955	3.9980413	9988	3.9994785
9923	3.9966430	9956	3.9980849	9989	3.9995220
9924	3.9966868	9957	3.9981285	9990	3.9995655
9925	3.9967305	9958	3.9981721	9991	3.9996090
9926	3.9967743	9959	3.9982157	9992	3.9996524
9927	3.9968180	9960	3.9982593	9993	3.9996959
9928	3.9968618	9961	3.9983029	9994	3.9997393
9929	3.9969055	9962	3.9983465	9995	3.9997828
9930	3.9969492	9963	3.9983901	9996	3.9998262
9931	3.9969930	9964	3.9984337	9997	3.9998697
9932	3.9970367	9965	3.9984773	9998	3.9999131
9933	3.9970804	9966	3.9985209	9999	3.9999566
9934	3.9971242	9967	3.9985645	10000	4.0000000

FINIS.